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ELEMENTARY ARITHMETIC

AND HOW TO TEACH IT

ELEMENTARY
ARITHMETIC

AND HOW TO TEACH IT

BY GEORGE RICKS, B.SC. (LOND.)
INSPECTOR OF SCHOOLS, SCHOOL BOARD FOR LONDON

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ANSWERS TO THE EXERCISES.

ELEMENTARY ARITHMETIC,

AND HOW TO TEACH IT.

PART I.

FOR INFANTS' SCHOOLS.

CHAPTER I.

THE Abacus, or Ball-frame, is almost invariably used in giving the first lessons in number to the lowest or "baby" class of children in the English Infants' Schools.

It is seldom, however, we find that *all* the children in the class are listening to the teacher and following the lesson. More often than not the majority are inattentive, especially if the teacher is inexperienced.

We have not far to search for the cause of this inattention. Young children have the faculty of attention but slightly developed; and the lesson from the abacus presupposes a capacity for attention on the part of very young children which they do not possess, and which therefore ought not to be expected from them.

On the other hand, the faculty of imitation in young children is very strong, and of this natural gift every advantage should be taken. The children must have something to *do*, something to *imitate;* and the *work* must be *done* by the scholars themselves, and not by the teacher for them.

The principles of the Kinder Garten system may be applied with special advantage in the teaching of number.

For the earliest lessons, provide for *each* child a number of *sticks*, or small *blocks of wood*, or *marbles*, or other suitable objects. The cubes of the 3rd "Gift"* in the Kinder Garten exercises will answer admirably. In the following lessons we shall suppose that these cubes are used. Of course the *principle* will be the same, whatever object may be selected.

NOTE.—For this exercise it is most convenient to have the children seated in their desks, or in a square class with forms or low tables in front of them. If the gallery is used, and there are no flaps to the backs of the seats, the teacher must of necessity use *sticks* as the most convenient objects for hand use only.

THE NUMBERS ONE, TWO, THREE.

Three cubes being placed on the desks or forms in front of each child, and a set on a table or desk in front of the whole class, the teacher should proceed thus:—

Take up one cube and, placing it on one side, say *one cube*, *all* the children imitating and repeating simultaneously. Then take up a second cube and say *two cubes*, the children imitating and repeating as before. Lastly, take up the third cube and say *three cubes*, the children carefully following.

The cubes may now be returned *one by one* to their original position, counting as at the first.

This exercise must be repeated again and again, till all the children in the class can count accurately from *one* cube to *three* cubes.

The children may now be directed to the counting of objects in the room. Three boys or three girls should be brought to the front of the class and counted one at a time, in a similar manner to the cubes. Thus, *one boy, two boys, three boys;* or *one girl, two girls, three girls.* Three should also be counted accompanied by clapping of hands.

---

* This "Gift" consists of one large cube divided into eight smaller ones.

Lastly, the counting should be learnt without reference to any object, thus:—*one, two, three; three, two, one.*

In the SECOND place we have to show how the numbers *two* and *three* are built up.

The cubes must be placed in front of the children as before.

The teacher should hold up one cube and say *one cube,* the children imitating and repeating in chorus. Then a second cube is to be placed in a similar way by the side of the other.

We see that *two* is *one* and *one,* and means two *ones;* and that *one* taken from *two* leaves *one.*

We may call two a *pair* or a *couple.* A boy has a *pair* of eyes, *two* ears, a *couple* of legs, *two* arms, a *pair* of hands, and a *pair* of feet. A cow has *two* horns. A bird has *two* legs and *two* wings. *Two* halfpennies make a penny.

The teacher should vary the exercise on the number *two* by directing the children to find out all the *pairs* of objects in the room; by calling individual scholars to the front of the class to select a *couple* of pencils, a *pair* of balls, or *two* books, &c.; by asking other scholars to take *one* from the *pair,* and so on.

We have now to show that *three* is made up of *one* and *one* and *one,* or *two* and *one,* or *one* and *two,* and this must be done by the children.

Let each child take three single cubes and place them in a column: thus we have a column or group of *three* cubes made up of *three* singles. Take *two* of these cubes in the right hand and *one* in the left. We see that the group of *three* is the same as *two* and *one,* or *one* and *two:* so that *two* cubes and *one* cube make *three* cubes; *one* apple and *two* apples make *three* apples; *two* marbles and *one* marble make *three* marbles; a *pair* of nuts and *one* nut make *three* nuts; *one* slate and a *couple* of slates make *three* slates.

Again, a *pair* of books taken from *three* books leaves *one* book; *one* pencil taken from *three* pencils leaves *two* pencils; *one* ball from *three* balls leaves a *couple* of balls.

It should also be *shown* that there are *three* sides to a triangle, and that *three* feet make one yard.

The teacher should now exercise her ingenuity in giving examples from objects near at hand, always calling on the children to do the work.

Thus, call one child to take *three* balls from a box, and another to take away a *pair* or a *couple* of these.

Send one child to fetch a *pair* of slates, and another for a *couple* of pointers.

Give another child a foot-rule to measure a yard, and another three rulers or pencils of equal lengths to make a triangle.

Place a few pennies and a few halfpennies on the table in separate heaps: ask one child to take one penny from the first heap, and another to take its value in halfpennies from the second heap, and vice versâ.

In the THIRD place we have to teach the *figures* or *symbols* which stand for the numbers. For this exercise the children should be provided with slates, and the teacher with a black-board.*

On the black-board make groups of large dots, or draw lines or squares to represent the numbers one, two, three: write beneath each the figure (Arabic and Roman) which stands for the number of squares or dots, thus:—

□	□□	□□□
1	2	3
I.	II.	III.

or thus:—

●	●●	●●●
1	2	3
I.	II.	III.

---

* If the children are very young the figuring may be postponed until the counting and building up of the numbers up to 6 have been fairly mastered.

The drawing of these groups of squares, and the writing of the proper figures under each group, will form good slate exercises for the children, firstly as copies from the blackboard, and secondly as exercises from memory.

Exercises in every conceivable shape and form should be given on these numbers until they are thoroughly mastered by the "baby" members of the class. It is better not to allow the use of the fingers in counting, for, when once the habit is formed, it is difficult for children to break it off, and it is moreover a temptation to them to shirk, later on, the mental exercise of addition and subtraction, and to substitute the use of the fingers.

We subjoin a few more examples of exercises to be given at this stage; the intelligent teacher will multiply them tenfold. Let no one think that the time given to such "baby" exercises is wasted. In the first stages of instruction the child should be scarcely conscious of effort. Constant but endlessly varied practice in the very simplest operations is the true foundation for rapid progress and accuracy in the later stages.

EXERCISES.

I. *With the cubes or other objects (to be worked simultaneously by the whole class).*

1. Place two cubes the one on the other: add one more; how many have you now? Take away a pair; how many are left?

2. Take a couple of cubes in the right hand and one in the left. Place one in the right hand and two in the left.

3. Hold up one finger on each hand; two fingers on each hand; one finger on the right hand and two on the left; two fingers on the right hand and one on the left.

4. Hold up one finger on each hand; point to each ear, to each eye, to each shoulder.

5. With your hands clap twice, once, three times.

II. *With the black-board.*

1. Look at these strokes: | |. How many are there? How many have I added to make | | |?

2.* Write three strokes: make the figure 3 beneath. Make the three strokes into two strokes: write the figure 2 below.

3. Make two crosses; add one cross more. Write the figures 1, 2, 3.

4. Make three dots. With the duster rub off a couple of dots; how many are left?

III. *With the slates (simultaneous work).*

1. Draw three squares. Write the figure 3 below. Write the figure 2 three times, and the figure 3 twice over.

2. Make a pair of crosses and a couple of strokes. Make one cross, one dot, and one stroke. Write the figure below which stands for the number.

3. Write the figures 2, 1, 3, and place under each figure the number of dots for which it stands.

4. Make three groups of three dots each. Take away a pair of dots from each group.

IV. *Oral exercises.*†

1. I have three oranges: I give one to William, one to John, and one to Mary; how many have I left?

2. Divide three apples among three boys; how many will you give to each?

3. How many halfpennies in one penny? How many farthings in one halfpenny?

---

* Any individual child selected by the teacher to come to the front of the class and work on the black-board.

† These exercises should be given to the class as a whole. The children must be taught to hold up the right hand when they know the answer, and the teacher must select one child to give the answer. It is of great importance that this rule be strictly carried out. If the few brighter children of the class are allowed to shout the answer immediately the question is given, the remainder follow without taking the trouble to think, and the teacher is, in fact, teaching only three or four children instead of thirty or forty. By adopting the above rule the teacher sees at a glance the inattentive children, and also gives the duller scholar a little more time to think out his answer.

4. Balls are a halfpenny each; what shall I pay for two? and for three?

5. How many sides has a triangle? Baby is one year old, John is two years older than baby; how old is John?

6. What is the half of a penny? A pair and one make —? one and a couple make —? twice one are —?

7. Name two things you see in the room. Now three things. How many more are three than one? What must I add to one to make three?

Note.—It should be clearly understood that the chapters into which the first part of this little work is divided in no case represent the amount to be accomplished in one lesson. With very young children, this first chapter may occupy a considerable number of lessons; and in the next, each number, four, five, and six, may well give material for many separate lessons.

CHAPTER II.

THE NUMBERS FOUR, FIVE, SIX.

(*Each child to be provided with six cubes.*)

The teacher should commence counting—the children imitating—exactly in the same manner as described in the first chapter, but now proceeding to *six* instead of stopping at *three.*

The meaning of the numbers *four, five,* and *six,* and the different ways of building them up, must then be dwelt upon, and fully illustrated *by the work of the children themselves with the cubes.*

Four.—Four is made up of *four ones,* or *three* and *one,* or *two twos,* or *two pairs,* or *two couples.* Thus, *one* cube and *three* cubes make *four* cubes: *two pairs* of hands make *four* hands: *four ones* are *four: twice two* are *four*: two boxes taken from *four* boxes leave *two* boxes: *one* apple taken from *four* apples leaves *three* apples, and so on.

There are *four* sides and *four* corners to a square: *four* legs to a chair: *four* seasons in a year: and *four* farthings make a penny.

Numerous exercises must now be given, using objects in the room, not exceeding four in number.

Thus, call A to count the legs of a chair, and show how many *pairs* of legs there are to one chair. Send B to fetch a *couple* of books, and C to get sufficient to make *four*. Let D count the sides and corners of the black-board, and E the sides and corners of a slate. Let one child arrange balls of cotton in groups of *three* each, another in groups of *four* each. Books, pencils, slates, pennies, &c. may be similarly arranged.

FIVE.—Five must be shown to be made up of *five* ones, or *four* and *one*, or *three* and *two*, or *two* pairs and *one*.

Thus, *one* cube and *four* cubes make *five* cubes: a *pair* of cubes and *three* cubes make *five* cubes: *three* cubes taken from *five* cubes leave a *pair* or a *couple* of cubes: *two pairs* of horses and *one* horse make *five* horses: *one* penny taken from *five* pennies leaves *four* pennies: *two* pennies from *five* pennies leave *three* pennies.

There are *five* fingers on each hand, and *five* toes on each foot.

We call *five* shillings a crown.

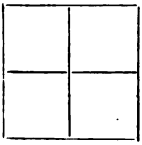
Numerous similar examples on the number *five* must now be given, as under *four*, using objects in the room as much as possible.

SIX.—Six is made up of *six* ones, or *five* and *one*, or *four* and *two*, or *three twos*, or *three pairs;* and the children must arrange their six cubes to show these combinations.

It must be *shown* also that the half of *six* is *three;* that one-third of *six* is *two;* that *twice three* are *six;* that *three* times *two* are *six;* that *two* is contained in *six three* times; and that *three* is contained in *six two* times, or *twice*.

A cube has *six* sides. A fly has *six* legs. There are *six* working days in the week.

Lastly, the symbols 4, 5, 6 must be taught by the use of the black-board as before shown, and numerous slate exercises must be given.



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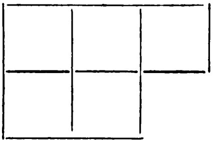

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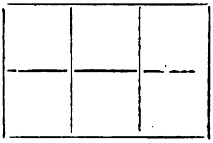

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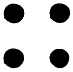

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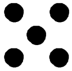

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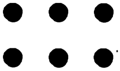

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VI.

EXERCISES.

I. *With the cubes or other objects* (*simultaneous work*).

1. Make a group of four cubes. Halve this group; how many cubes in each half? From three cubes take a pair of cubes; how many are left?

2. Show that two twos, or twice two, are four. How many couples can you make out of five cubes, and how many cubes are there over?

3. Divide six cubes into three pairs; and also into two groups of three cubes each.

4. Divide a group of five cubes to show that five is made up of three and two.

5. Hold two cubes in one hand and three in the other: put them together; how many cubes altogether?

6. Hold up three fingers on each hand; now two; then five.

7. With your hands clap three times, five times, four times, six times.

II. *With the black-board.*

1. Here are strokes: ||||. How many are there? How many pairs are there? How many pairs must I add to make up six strokes?

2. Write five strokes. Make the number of strokes six. Now make it three. How many have I rubbed off to leave three? Therefore three from six leaves — ? Write the figure 3.

3. Make three pairs of dots. Take away one pair; how many dots remain? Write the figure 4.

4. Make groups of three, four, five, and six dots, and write the proper figure under each.

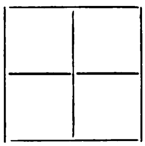
III. *With the slates (simultaneous work).*

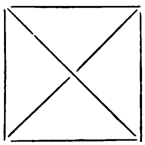
1. Make five strokes and write the figure 5 below. Also six strokes and write the figure 6.

2. Write the figure 4 three times, the figure 6 five times, and the figure 5 four times.

3. Write the figure 2 six times. Make a couple of each of the figures up to 6.

4. Draw a large square and divide it into four squares. Draw a second large square and divide it into four triangles.





IV. *Oral exercises.**

1. Five boys were standing: two sat down; how many were left standing?

2. Three girls are sitting and three standing; how many girls in all?

---

* See Note, p. 6.

3. Five children were sitting in a desk; three of them were boys; how many were girls?

4. John has three apples, Mary has two apples; how many have they together? If John gives one of his apples to Mary, how many will each then have?

5.* Six birds were sitting on a tree; two flew away; how many couples were left? How many wings have three birds?

6. A boy had to go six miles; he rode three miles by train and one by cart; how far had he to walk? If the boy could walk three miles an hour, how long would it take him to walk the whole distance?

7. How many fingers are there on one hand? Without the thumb, how many pairs of fingers on one hand?

8. I see five children—two of them are girls; how many are boys? How much money will it take to give each boy and girl one penny?

9. How many sides are there to a brick? How many working days in a week?

10. Divide six apples among three children; how many ought each to receive? If each child ate one apple, how many would be left altogether?

11. When oranges are one penny each, how many can I buy for sixpence? If twopence each, how many can I buy; and if threepence each, how many can I buy for sixpence?

12. How many penny stamps can I buy for fivepence? How many halfpenny stamps for twopence?

13. John has twopence; how much must I give him to make fivepence? If he buys a penny ball and a pennyworth of marbles, how much money will he have left?

14. An egg cost three halfpence; what is the cost of two eggs? How many eggs can I buy for threepence at the same price?

15. Annie carried sixpence to the shop to buy a pound of rice at twopence, and a pound of sugar at threepence; what change ought she to receive?

---

* Such questions as this show the importance of teaching thoroughly the composition of each separate number. The child who has seen repeatedly how six contains three pairs will answer this question at once; not otherwise.

16. What is the half of six? What is the half of four? What is the half of six and the half of four added together?

17. Three and two are —? Four and two are —? Three from four leaves —? Three from six leaves —?

18. Add one and two and three. Add three and two, and take one away. Add five and one, and take three away.

19. How many pairs are there in six? How many threes in six?

20. Three times two are —? Twice two are —? Six times one are —? Five times one are —?

Note.—Examples of this kind must be given until all the children in the class answer readily and correctly.

CHAPTER III.

THE NUMBERS SEVEN, EIGHT, NINE.

(*Each child to have nine cubes.*)

It will not be necessary here to do more than point out that the same plan should be followed in teaching the numbers *seven*, *eight*, and *nine*, as that adopted for teaching the numbers *one* to *six*. But the meaning and composition of each new number should be thoroughly learned before passing on to the next.

Note.—A cube has eight angles or corners, and a spider has eight legs. Nine, like four, is a square number. Show this with the cubes.

EXERCISES.

I. *With the cubes or other objects (simultaneous work).*

1. Take seven cubes and arrange them to show that seven is made up of five and two, or four and three, or three twos and one.

2. Arrange eight in two fours, and in four pairs.

3. Show that when three cubes are taken from a group of seven, two pairs are left.

4. Show that four twos are eight, and that twice four are eight.

5. Show that nine is made up of three threes.

6. Hold up four fingers on one hand, and three on the other; how many altogether?

7. Clap five times, three times, seven times, nine times, eight times.

II. *With the black-board.*

1. Name the figures I write on the black-board—thus, 4, 3, 5, 9, 7, 8, 1, 2, 7, 6, 8, 9.

2. How many dots have I made—: : : :? How many twos are there? how many fours?

3. How many must I add to this group : : to make seven? How many couples are there in seven? and how many ones over?

4. How many crosses in this group $\times_{\times}\times$? how many in this, $\begin{smallmatrix}\times&\times&\times\\\times&\times&\times\end{smallmatrix}$? How many in the two together? Take two away from each group; how many are left?

5. Make four dots: add two more; how many are there now? What is the half of six?

6. Write the figures 2, 4, 6, 8. Make dots under each figure to show the number for which it stands. Divide each group of dots into two equal parts, by drawing a line between them.

7. Draw three squares and nine squares. Show, by dividing the nine squares, that nine is three threes.

III. *With the slates (simultaneous work).*

1. Write the figures one to eight.

2. Make groups of five, seven, six, eight, and nine dots, writing the proper figures beneath.

3. Write the figure 7 four times, the figure 8 five times, and the figure 9 six times.

4. Draw a large square. Divide it into four squares. Divide each square into two triangles; how many triangles are there?

5. Draw a square, and divide it into nine equal squares.

IV. *Oral.*

1. How many days are there in a week? A boy saves a penny a day; how much is that in a week?

2. How many legs have two chairs? How many legs have four birds? and how many have three boys?

3. William had eight marbles, he gave his brother three, and lost two at play; how many had he left?

4. John is now nine years old, and four years older than his sister; how old is his sister?

5. How many sides has a square garden? If a tree is planted in each side and one in each corner, how many are planted altogether?

6. Divide eight apples equally among four boys; how many will each receive? If there are two boys only, how many ought each to receive? Divide nine apples equally among three boys.

7. A cat has a halfpenny worth of meat per day; what is the cost per week?

8. When cheese is 8*d.* per lb. how much must I pay for ½ lb.? What change ought I to have out of 6*d.*?

9. Apples are six a penny; how many is that for a halfpenny? If they are four a penny, how many can I buy for 2*d.*?

10. A girl pays her school fee of 3*d.*, and buys a slate for 2*d.*; how much change ought she to have out of sixpence?

11. How many penny pencils can I buy for a penny; and how many twopenny ones for 8*d.*?

12. A copy-book and slate together cost 7*d.* The copy-book cost 3*d.*; what was the price of the slate?

13. I bought a book for 5*d.* and sold it for 7*d.*; what did I gain?

14. Name any pairs of numbers which make seven, and any pairs which make eight.

15. How many fourpenny pieces are there in 8*d.*? What two small silver coins added together make 7*d.*?

16. What is the half of eight? and the half of six? How much is the half of eight more than the half of six? Add the half of four to the half of six.

17. What must be added to 5*d.* to make 8*d.*? and to 2*d.* to make 5*d.*?

18. A little girl bought a pound of raisins for 4*d.*, a pound of sugar for 3*d.*, and an egg for 1*d.*; what did she spend altogether?

19. How many threepenny pieces in ninepence? how many in sixpence? Take a threepenny piece from ninepence; how much is left? What two silver coins together amount to ninepence?

20. A window has four rows of two panes of glass each; how many panes of glass are there in the window?

21. Mary had fourpence, she lost threepence, and found twopence; what had she then?

22. I bought six balls for fourpence, and sold them at a penny each; how much did I gain?

23. Add 5*s.* and 3*s.*; 6*s.* and 2*s.*; 4*d.* and 5*d.*; 3*d.* and 4*d.*

24. From 8*s.* take 4*s.*; from 9*s.* take 3*s.*; from 7*d.* take 2*d.*; from 6*d.* take 1*d.*

25. How much must be added to 2*s.* to make 5*s.*? to make 7*s.*? to make 9*s.*?

26. How much must be taken from 7*s.* to leave 5*s.*? to leave 3*s.*? to leave 1*s.*?

27. I have four apples in one pocket, three in another, and two in a third pocket; how many apples have I in all?

28. Tom goes to school at 9 o'clock and leaves at 12; again at 2 and leaves at 4; how many hours is he in school per day?

NOTE.—Children should not be allowed to pass on to the next stage until all questions requiring addition, subtraction, multiplication, and division of numbers under ten are quickly and accurately answered.

CHAPTER IV.

THE NUMBERS TEN, ELEVEN, TWELVE.

WE have now dealt with all the figures or signs used for numbers except the figure 0, which stands for naught or nothing.

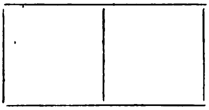
Attention must be directed to this sign, and the children must be taught to call it *naught* and not *aught*, the more common but incorrect name.

The counting and the building up of the numbers *ten*, *eleven*, and *twelve*, by means of pairs must be done by the children, as in the case of the preceding numbers; but the *figuring*, that is the *notation*, of these numbers demands the most careful attention.

The number ten is the foundation of the decimal system of notation, now almost universally adopted, being, in fact, the foundation of the science of number.

Notation appears to be the stumbling-block in the earlier standards; and, as a correct knowledge of notation is absolutely necessary to the proper understanding of the elementary processes of addition, subtraction, multiplication, and division, particular attention should be given to this and the following chapters.

If practicable, let two squares be drawn with chalk on the desk in front of each boy, about six inches in the side, thus:—



If this is not practicable, let the squares be drawn on a sloping table* in front of the class.

We will take the number eleven first. Eleven cubes consist of ten cubes and a single cube. Put the ten cubes in a bag,† and place the bag in the left-hand square, and the single cube in right-hand square. In the left square then we have a bag containing ten cubes or *one ten*, and in the right square a single cube or *one*, viz. a *unit*.

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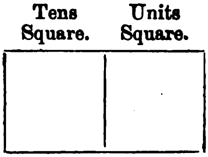
* This may be formed of a desk or sloping black-board. If it can be covered with a piece of cloth or baize, to prevent the sliding of the cubes, so much the better.

† If sticks are used ten must be tied in a bundle.

As we shall be constantly using these squares, and as we shall want at all times carefully to distinguish the one from the other, let us give them names. Let us call the one on the right the Units Square, meaning that we agree always to put any number of single cubes in it up to nine, and no more; and that on the left let us call the Tens Square, meaning that we will only put bags of tens in it.

We have no new figure which stands for eleven, and we must use over again those we have already learnt.

We will draw two squares on the black-board, and write their names over, thus:—



Now, as we have one bag containing ten cubes in the tens square on the table, we will put the figure 1 (one) in our tens square on the black-board, and this 1 means (and this is *the* important point), not a single one, but one *ten*.

Also we have a single cube in the units square, and so we put 1 (one) in the units square on the black-board to stand for it, so that now we have the figure 1 in the tens square, and the figure 1 in the units square; the figure 1 in the tens square meaning one *ten*, and the figure 1 in the units square meaning one *unit* or a single, and these together stand for eleven.

In the same way it must be shown that as twelve is made up of one ten and two singles, the bag of ten is placed in the tens space on the desk or table, and the two singles are placed in the units space. Also on the black-board we write 1 in the tens square and 2 in the units square.

The number ten is somewhat different. Ten forms only one group of ten: these are all put in the one bag, and this bag is put into the tens space, and so there are no singles for the units space. On the black-board, therefore, we write 1 in the tens square to stand for this one bag, and 0 in the units square, meaning that we have no single cubes or units of cubes.

NOTE.—Twelve is sometimes called a *dozen;* a year consists of *twelve* months; *twelve* pence make one shilling, and *twelve* inches one foot.

EXERCISES.

I. *With the cubes or other objects (simultaneous work).*

1. Take ten cubes and arrange them to show that a group of ten cubes is the same as two groups of five cubes. Divide ten cubes into five pairs of cubes.

2. Show that eleven cubes are made up of ten and one, and twelve of ten and two.

3. Arrange twelve cubes in groups of two each, in groups of three, of four, and of six.

4. Show that eleven is made up of six and five, or four and seven.

II. *With the black-board.*

1. I draw two squares side by side; what name shall I give to the left square, and why? and what name to the right square, and why?

2. Here I put 1 in the left square; what does it stand for? What figure must I also put in the right square to make twelve, to make eleven, to make ten?

3. I put 8 in the right square, without any figure in the left square; what does it stand for? I now put 0 in the left square; what does this figure stand for? *

III. *With the slates.*

1. Write the figure 9 four times; the figure 7 ten times; and the figure 8 twelve times.

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* A large number of exercises of this kind should be given.

2. Draw two squares side by side; place the figure 1 in each square; what does the 1 in the tens square stand for, and also the 1 in the units square? What do the figures stand for together?

3. Put 1 in the tens square; now place in the units square the figure required to complete the number twelve.

4. Write 10 in the squares. What does the 0 stand for? If I put 0 in the tens square, and 1 in the units square, what number do the two figures stand for?

IV. *Oral.*

1. How many fingers on both hands? How many toes on two feet? How many pairs of fingers can you make without the thumbs?

2. I walked eleven miles before dinner; five of these were walked before breakfast; how many did I walk between breakfast and dinner?

3. How many eggs make a dozen? How many apples make half a dozen?

4. A tailor had eleven yards of cloth; he sold five; how many had he left?

5. Tom and Harry had eleven marbles between them. Tom had three; how many had Harry?

6. A boy, playing at nine-pins, knocked down three the first throw, and four the second; how many were left standing?

7. If a slate pencil is 6 inches long, how many placed end to will measure a foot? If the pencils are only 4 inches long, how many will it take to make a foot?

8. Divide a shilling equally among three boys. What is the third of a shilling?

9. Divide a shilling among four girls; how much must I give to each? What is the quarter of a shilling?

10. A boy spent 2*d.*, 5*d.*, and 3*d.* out of a shilling; what had he left?

11. A schoolboy had sixpence. After paying his school fee he had 2*d.* left; what was the school fee?

12. What is the price of half a dozen eggs at 2*d.* each? and at 1*d.* each?

13. Oranges are 1*s.* per dozen. What is the price of one? of four? and of half a dozen?

14. A boy had a dozen eggs in a basket. He let the basket fall, seven only were found whole; how many were broken? If the eggs were worth 2*d.* each, what was the loss?

15. When rice is 2*d.* per lb. how many lbs. can I buy for 6*d.*? for 10*d.*? and for 1*s.*?

16. A boy bought a book; he gave 1*s.* for payment, and received 5*d.* change; what was the price of the book?

17. Bought a lb. of raisins at 5*d.*, and a lb. of currants at 4*d.*; what was the change out of 1*s.*?

18. Add together a third of a dozen, and a fourth of a dozen; take this number from a dozen; how many are left?

19. What is the difference between 5 and 12? between 4 and 11? and between 3 and 9?

20. A pair of numbers added together make 11; one of the numbers is 5, what is the other?

21. Add 7, 2, and 3. Add 8 and 2, and take away 4. Add 2, 5, and 4, and take away 8.

22. How many threes in twelve? How many fours in twelve? How many sixes in twelve?

CHAPTER V.

THE NUMBERS THIRTEEN TO NINETEEN.

The children ought by this time to have acquired the habit of attention, and the teacher alone may use the cubes or sticks or whatever other suitable objects are selected.

If the figuring of the numbers ten, eleven, and twelve has been thoroughly understood, the children will have little or no difficulty with the numbers up to nineteen, and it will be unnecessary to deal with all the pairs of which the numbers are built up.

It must, however, be shown that

13 cubes = 1 group of ten cubes and 3 single cubes.
14 „ = 1 „ „ „ 4 „ „
15 „ = 1 „ „ „ 5 „ „
16 „ = 1 „ „ „ 6 „ „
17 „ = 1 „ „ „ 7 „ „
18 „ = 1 „ „ „ 8 „ „
19 „ = 1 „ „ „ 9 „ „

These divisions of the cubes into tens and singles must be shown on the table as in the preceding chapter, and the figures must be written in their proper squares on the black-board.

NOTE.—"Teen" stands for the words "and ten"—thus, sixteen means six and ten; nineteen means nine and ten, and so on.

In dealing with the number 16, note that 16 oz. make a lb. (except with gold and silver).

EXERCISES.

I. *With the slates or black-board.*

1. Write the figure 2 thirteen times, the figure 7 fourteen times, the figure 9 sixteen times, and the figure 8 nineteen times.

2. Put the figures in their proper squares, which stand for fifteen, nineteen, seventeen, sixteen, fourteen, thirteen, and eighteen.

3. What number is represented by 1 in the tens square and 3 in the units square? Also 1 in the tens square and 7 in the units square?

4. Write ten, sixteen, nineteen, thirteen, eight, four, eleven, twelve, putting the figures in their proper squares.*

---

* A large number of exercises of this kind should be given.

II. *Oral.*

1. If I divide 16 shillings among four people, how many shillings do I give to each?

2. A horse trotted 18 miles in 2 hours; how much was that per hour?

3. If I spend 6*d.* per day, how long will it take me to spend 18*d.*?

4. If I spend 2*s.* per day, how much do I spend in 6 days? If I earn 3*s.* per day and spend 2*s.*, how much do I save in a week?

5. If 3 boys can be seated in one desk, how many can be seated in 4, in 5, and in 6 desks respectively?

6. I bought 16 yards of braid at ½*d.* per yard; what was the cost, and what was my change out of 1*s.*?

7. Seven buns for 6*d.*, how many for 1*s.*? Ten eggs for 1*s.*, how many for 6*d.*?

8. John had 3*d.* more than his sister; he gave her 1*d.*; how much had John then more than his sister?

9. Eighteen loaves are to be given to 6 men and 6 women; to each man 2 loaves, and to each woman 1 loaf; how many loaves will the men receive more than the women?

10. I bought a dozen and a half-dozen of eggs and sold 7; how many had I left?

11. How many ounces in half a pound of butter? When butter is 16*d.* per lb., what is that per ounce? and what per quarter-pound and per half-pound?

12. How many threepenny pieces in 1*s.*? fourpenny pieces in 16*d.*? and sixpences in 18*d.*?

13. Divide 15 marbles equally among 5 boys, and 18 equally among 6 boys.

14. What is the meaning of *teen* in fourteen? How many must be added to ten to make 15? to make 17? and to make 19?

15. Six added to 5 make how many less than 14?

16. What is the half of 14? the third of 15? the fourth of 16? and the sixth of 18?

17. What is the difference between 9 and 15? 6 and 17? 12 and 19? and between 8 and 18?

18. How many groups of 5 can you make with 15 ? and how many groups of 4 with 16 ?

19. Eggs are 1¼*d.* each ; how many can I buy for 5*d.* ?

20. I bought 4 eggs for 6*d.* ; what was the price per egg ?

21. I bought two loaves at 4½*d.* each ; what was the change out of a shilling ?

22. Which will cost the most, and by how much—2 lbs. of rice at 2½*d.* per lb., or ½ lb. raisins at 8*d.* per lb.?

CHAPTER VI.

THE NUMBERS TWENTY TO NINETY-NINE.

THE children may now be taught to count up to ninety-nine forwards and backwards.

It must then be shown with the cubes that

20 cubes = 2 groups of 10
30 ,, = 3 ,, 10
40 ,, = 4 ,, 10
50 ,, = 5 ,, 10
60 ,, = 6 ,, 10
70 ,, = 7 ,, 10
80 ,, = 8 ,, 10
90 ,, = 9 ,, 10

and a sufficient quantity of the other numbers must be taken to make the children thoroughly conversant with the composition of all the numbers up to 99.

The composition of the numbers must be demonstrated in the squares on the table, and the figuring on the blackboard.

Attention must be called to the following :—

1. The numbers 25, 36, 49, 64, and 81 are square numbers.*

2. 20 is called a score.
   20 cwt. = one ton.
   20*s.* = £1.
   20 quires = 1 ream of paper.

3. A day has 24 hours.
   A quire of paper has 24 sheets.

4. 30*d.* = half a crown.
   60*d.* = a crown.

5. 60 minutes = one hour.
   60 seconds = one minute.

EXERCISES.

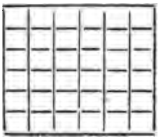
I. *With the slates or black-board.*

1. Write twenty in figures, placing the figures in their proper squares. What does the figure 2 stand for? Write twenty-two in figures; what is the difference between the two twos?

2. I write 0 in the units square, and 3 in the tens square; what number do these figures represent? What will be the number if I put 7 in the tens square?

3. Write 55, 77, 88, and 99 in the squares.

* If necessary this may be shown with the cubes, or with squares on the black-board, thus the square of 6 = 36.



4. Write 25, 37, 59, 74, 63 in the squares, and explain the meaning of the figures in each case.

5. I put 33 in the squares; what must I add to make 53?

NOTE.—The answer should be given in the form—"two tens."

6. I put 32 in the squares; what must I add to make 72? to make 52? to make 92?

7. I have 74 in the squares; what must I take away to leave 54? to leave 34? to leave 14?

8. Write 42 in the squares; what must I add to make it 64?

NOTE.—The answer should be given in the form—"two tens and two units."

9. I have 96; what must I take away to leave 64? to leave 43? to leave 14?

II. *Oral.*

1. How many fingers have five boys? Take away the thumbs; how many fingers are left?

2. How many wings have six birds more than four birds?

3. An owl, a goat, and a donkey have how many legs together?

4. A girl had a dozen eggs in a basket; she let the basket fall. On picking up the eggs she found only seven whole; how many were broken? If the eggs were worth 1*d.* each, what was the loss?

5. Tom's cap cost 30*d.*; Henry's cost 24*d.*; what was the difference in price?

6. Three boys of the same family go to one school—the eldest pays 4*d.* per week, and the other two 2*d.* each; how much do the three boys pay per week?

7. How many days in three weeks? How many Mondays, Tuesdays, and Wednesdays in three weeks?

8. In a class of 24 scholars, half a dozen came late; how many were punctual?

9. There are 60 seconds in a minute; how many seconds are there in half a minute? How many minutes are there in half an hour?

10. A boy in walking takes two steps in a second; how many steps will he take in half a minute?

11. How many score of apples are there in 40, in 60, and in 80 apples?

12. A pound is called a sovereign, and has a value of 20 shillings; how many shillings in half a sovereign?

13. A crown is 5 shillings; how many crowns in a sovereign? in half a sovereign?

14. A ton consists of 20 cwt.; how many cwt. in half a ton? in a quarter of a ton?

15. If coal is 1*s*. per cwt., what is that per ton?

16. How many pence in a crown, and in half a crown? How many pence must be added to 2*s*. to make half a crown?

17. The longest day in summer is 16 hours; how long is the night? The shortest day in winter is eight hours; how long is the night? When the day is 12 hours long, how long is the night?

18. I wish to give £5 to 10 men; what should I give to each?

19. How many months in 2 years? A boy saves 6*d*. per month; what does that amount to in 2 years?

20. Divide half a crown among 5 boys; what will each receive? Three boys have 1*s*. 6*d*. each, and two boys 9*d*. each; what have they altogether?

21. If I save 4*d*. per day, how long will it take me to save a crown?

22. Harry was sent to shop with a sovereign; he bought a coat for 10*s*. and a vest for 5*s*. He received 4*s*. change. Was this correct? If not, what was the mistake?

23. I have to travel 20 miles by rail; the 3rd Class fare is 1*d*. per mile, the 2nd Class 1½*d*.; what is the difference in the fare?

24. In the numbers 18, 84, what does the figure 8 stand for?

CHAPTER VII.

ADDITION AND SUBTRACTION OF NUMBERS NOT EXCEEDING NINETY-NINE.

ADDITION.

The teacher must have the usual sloping table, so that the addition may be performed *practically*, and a black-board on an easel for the *figuring*.

The scholars should draw their usual squares, and then erase the bottom line and produce side lines, so as to form columns. It will now be well to call the columns the tens and units *columns*, or tens and units *places*.

Let the number of cubes to be added be 26 and 39. Place the numbers in the columns on the black-board and draw a line beneath as here shown.

Make spaces for the units and tens on the sloping table. Arrange two bags of 10 in the tens space, and a group of 6 in the units space, and under these three bags of 10, and a group of 9.

Add the units groups together and rearrange them as one ten and five singles, and put the ten in a bag. Place the bag of ten in the tens space with the other bags of ten and add. We thus get 6 tens and 5 units, or 65.

Column for tens only.	Column for units only.
2 3	6 9
6	5

Now turn to the figures on the black-board—just as with the cubes, 9 added to 6 make 15, that is one ten and five units, and we place the 5 in the units column, and *carry* the *one ten* on to the tens column and add; thus we have 1, 3, and 2 tens, making 6 tens; set down the 6 in the tens place: we thus have 6 tens and 5 units, or 65.

A large number of examples should be similarly worked, and the scholars should be called alternately from the class to do the *actual* addition, carrying, &c., *with the objects,* whilst another scholar should write the figures in the columns on the black-board.

SUBTRACTION.

Example 1.—From thirty-seven cubes subtract twelve cubes.

Arrange the three bags of ten and the seven units in their proper spaces on the table; and place the figures in their proper columns on the black-board, thus:—

Column for tens.	Column for units.
3 1	7 2
2	5

Call a boy to take 2 units from the 7 units, and put on one side; 5 are left. Place the figure 5 in its proper place on the black-board. Next take away one of the bags of 10 and put on one side; how many tens are left? Write the 2 in its proper column on the black-board. There are 2 tens and 5 singles left, that is 25.

After allowing half a dozen scholars to perform the operation of subtraction thus *practically,* taking only such examples as require no "borrowing," to use a common but erroneous term—put to the class such a question as the following:—

From thirty-two subtract seventeen.

Place the cubes representing the 32 in their proper spaces on the table, and ask a boy to take 7 units from the units space; of course he cannot do it: ask another and another to try until the difficulty is brought home to every child.

Now ask, "Suppose a man has to pay *5d.* and he has a sixpence and two pennies in his pocket, what will he do?" "He will change the sixpence to pennies." From an example like this it is quite possible that some child may suggest what to do in the case before us; but if not, simply take one of the bags of ten and *change* it or *transfer* it to the units space. With the two cubes previously there we shall now have 12 cubes in the units place. In changing ten from one place to another, have we altered the number of cubes on the table? Certainly not; we have neither added to nor taken away.

It is very important for the children to understand that to take a bag of ten from the tens space and transfer it to the units space, does not alter the number.

The subtraction is now precisely as in the former example: the child can take seven cubes from the twelve cubes and five will remain, and one bag of ten cubes from the two bags, and one bag of ten will remain, or in all fifteen cubes.

Column for tens.	Column for units.
$\overset{\times}{3}$	2
1	7
1	5

The process must now be shown on the black-board. We cannot take 7 units from 2 units, we therefore take 1 of the 3 tens and transfer it to the units place, that is, we now have 12 units, and 7 may be taken from 12 and 5 placed below. We have then to take 1 ten, not from 3 tens,* because we have used 1 ten before, but from 2 tens, and 1 ten remains, that is, as we saw with the cubes, 15 remain.

It is a common practice for teachers to say, *without any explanation*, that because we *borrow* one from the top line, we must pay one back to the lower; in other words, if I borrow a penny from Richard I must pay it back to John, which must appear to the children a ridiculous sort of proceeding.

In working the following and similar examples the lines may be discarded at the discretion of the teacher.

EXERCISES.

(*For paper or slate.*)

1. Add together 19 pebbles, 23 pebbles, and 31 pebbles.
2. If I add 25 apples to 36 apples, how many shall I have in all?
3. From 21 oranges take 15.
4. Add 23 nuts, 17 nuts, and 31 nuts.
5. To 62 shillings add 17 more.
6. From 62 shillings take 17 shillings.
7. Add £15, £19, £24, and £16.
8. From £37 take £29.
9. Take 35 books from 71 books.
10. A man walked 29 miles on one day and 32 on another; how many miles did he walk in the two days?
11. There are 39 cap-pegs in one room, and 57 in another; how many pegs are there in the two rooms?

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* A cross (×) may be placed over the 3, to indicate that one has been taken away.

12. There were 72 children in school, 26 were sent into the playground; how many were left in school?

13. Henry has 9 marbles, John 21, William 17, and Tom 15; how many have they in all?

14. William had 37 marbles, but he lost 29; how many had he left?

15. There were 60 trees in a garden, 29 of them were cut down; how many were left?

16. In a class of 45 infants, 26 were boys; how many were girls?

17. A girl paid 1*s*. for gloves, 3*s*. for a hat, and 5*s*. for a pair of shoes; how many shillings did she pay in all? and what change ought she to receive out of a sovereign?

18. There were 52 boys in the second class, 19 were promoted to the first; how many were left in the second class?

19. A street contained 73 houses, there were 37 on one side; how many were there on the other?

20. There are 52 weeks in a year, we have 7 weeks' holiday; how many school weeks are left?

21. Of 37 scholars in a class, 19 only are provided with books; how many have no books?

22. From seventy-five subtract thirty-seven. From forty subtract seventeen.

23. Add thirty-one, forty-seven, and nineteen. From the sum take away thirty-eight.

24. What is the difference between thirty-nine and seventy-four?

25. A farmer bought twelve sheep for £36, and sold them for £41; what did he gain?

26. A grocer bought 96 lbs. of sugar, and sold 27 lbs. and 31 lbs.; what had he remaining?

27. William had 2 shillings in his money-box; he put in 3 shillings more, and took out 2 sixpences; what sum had he left?

28. Fourteen buns for one shilling; how many for 4 shillings?

29. A boy bought 12 newspapers for 9*d*. and sold them at a penny each; how much money did he gain?

30. Henry is 14 years old; his father is 41; how much older is the father than the son?

81. I bought a book for 27 pence, and another for 32 pence; how much change shall I have out of 80 pence?

82. Add together thirty-eight and forty-seven. From the sum take away fifty-six.

83. Find the difference between fifty and thirty-nine.

84. There are in a school 56 framed slates, and 29 without frames; how many slates are there in all? and how many framed more than unframed?

85. A tea-dealer bought a chest of tea weighing 54 lbs.; he sold 9 lbs. on one day and 16 lbs. on another day; how many lbs. were left?

86. There are 37 trees standing in a garden, and 19 have been cut down; how many were there at first?

87. Add 15, 16, 17, 18, and 19.

88. From 27 added to 25 subtract 19 added to 21.

PART II.

FOR JUNIOR SCHOOLS.

THE SIMPLE RULES.

CHAPTER VIII.

THE operations and calculations as contained in Part I. of this little work will probably be as much as may fairly be expected from a good Infants' School, where the children are not retained over *seven* years of age.

A preparation for the *First* Standard examination will necessitate the study of this and the three following chapters.

It often unfortunately happens that a large number of children enter the Junior School over seven years of age who are unable to perform the simplest calculations.

When this is the case, sufficient of the preceding chapters must be gone through to enable the children to understand the notation of tens and units, and to add and subtract with numbers not exceeding ninety-nine.

In the Junior School we lack the cubes of the Kinder Garten, and a substitute must be found: for at this stage, as well as in the Infants' School, if we mean to teach numbers and not mere ciphers, *we must appeal to the evidence of the senses. We must teach through the eye and hand.* The hundreds and the tens and the units must be handled, and the additions and subtractions must be realities and not mechanical processes.

Given five or six hundred marbles, four or five dozen small holland bags, just sufficiently large to hold ten marbles each, and a dozen or so of larger bags of sufficient size to contain ten of the small bags; also a table and a black-board, and we have all the materials necessary for our purpose at a minimum cost.*

The table should be placed sufficiently high and with a little slope, so that the surface may be visible to the whole class.

The surface should be divided by chalk lines, or otherwise, into three parts, for the hundreds, tens, and units spaces respectively. A shallow box placed in the units space will be useful to hold the single marbles.

The black-board must be placed on an easel in front of the class.

The sloping table is to serve for the *practical* operations, and the black-board to show the *figuring* of the same.

THE NOTATION OF THE NUMBERS ONE HUNDRED TO NINE HUNDRED AND NINETY-NINE.

In counting upwards from ninety-nine we come next to one hundred, that is, one added to ninety-nine makes one hundred.

Arrange ninety-nine marbles on the table, nine small bags with ten marbles in each, to be placed in the tens space, and nine single marbles in the units space.

Call one child to the front to add one marble to the nine in the units space. Thus we have ten marbles in the units space and we may put these in a tens bag, and transfer or change it to the tens space; this gives ten bags of ten in the tens space, and there is nothing left in the units space.

---

* Bundles of sticks tied in tens, and the tens in hundreds, will answer equally well and cost still less.

We have shown that ninety-nine and one—or, which is the same thing, ten tens—make *one hundred*, and the point which has to be impressed on the children is this, that, just as when we get ten single marbles in the units space, we put them together and call them *one ten*, so, when we get ten tens in the tens space we put them all together and call them *one hundred.*

We will put the ten small bags into a large one, and calling it a hundreds bag we will carry it on to the space next to the tens on the left, and because this space has to hold hundreds only we call it the hundreds space.

We must now show how one hundred is represented with figures on the black-board.

Draw three columns and write the names above, thus:—

Place for hundreds.	Place for tens.	Place for units.
1	0	0

Look at the table; how many single marbles have we in the units space? We have none, we must therefore place 0 on the black-board in the *place for the units.* For a similar reason we must write 0 in the *place for the tens.* In the hundreds space on the table we have a large bag of *one* hundred, hence we write the figure 1 in the *place for the hundreds.*

One hundred then is shown in figures by 1 in the hundreds place, 0 in the tens place, and 0 in the units place.

In the next place let us see how the number *one hundred and eleven* is figured. Retain the hundred in its place on the table and add a bag of ten and a single, of course putting each in its proper space. This is one hundred and eleven.

Turn to the black-board: we have to write 1 i hundreds place, 1 in the tens place, and 1 in the place. In other words, three ones placed side by side for one hundred and eleven.

Again, how shall we figure a number like two hu and thirty-four? Two hundred and thirty-four mean hundreds, three tens, and four units. We shall the require two large or hundreds bags in the hundreds three bags of ten in the tens space, and four s in the units space, and, as before, we write the correspo figures, viz. 2, 3, and 4, in their proper places on the board.

Lastly, how do we figure a number like two hundre seventy, or two hundred and seven?

Two hundred and seventy means two hundreds, seven and no units; two hundred and seven means two hun no tens, and seven units.

Arrange the marbles as before in their proper bag spaces, and we find that in the first example we nothing in the units space, and in the second examp have nothing in the tens place, so when we figure o black-board we *always* place a 0 to show that there a units or tens as the case may be.

This visible exhibition of the principle of decimal tion, by means of marbles and bags, will be found to pr much perplexity when the children come to deal with nu like 2001 or 1020.

EXERCISES.

(*For the slate or black-board.*)

1. Write one and thirty-one in figures. What is the diff between the two *ones* here?

2. Write in figures, and place in their proper columns each other—

(1) Two hundred and fifty.
(2) Four hundred and one.
(3) Seventy-eight.
(4) One hundred and ten.
(5) Eight hundred and thirty-seven.
(6) One hundred and one.

3. Write in words—42, 30, 120, 209, 600, 740, 999.

4. How many tens in two hundred units? in one hundred and sixty? and in sixty?

5. What does the figure 5 stand for in each of the following numbers—250, 505, 175, 154.

NOTE.—Numerous examples of this kind should be given before proceeding to the next chapter.

CHAPTER IX.

ADDITION AND SUBTRACTION OF NUMBERS NOT EXCEEDING NINE HUNDRED AND NINETY-NINE.

ADDITION.

LET the numbers of marbles to be added be thirty-five, two hundred and fifty-one, one hundred and forty-seven, and nineteen. Thirty-five means three tens and five singles; we therefore put three bags of ten in the tens space, and five singles in the units space.

Two hundred and fifty-one means two hundreds, five tens, and one single; we therefore put two bags of one hundred in the hundreds space, five bags of ten in the tens space, and one single in the units space.

The other numbers must be similarly dealt with.

Now collect all the singles or units, and we get twenty-two, that is, two tens and two singles. Place the tens in two bags and transfer them or *carry* them on to the tens space, leaving the two singles in the units space. Next collect the tens: they number fifteen tens, that is, ten tens and five tens; place the ten tens, that is one hundred, in a large bag, and transfer or *carry* the one bag to the hundreds space, leaving the five tens in the tens space.

Lastly, collect the hundreds—we find four hundreds. Now what is it we have done? We have simply rearranged the marbles on the table, or collected them together; or in other words we have added them, and the result we get is four hundreds, five tens, and two units, or four hundred and fifty-two.

Now turn to the black-board and work the example with the figures. Of course we must be careful to put the hundreds and tens and units in their proper places, thus:—

Place for hundreds.	Place for tens.	Place for units.
	3	5
2	5	1
1	4	7
	1	9
4	5	2

Draw a line under the numbers to be added.

Add the units first—nine and seven are sixteen, and one seventeen, and five are twenty-two.

Twenty-two means two tens and two units. Put down the 2 units in the units column under the line, and transfer or *carry* the 2 tens to the tens column. Adding the tens with the two carried we get fifteen tens. Fifteen tens means ten tens and five tens, or one hundred and five tens. Put down the 5 tens in the tens column under the line, and carry on the 1 hundred to the hundreds column.

Add the hundreds column, we get four hundred. Put down the 4 in the hundreds column under the line.

Our result is four hundreds, five tens, and two units. Compare this with the marbles on the table, we also find there four hundreds, five tens, and two units. The answer is 452.

SUBTRACTION.

Example.—From two hundred and thirteen marbles subtract one hundred and fifty-four.

Arrange two bags of one hundred, one bag of ten, and three singles in their proper spaces on the table. From these we have to take away one hundred, five tens, and four units.

Firstly, we have to take away four single marbles from the units space, but we have only three marbles there; we therefore take the bag of ten from the tens space, and empty it into the units space. This leaves us the tens space empty, but gives us thirteen in the units space; from this thirteen we take away four and put on one side, and nine are left.

Secondly, we have to take five tens from the tens space; at present we have no marbles there, so we take one of the hundreds bags and empty its *ten* small bags into the tens space; we can now take away five tens, and five tens remain. Lastly, we take away the one hundred left; and we have fifty-nine on the table after taking away one hundred and fifty-four from two hundred and thirteen.

To show the working with the figures, write the figures in their proper places on the black-board.

Two hundred and thirteen means two hundreds, one ten, and three; we therefore write 2 in the hundreds place, 1 in the tens place, and 3 in the units place, and under these we write 1 hundred, 5 tens, and 4 units, which stand for one hundred and fifty-four. Draw a line under the figures.

Place for hundreds.	Place for tens.	Place for units.
×	×	
2	1	3
1	5	4
	5	9

Four units from three units is impossible. I therefore transfer one ten from the tens place, and add this ten to the three units, making thirteen units. Four from thirteen leaves nine. Write the figure 9 below the line in the units place.

Next, we cannot take five tens from nothing, so we take one from the hundreds place, that is ten tens, and put it in the tens place; we now subtract five tens from ten tens, and five tens are left. Set down the 5 remaining in the tens place below the line.

Lastly, one hundred from one hundred leaves nothing. That is, our result is 5 tens and 9 units, or 59.

Examples in addition and subtraction must be worked by the children on the table, and on the black-board and slates, until they thoroughly understand the process of *carrying* in addition, and of *borrowing* in subtraction.

NOTE.—On no account should the scholars be allowed to add and subtract with the fingers, or with strokes on their slates.

The numbers should always be concrete quantities. Only in this way can the children see the *reason* for learning these processes.

It may be thought by some teachers that problems ought not to be attempted in the work of the 1st and 2nd Standards. But why not? Are the children when promoted to the Junior School less intelligent than they were in the Infants' Department? In the Infants' Department they began and ended with problems. They were constantly being plied with such questions as, "John had seven marbles and lost four; how many had he left?" or, "A boy bought an orange for 1*d.* and a slate for 2*d.*; what change had he left out of 6*d.*?" And these and similar questions were answered readily enough. Then why—having learnt addition and subtraction with figures—do the children of the 1st and 2nd Standards look aghast at such a simple problem as, "A woodcutter cut down 270 trees from an estate: it was found that 230 were left; how many were there at first?"

There can be no *valid* reason for this. But does not the cause lie in the fact that the child has *not* been taught to translate the language of the mind into the language of words? Has he been encouraged to describe in simple language the process that went on in his mind which enabled him to arrive at a certain result?

If a boy—having of course learnt the process of subtraction—can answer such a question as this: "*A gardener cut down* 2 *trees from a garden containing* 5 *trees; how many trees were left?*" he ought certainly to be able to work such a problem as, *A woodcutter cut down* 537 *trees from a wood containing* 951; *how many were left standing?*

There is no new process of reasoning required in the second example, and if the child had been taught to put his mental process into words, thus: "The gardener cut down two trees, there were five at first, so there must now be two less, and I must therefore subtract two from five;" he would find little difficulty in applying the same reasoning in the second ase.

It is an excellent practice to give the class a simple problem to work mentally prior to giving a similar problem to be worked on the slates, and to require that the reasoning be given in words.

A few of such problems are given in the following examples.

EXERCISES.*

1. There are 6 boys in one desk, 4 in another, and 5 in a third; how many boys are there in the 3 desks? There are 27 boys in the 1st Class, 29 in the 2nd, 33 in the 3rd, and 37 in the 4th; how many boys are there in the four classes?

2. I see 10 children, 6 of them are boys; how many are girls? In an infant school there are 254 scholars, 149 of them are girls; how many boys are there in the school?

3. A man bought 3 bars of soap—the first bar weighed 8 lbs., the second 4 lbs., and the third 5 lbs.; how many lbs. did the three bars of soap weigh? A grocer bought 3 boxes of soap—the first box contained 140 lbs. of soap, the second 119 lbs., and the third 210 lbs.; what was the total weight of soap?

4. Mother bought 9 lbs. of beef, and gave 3 lbs. away; how many lbs. had she left? A butcher bought 235 lbs. of meat, and sold 226 lbs.; how much had he left?

5. Here are 7 pins and needles—3 are pins; how many are needles? In a box containing 785 pins and needles, 429 are needles; how many pins are there?

6. William is 19 years old, John is 13 years; how much older is William than John? John's father is 52; how many years is the father older than the son?

7. John received 3*s.* on Monday, 5*s.* on Tuesday, and 4*s.* on Wednesday; what did he receive in the three days? A shopman took 112 shillings on Monday, 184 on Tuesday, 91 on Wednesday, 47 on Thursday, 56 on Friday, and 237 on Saturday; how many shillings did he take during the week?

8. I have 124 apples; how many do I want to make 310?

9. I require 36 oranges to make up 78; how many have I now?

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* Most of these and similar examples should be written on the black-board *for this Standard,* and if necessary the teacher should read and explain.

10. A farmer bought at market 19 cows, 157 sheep, and 56 pigs; how many animals did he buy altogether?

11. Harry walked 21 miles each day for a week; how many miles did he walk in the week? (week = 6 days).

12. England has 52 counties and Scotland 33; how many counties are there in England more than in Scotland?

13. I have 3 shelves of books—there are 124 on the first shelf, 219 on the second, and 176 on the third; how many books have I in all?

14. A railway train has 5 carriages. In the first carriage are 29 persons, in the second 44, in the third 92, in the fourth 68, in the fifth 114; how many persons are there in the train?

15. A man owes £130. He has only £75 with which to pay; how much more money does he want to enable him to pay the whole bill?

16. January has 31 days, February 28, March 31, April 30, May 31, June 30, July 31, August 31, September 30, October 31, November 30, and December, 31; how many days are there in the year?

17. I was born in the year 1869; how old was I in 1877? My brother is 7 years older than I; how old is my brother?

18. I borrowed £350. I have repaid £179; how much do I still owe? If I pay £57 per year of the remainder, how long will it take me to clear off the debt?

19. I bought two horses for £110; at what price must I sell them to gain £27?

20. How much is four hundred and twenty-one greater than three hundred and twenty-nine?

21. Take thirty-seven from the sum of fifty-one and sixty.

22. To the difference between one hundred and ten and ninety-seven add thirty-seven.

23. How many must be added to forty-seven to make ninety-four?

24. How many must be taken from sixty-five to leave twenty-seven?

CHAPTER X.

THE NUMBERS ONE THOUSAND TO TEN THOUSAND.

It may suffice to explain to the children that just as when we get *ten* bags of *ten* marbles each, we put them in a larger bag and call the number one *hundred*, so when we get *ten* bags of one *hundred* each, we again put these in a still larger bag and call the new number a *thousand;* but as children can have but the faintest idea of the size of a thousand articles compared with one, it is very desirable that the composition and size of one thousand should be shown practically.

For this purpose one hundred small bags for tens, ten larger for hundreds, and one larger still for the thousands will be required.* The table should also be divided into *four* spaces.

Place nine marbles in the units space, nine bags of ten in the tens space, and nine bags of a hundred in the hundreds space. This stands for nine hundred and ninety-nine; one more will make a thousand.

Add *one* marble to the units, this makes ten, and so we put this ten in a bag and transfer as *one* ten to the tens space. We thus get *ten* bags in the tens space, and we put these in a hundreds bag and transfer this *one* hundred to the hundreds space. Again, we now get *ten* hundreds bags in the hundreds space, and we collect them together and put them in a larger bag and transfer this bag to the space on the left (the fourth space from the units) and we call the number a *thousand.*

We also name the space into which we put the thousands *only* the "*thousands space.*"

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* *If* there is any difficulty in getting so many bags, use bundles of sticks.

To figure a thousand draw four columns, and name them thus:—

Place for thousands.	Place for hundreds.	Place for tens.	Place for units.
1	0	0	0

Now look at the table; on the table we have a thousand marbles. How many are there in the units space? How many in the tens? How many in the hundreds? We have nothing in either space; we must therefore write the figure 0 in each of the columns for units, tens, and hundreds.

In the thousands space we have *one* bag of a thousand, we therefore write 1 in the thousands column or place.

What then is one thousand? Empty the bag. Ten bags of one hundred is the result. Empty each of these bags. One hundred bags of ten each lie on the table. Empty each of these. We get a heap of one thousand single marbles.

A thousand, then, is

ten hundreds,
or, a hundred tens,
or, a thousand singles.

We may have in our thousands space on the table any number of thousands bags up to *nine*, but when we arrive at *ten* we should put them in a larger bag again and call this bag a ten-thousands bag, and we should place this bag in a space to the left of the thousands. And just in a similar way when we arrive at *ten* ten-thousands bags, we should put them in a still larger bag and call it the hundred-thousands bag, and we should again place this in a space to the *left* of the ten-thousands space.

So then on the black-board we must make a 5th column for ten-thousands, and a 6th column for the hundred thousands; thus:—

6th	5th	4th	3rd	2nd	1st
Place for hundred thousands.	Place for ten thousands.	Place for thousands.	Place for hundreds.	Place for tens.	Place for units.
1	0	0	0	0	0

One Hundred Thousand.

NOTE.—If the scholars have mastered what we have before called the *visible exhibition of notation* there can be no possible difficulty in their writing any number in figures from dictation.

But it may be well to add that we have seen the following mechanical plan of teaching notation adopted in several schools, and, so far as mechanical work can be considered satisfactory, with very gratifying results.

Let the children draw six columns on their slates with a *double* line in the centre and write the names above, thus:—

		Thousands.				
	Place for hundreds.	Place for tens.	Place for units.	Place for hundreds.	Place for tens.	Place for units.
(1)	1	2	5	6	4	4
(2)	2	0	4	1	4	5
(3)		6	6	2	2	0
(4)			5	0	2	5
(5)	8	9	0	0	7	0

First, practice them in writing numbers consisting only of hundreds, tens, and units; but placing the numbers on *either* side of the middle line.

Secondly, write numbers on each side of the middle line, *but placing* them opposite to each other—thus: write one hundred and twenty-five on the left side, and six hundred and forty-four on the right side *opposite to* the former number [see (1) above].

Thirdly, give the left side its name of thousands side instead of left side, and practice well with such examples as the following:—

(*a*) "Set down two hundred and four on the thousands side, and one hundred and forty-five on the right side" [see (2)].

(*b*) "Set down sixty-six on the thousands side, and two hundred and twenty on the other side" [see (3)].

(*c*) "Set down five on the thousands side, and twenty-five on the right side" [see (4)].

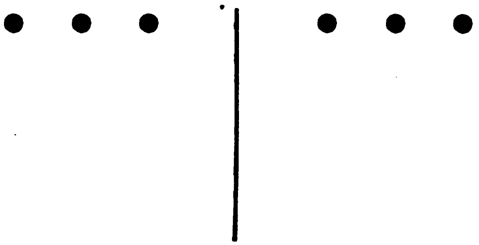
Here we have no figure in the hundreds place—right side—and we fill in with a naught.

(*d*) "Set down eight hundred and ninety on the thousands side, and seventy on the right side" [see (5)].

Fill up the vacant places with ciphers.

Fourthly, drop the word *side* altogether and proceed thus: "Set down two hundred and fifty-one thousands"—see that this is written on the left side before proceeding further —"one hundred and seven." See that the tens place is occupied by a naught.

Lastly, gradually reduce the mechanical helps. First, take away the lines between units and tens, and between tens and hundreds. Then take away the names at the top and substitute marks to indicate position.



Next take away the line and substitute commas, and lastly take away the position marks.

NOTE.—It cannot be too strongly impressed on the minds of the scholars that though ordinarily we use no lines when we write numbers, yet they are understood: the hundreds and tens and thousands, &c., all have their proper *places*, and no others. We cannot write hundreds in the *fourth* place, or thousands in the *third* place. Thus, if we write a single figure, say 8, this stands for eight units or ones only, and nothing else. If we put a naught on the right-hand side so that it now occupies the second place, it means eight tens, and nothing else. If we put another naught on the right side, so that the eight occupies the third place, it stands for eight hundreds, and nothing else; and so constantly if we change the *place* of a figure we as constantly change its value.

CHAPTER XI.

ADDITION AND SUBTRACTION.

HIGHER NUMBERS.

THE addition and subtraction of numbers involving* thousands is so exactly the same as the addition and subtraction of hundreds and tens, that we need only indicate that the same plans should be followed as are given on pages 37 to 42.

NOTE.—The terms addition, subtraction, sum, difference, subtrahend, minuend, and the signs +, —, =, must be explained as opportunity offers.

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* Unless compelled by the necessity of examination it is not advisable to go beyond thousands in the 1st Standard.

EXERCISES.

1. The first class in a school contains 35 children, the second 29, and the third 42; how many children are there in the three classes?

2. The first class in a boys' school contains 51 scholars, and the first class in a girls' school 37 scholars; how many more boys are there than girls in these classes?

3. (*a*) A gardener planted 4 trees on Monday, 6 on Tuesday, and 5 on Wednesday; if he charged 1*s*. for planting each tree, what did he earn?

(*b*) Some foresters cut down 215 ash-trees on Monday, 75 elms on Tuesday, 349 oaks on Wednesday, and 117 beeches on Thursday; if a shilling per tree was paid for cutting, how many shillings did they earn?

4. (*a*) I had 12 apples; I gave 3 to Tom, and 6 to Harry; how many had I left?

(*b*) A woman had 306 eggs in her basket; she sold 139, gave 12 away, and broke 7; how many had she left?

5. A farmer had 978 sheep in one field, 429 in another, and 63 in a third; he put all into one field; how many were there?

6. (*a*) I have 9 marbles; how many more do I require to make 14 marbles?

(*b*) I have 325 cows on my farm; how many more do I want to make up 500?

7. An apple-tree had upon it 509 apples; I picked 374; how many were left?

8. (*a*) What number is 7 less than 16?

(*b*) What number is 95 less than 1,003?

9. A woman sold 304 pears on one day, and 250 the next day; she found she had 108 left; how many had she at first?

10. There were 1,000 fruit-trees in a market-garden; of these 307 were apple-trees, 204 plum-trees, 246 pear-trees, the rest were cherry-trees; how many cherry-trees were there?

11. A butcher sold 215 quarters of beef in January, 468 quarters in February, and 579 quarters in March; how many quarters did he sell in the three months?

12. A baker sold 217 penny rolls on Thursday, 546 on Friday,

and 372 on Saturday; how many pennies should he receive for the whole?

13. Charlie began to play with 48 marbles; he won 38 in the morning, and lost 27 in the afternoon; how many had he at the end of the day?

14. Harry began to play with 196 marbles; he lost 67 on the first day, and won 84 on the second day; how many marbles had he at the end of the second day?

15. (*a*) I had 12 apple-trees in my garden; three of them were destroyed; how many more do I require to make up 20?

(*b*) I had 785 stamps, but I lost 18 of them; how many more do I require to make up 1,000?

16. I read 69 pages of a book on Tuesday, 75 pages on Wednesday, and the remainder on Thursday. The book contained 208 pages; how many pages did I read on Thursday?

17. If I sell 95 eggs this week, and 84 next week, how many shall I have left out of 200?

18. How many must be added to 598 to make 1,000?

19. I travelled 216 miles on the first day, on the second day I travelled 15 more miles than on the first, and on the third day 15 miles more than on the second; how many miles did I travel in the three days?

20. If there are 306 beans in one bag and 75 in another, and in a third as many as in the other two, how many beans are there in the three bags?

21. I have purchased 292 yards of netting, but I require in all 500 yards; how many more yards must I buy?

22. After a robbery a banker finds he has 396 sovereigns; he had 1,050 sovereigns at first; of how many sovereigns was he robbed?

23. Add 3,705 to itself three times.

24. How many less than 1,000 is the number 897?

CHAPTER XII.

EXAMINATION PAPERS.

STANDARD I.

I.

1. Write in figures four thousand three hundred and seven. What is the value of the figure 3 in this number?

2. Out of 25 apples, 17 pears, and 31 plums, 7 of each were rotten; how many sound fruit had I in all?

3. A woman took a basket of eggs to market; she sold 87, broke 5, and took home 43; how many eggs were there in the basket?

4. Tom has 504 apples and 606 pears; how many pears has he more than apples?

II.

1. Write in figures five thousand eight hundred and seventy; what is the value of the 7 in this number?

2. Add together three hundred and ninety, fifty-seven, one hundred and ten, and five hundred and eighty-six.

3. I gave 36 marbles to William, 65 to Henry, 94 to James, and have 127 left; how many had I at the first?

4. One box of figs contained 55 and another 75; how many less than 200 were there in both boxes?

III.

1. Add together six hundred and ninety-five, one thousand and fifty-four, two hundred and seventy, and eighty-one; how many *hundreds* in the answer?

2. Find the difference between one thousand and three, and one hundred and three.

3. After selling 184 oranges I have 96 left; how many had I at first?

4. There are 1,760 yards in a mile; after travelling 706 yards, how many yards have I to go to finish the mile?

IV.

1. How much less is three hundred and five than four hundred and one? For what does the 9 stand in the answer?

2. Find the number of trees in an orchard containing 86 apple, 109 pear, 29 cherry, and 25 plum trees.

3. Take 59*d.* from the sum of 48*d.*, 67*d.*, and 109*d.*

4. Write in words the difference between two thousand, and one thousand and ninety-four.

V.

1. Write in words 3,040 and 1,207.

2. Add two hundred and sixty, eighty-seven, nine hundred and three, three thousand and sixty-four, and twenty-seven.

3. How many acres of land has a farmer who owns four fields, the first field containing 95 acres, the second 65, the third 64, and the fourth 37 acres?

4. A tailor had a piece of cloth which measured 65 yards; he used 15 yards on Monday, 13 on Wednesday, and 17 on Saturday; what length of cloth remains?

VI.

1. From one thousand six hundred and one subtract nine hundred and six.

2. A labourer earns 21 shillings per week, and spends 16 shillings; what does he save in four weeks?

3. 17 gallons, 19 gallons, 25 gallons, and 13 gallons of water were drawn from a cask containing 100 gallons; how many gallons were left?

4. There are 570 children in a school of boys and girls; 291 are boys; how many are girls?

VII.

1. Add 310, 694, 7,014, and 657; from this sum subtract 5,029.

2. A man owes £5, £9, £13, £19, but he has only £29; how much more money does he want to enable him to pay his debts?

3. There were 1,106 people in a public building; 507 were men, 310 were women, and the remainder children; how many children were there?

4. A fruiterer bought 4 boxes of oranges, each containing 396 *oranges*; it was found that in the first box 15 were bad, in the

second 39, in the third 28, and in the fourth box 17; how many sound oranges were there in all?

VIII.

1. Find the sum of three hundred and one, four thousand and sixty-nine, seven hundred and eighty-five, and one thousand and eight.

2. Find the difference between one hundred and two, and one thousand and twenty.

3. Harry and John played a game at marbles. Each had 17 at the beginning. At the end of the game John had won 6. How many had each?

4. There are 560 oranges in one box, 40 more than this in a second, and in a third as many as in the other two. How many oranges in the three boxes?

CHAPTER XIII.

MULTIPLICATION TABLE.[*]

Example (*for the black-board*).—"I bought six boxes of apples; there were 245 apples in each box; how many apples were there in all?"

Work the exercise by addition.

$$\begin{array}{r}245\\245\\245\\245\\245\\245\\ \hline 1,470\\ \hline\hline\end{array}$$

We first add the units and find the amount to be 30, that is, six fives added together make 30. Next we add the tens, and find that six fours make 24, and 3 we carried make 27. Lastly six twos in the hundreds place added make 12, and 2 we carried make 14. Now had we known, *without adding*, that six fives made thirty, that six fours made twenty-four, and that six twos made twelve, we might have saved the long addition, and said at once six fives are thirty, set down naught and carry three; six fours are twenty-fo and three are twenty-seven, set down seven and carry tw and six twos are twelve and two are fourteen, set dow fourteen.

---

[*] If the scholars have been well grounded in notation the objects may now be discarded.

This short method of adding equal numbers we ca *Multiplication.*

Write on the black-board the following exercises in add tion, and let the children add:—

2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2
	4	2	2	2	2	2	2	2	2	2	2
		6	2	2	2	2	2	2	2	2	2
			8	2	2	2	2	2	2	2	2
				10	2	2	2	2	2	2	2
					12	2	2	2	2	2	2
						14	2	2	2	2	2
							16	2	2	2	2
								18	2	2	2
									20	2	2
										22	2
											24

From this we see that

one two or once two are two
two twos or twice two are four
three twos or three times two are six
four twos or four times two are eight
five twos or five times two are ten
six twos or six times two are twelve
seven twos or seven times two are fourteen
eight twos or eight times two are sixteen
nine twos or nine times two are eighteen

ten twos or ten times two are twenty
eleven twos or eleven times two are twenty-two
twelve twos or twelve times two are twenty-four

This column we have written is the first column of the "Multiplication Table." It is usually written—

Twice	1	are	2
,,	2	,,	4
,,	3	,,	6
,,	4	,,	8
,,	5	,,	10
,,	6	,,	12
,,	7	,,	14
,,	8	,,	16
,,	9	,,	18
,,	10	,,	20
,,	11	,,	22
,,	12	,,	24

The teacher should now take another table, say *five times*, and treat it in the same way, and then, directing attention to the last column in each table, should elicit from the children that in the "*twice*" table the numbers rise by additions of *two*, and that in the "*five*" table the numbers rise by additions of *five*. And so if we construct other tables: in the "*four*" table the successive additions will be by *fours*, in the "*six*" table by *sixes*, and so on.

In this way, the children may be taught how to build up each column of the Multiplication Table without any previous knowledge of the table.

The Multiplication Table having been thus constructed by the children themselves, must be committed to memory, and must be so thoroughly mastered that the mere mention of certain factors* shall invariably call up the product, and the mention of a product shall immediately connect itself in the minds of the children with its factors. Nine times four must be inseparably connected with thirty-six, and thirty-six must be as closely associated with nine times four, or twelve times three, or six times six.

---

* We say that 6 multiplied by 4, or 4 multiplied by 6, makes 24; hence we may call the 6 and 4 makers or *factors* of 24. And the 24 is called the *product* of the factors 6 and 4.

The Multiplication Table is to Standard II. what notation is to Standard I., and unless it is perfectly learnt by heart, no satisfactory progress can be made; but when mastered progress is easy and rapid.

Of course it is not necessary that the *whole* table should be learnt before commencing slate exercises, but whatever portion has been attempted should be well done.

ORAL EXERCISES ON THE MULTIPLICATION TABLE.

1.	(1)	$3 \times 4 =$	$12 \div 4 =$	$12 \div 3 =$
	(2)	$5 \times 6 =$	$30 \div 5 =$	$30 \div 6 =$
	(3)	$7 \times 4 =$	$28 \div 7 =$	$28 \div 4 =$
	(4)	$6 \times 8 =$	$48 \div 8 =$	$48 \div 6 =$
	(5)	$9 \times 5 =$	$45 \div 9 =$	$45 \div 5 =$
	(6)	$8 \times 9 =$	$72 \div 9 =$	$72 \div 8 =$
2.	(1)	$32 \div 8 =$	$4 \times 8 =$	$108 \div 9 =$
	(2)	$64 \div 8 =$	$8 \times 8 =$	$120 \div 12 =$
	(3)	$81 \div 9 =$	$9 \times 9 =$	$132 \div 11 =$
	(4)	$63 \div 7 =$	$7 \times 9 =$	$144 \div 12 =$
	(5)	$56 \div 7 =$	$8 \times 7 =$	$132 \div 12 =$
	(6)	$54 \div 9 =$	$6 \times 9 =$	$120 \div 10 =$

3.* What is the value of—

(1) 4 lbs. of sugar at 5*d*. per lb.?
(2) 6 chairs at 5*s*. each?
(3) 7 pairs of gloves at 4*s*. each?
(4) 9 slates at 8*d*. each?
(5) 11 books at 5*d*. each?
(6) 8 pears at 3*d*. each?

---

* The answer to be given in pence, or shillings, or pounds, *without* any *reduction*.

4. What is the value of—

(1) One lb. of tea, when 5 lbs. cost 20*s.*?
(2) One lb. of raisins, when 8 lbs. cost 40*d.*?
(3) One basket, when 9 baskets are worth 54*s.*?
(4) One sheep, when 7 sheep are worth £21?
(5) One cap, when 4 caps are worth 12*s.*?
(6) One ball, when 11 balls are worth 55*d.*?

5. What number multiplied—

(1) By 9 will produce 54?
(2) „ 8 „ „ 64?
(3) „ 7 „ „ 63?
(4) „ 6 „ „ 42?
(5) „ 5 „ „ 40?
(6) „ 4 „ „ 32?

6. (1) What is 5th part of 30?
(2) „ 6th „ 54?
(3) „ 9th „ 72?
(4) „ 8th „ 72?
(5) „ 7th „ 56?
(6) „ 4th „ 32?

7. Name all the factors of 32, 24, 28, 36, 42, 48.

8. What factors will produce the following products:—72, 64, 96, 81, 54, 63?

CHAPTER XIV.

SHORT MULTIPLICATION.

Example 1 (*for the black-board*).—"There are 5 classes in the school, and in each class 34 boys; how many boys are there in the school?"

Work the exercise first in the addition form, thus:—

$$\begin{array}{r}34\\34\\34\\34\\34\\\hline 170\end{array}$$

Five times 4 units = 20 units = 2 tens. Set down 0 and carry 2 tens.

Five times 3 tens = 15 tens, and adding the 2 tens carried we have 17 tens = 1 hundred and 7 tens. Set down 7 in the tens place, and 1 in the hundreds place.

It is more conveniently worked as follows:—

$$\begin{array}{r}34\\5\\\hline 170\end{array}$$

Write down 34, and place the *multiplier* 5 under the 4 units; draw a line below, and proceed thus: five times four units are twenty, viz. 2 tens and 0 units, set down 0 under the units, and carry 2 to the tens place—we add this 2 presently—five times three tens are fifteen, viz. 1 hundred, and 5 tens; add the 2 tens we carried from the units, and we get 1 hundred, 7 tens, and 0 units, or 170.

Prove this sum by addition, and thus again show that multiplication is a short method of adding equal numbers.

After the children have had sufficient practice to enable them to understand the method and theory of multiplication, they may omit all reference to hundreds, tens, units, &c., and proceed as in the next example.

Example 2.—"There are 6 bags, and in each bag 2,047 marbles, how many marbles are there in all?"

$$\begin{array}{r}2{,}047\\6\\\hline 12{,}282\end{array}$$

Six times 7 are 42, put down 2 and carry 4. Six times 4 are 24, and 4 we carried are 28. Set down 8 and carry 2. Six times 0 is nothing, set down the 2 we carried. Six times 2 are 12, set down 12.

EXERCISES IN SHORT MULTIPLICATION.

1. How many legs has a flock of 1,432 sheep?

2. In a school of 537 boys I gave five oranges to each boy; *how many oranges* did I give away?

3. There are 313 working days in a year; if a boy earns 4*d.* per day, how many pence does he earn in a year?

4. If there are six panes of glass in each window of a street containing 564 windows, how many panes of glass are there in the street?

5. A parish has 1,047 houses, and each house contains 7 persons; how many people are there in the parish?

6. What number of pence will pay for 3,269 mackerel at 6*d.* each?

7. A market-gardener planted 639 rows of cabbage-plants, putting 8 in a row; how many were planted?

8. A yard is three feet; how many feet are there in 84 yards? How many feet in 1,760 yards?

9. How many days are there in 3,094 weeks?

10. How many pennies should I require to pay 756 boys 6*d.* each? and how many to pay 3,807 girls 9*d.* each?

11. How many days are there in four years? How many in seven years? How many pence will 3*d.* per day amount to in five years?

12. A shilling is worth 12 pence; how many pence can I get for 36 shillings? how many for 40 shillings? and how many for 762 shillings? and how many for all put together?

13. A nurseryman sold 11 trees each day for 313 days; how many did he sell in all?

14. In each bag there are 8 pebbles; how many pebbles in 596 bags?

15. A garden contains 7 beds of tulips with 144 in each bed, and 9 beds of hyacinths with 157 in each bed; how many flowers are there in the garden?

16. I distributed nuts in a school, giving 375 boys 8 each, and 276 girls 7 each; how many nuts did I give away?

17. A man walks 9 miles per day, except Sundays; how far does he walk in a year?

18. In one acre there are 4,840 sq. yards; how many sq. yards are there in 7 acres?

19. A clerk earns £5 per week; what is his yearly income?

20. A sovereign weighs 123 grains; how many grains will 7 sovereigns weigh?

21. Which is the greater, and by how much, 4,309 × 9, or 5,470 × 8?

22. What is the difference between 5,037 × 7 and 6,207 × 6?

23. Which is the greater, and by how much, 5,707 × 5 or 3,675 × 8?

CHAPTER XV.

LONG MULTIPLICATION.

STEP I.

To multiply by 10, *or any power of* 10.

Example 1.—Multiply 324 by 10, and 3,240 by 10.

$$(1)\quad \begin{array}{r} 324 \\ 10 \\ \hline 3240 \\ \hline\hline \end{array} \qquad\qquad (2)\quad \begin{array}{r} 3240 \\ 10 \\ \hline 32400 \\ \hline\hline \end{array}$$

Call the attention of the class to the fact that we have in the first example the same figures in the product as in the multiplicand, with a cipher on the right-hand side. Hence, to multiply by 10, we have only to place a cipher to the right-hand of the multiplicand.

In the second example we have multiplied the product of the first by 10. In other words we have multiplied by 10 times 10 or 100. Here also we have the same figures with another cipher added. Hence to multiply by 100, add two ciphers to the right hand of the multiplicand.

In the same way to multiply by 1,000 we add three ciphers.

STEP II.

To multiply by factors.

Example 1.—Multiply 4 apples by 6.

4 apples × 6 = 24 apples,
but 4 apples × 2 = 8 apples,
and 8 apples × 3 = 24 apples.

Here we see that to multiply 4 apples by 2, and then the product of this, viz. 8 apples, by 3, produces the same result as 4 apples multiplied by 6, or 3 times 2.

We may therefore multiply by a number or by its factors, and the product will be the same.

Example 2.—Multiply 68 shillings by 12.

We may multiply either by 12 or by its factors, and the product must be the same.

The factors of 12 are 6 and 2, or 4 and 3.

(*a*)	(*b*)	(*c*)
68	68	68
12	6	4
816	408	272
	2	3
	816	816

The product is the same in each case.

Example 3.—How many shillings are there in £52?

There are 20*s.* in one £, therefore in £52 there are 52 times 20 shillings; we therefore multiply 20*s.* by 52, or, which is the same thing, 52 by 20. The factors of 20 are 10 and 2, or 4 and 5.

(*a*)	(*b*)
52	52
2	5
104	260
10	4
1040	1040

In the first case we first multiplied by 2 and then by 10; but to multiply by 10 is simply to add a cipher. Hence to multiply by 20 we may multiply by 2 and add a cipher:

Thus:—
$$\begin{array}{r} 52 \\ 20 \\ \hline 1040 \\ \hline \end{array}$$

In the same way to multiply by 50, multiply by 5 and add a cipher, and to multiply by 500, multiply by 5 and add two ciphers, and so on.

Example 4.—Multiply 4,275 by 500.

$$\begin{array}{r} 4275 \\ 500 \\ \hline 2137500 \\ \hline \end{array}$$

EXERCISES.

1. How many nuts are there in 64 bags, each bag containing 10 nuts; and how many in 76 bags, each containing 100 nuts?

2. 569 men are to receive 100 shillings each; how many shillings will it take to pay the whole?

3. A man uses 48 postage stamps per day; how many does he use in 20 days, in 50 days, and in 300 days?

4. How many days are there in 10 years, in 100 years, and in 1,000 years?

5. If a man can walk 35 miles per day, how far can he walk in 60 days, and how far in 80 days?

6. A shopman sells 136 marbles per week; how many does he sell in 24 weeks, also in 36 weeks?

7. A farmer has 154 sheep; how many has another who has 27 times as many?

8. In one box there are 239 oranges; how many in 500 boxes of the same size?

9. A bag of nuts contained 1,960; how many nuts in 63 such bags?

10. There are 60 pence in a crown; how many pence are there in 549 crowns?

11. A clock strikes 156 times per day; how many times does it strike in one week, in a fortnight, in five weeks, and in seven weeks?

12. Add 1,000 times 79 to 100 times 187.

13. How many fingers have 871 boys and 563 girls together?

14. Find the difference between 1,000 times 58 and 100 times 508.

15. A box of figs contains three dozen; how many figs are there in 160 such boxes?

16. How many shillings are there in £364?

17. A newsman sells 369 papers daily; how many does he sell in 28 days?

18. Find the cost in shillings of 250 yards of velvet at 16 shillings for one yard?

19. If a bird flies 982 yards in a minute, how far does it go in an hour?

20. An ear of corn contains 43 grains; how many grains are there in 396 ears of the same size?

21. How many soldiers are there in 53 companies, if each company is made up of 60 men?

22. I bought 126 baskets of oranges, each containing 54; and 145 baskets, each containing 36; how many oranges did I buy?

23. Find the difference between 18 times 604 and 64 times 180.

24. What is the sum of 8 × 10, 8 × 100, 8 × 1,000, 8 × 10,000?

STEP III.

To multiply by any number.

Example 1.—A cistern supplies 2,409 gallons of water daily, how many will it supply in a year?

There are 365 days in a year; we therefore multiply 2,409 gallons by 365, that is, by 3 hundreds, 6 tens, and 5 units.

We will multiply by each separately:

$$\begin{array}{r} 2409 \\ 5 \\ \hline 12045 \\ \hline\hline \end{array} \qquad \begin{array}{r} 2409 \\ 60 \\ \hline 144540 \\ \hline\hline \end{array} \qquad \begin{array}{r} 2409 \\ 300 \\ \hline 722700 \\ \hline\hline \end{array}$$

And then add the products:

$$\begin{array}{r} 12,045 \\ 144,540 \\ 722,700 \\ \hline 879,285 \\ \hline\hline \end{array}$$

This gives the entire product.

In practice it is not convenient to multiply by th
tens, and hundreds separately, so we arrange as follo

$$\begin{array}{r} 2,409 \\ 365 \\ \hline 12,045 \\ 144,540 \\ 722,700 \\ \hline 879,285 \\ \hline\hline \end{array}$$

The attention of the children must be directed
points. (1) The naughts or ciphers in the second
lowing products are usually omitted, because they
alter the value of the entire product. (2) The mult
is multiplied by each figure of the multiplier—usual
mencing with the units figure—*and the right-hand*
each product is placed in the same column as the multi
other words, when we multiply by tens, the first (righ
figure of the product is placed in the tens column, an
we multiply by hundreds, the first figure (right-h
placed in the hundreds column, and so on for higher n

Example 2.—There are 960 farthings in a £; ho
farthings in £372?

$$\begin{array}{r} 372 \\ 960 \\ \hline 22320 \\ 3348 \\ \hline 357120 \\ \hline\hline \end{array}$$

Note how the cipher is placed.

Example 3.—There are 806 soldiers in one regiment; how many soldiers in 157 such regiments?

$$\begin{array}{r} 157\\ 806\\ \hline 942\\ 1256\\ \hline \underline{\underline{126542}} \end{array}$$

Note the *effect* of the cipher.

EXERCISES.

1. A reading-book has 328 pages; how many pages in 39 of such books?

2. There are 32 lines in the page, and 9 words in a line; how many words in the page?

3. There are 24 hours in a day; how many hours in a year of 365 days?

4. A ton of coals weighs 2,240 lbs.; what is the weight of 17 tons?

5. There are 1,760 yards in a mile, and 36 inches in every yard; how many inches in a mile?

6. What will 8,036 cheeses weigh, if each weighs 57 lbs.?

7. Trees are planted in rows in the form of a square; if there are 65 in each side, how many trees are there in the square?

8. How many pence are there in 78 sovereigns? (One sovereign is worth 240*d.*)

9. How many days are there in 39 years?

10. What must I give for 76 horses at £120 per pair?

11. I counted 105 onions in a row; how many onions are there in 57 such rows?

12. In one copse there are 96 trees, in another 8 times as many as in the first, and in a third 9 times as many as in the second; how many trees in the three copses?

13. Add together 215 dozen and 79 score.

14. I travelled 29 miles an hour for 10 hours, and 37 miles an hour for the next 11 hours; how far did I travel in all?

15. Multiply the sum of 9,160 and 19 by 704.

16. Multiply the sum of 18 and 108 by their difference.

17. A postman delivers 694 letters each morning and 379 each evening, omitting Sundays; how many does he deliver in 17 weeks?

18. There are 24 sheets in a quire, and 20 quires in a ream of paper; how many sheets in 249 reams?

19. How many men are there in an army which consists of 96 regiments of 980 men each?

20. "*Good Words*" has a circulation of 98,600 per month; how many is that in one year; and in 12 years?

21. Multiply the sum of 1,769 and 1,723 by their difference.

22. A book has 192 pages, on each page there are 28 lines, and in each line 11 words; how many words in the book?

23. A terrace consists of 13 houses, each house contains 13 rooms, each room has 2 windows, and each window 9 panes of glass; how many panes of glass are there in all the houses in the terrace?

24. The product of a number multiplied by itself is called its square; what are the squares of 3, 5, 7, 9, 11, 16, 18, 19, and 25?

CHAPTER XVI.

SHORT DIVISION.

As multiplication is a short method of addition, so division is a short method of subtraction.

Example 1.—A man has 44 marbles, which he wishes to divide equally among his 4 sons.

This means that the father gives 11 marbles to each boy; in other words he subtracts 11 from the number he holds, *four times* over.

The father will naturally say there are 4 elevens in 44, and I must therefore give each boy 11.

Suppose now that 40 of the marbles were in 4 bags of ten each; how could the marbles be best divided? Each boy will take one bag of ten and one single.

We show this in figures thus:

	tens.	units.
4)	4	4
	1	1

that is, we first divide the tens by 4, and then the units.

Example 2.—Divide 963 apples between 3 girls, how many will you give to each?

Suppose the apples to be in boxes, 9 boxes of a hundred each, 6 boxes of ten each, and 3 singles over. We must give each girl 3 of the hundreds boxes, 2 of the tens boxes, and one single apple; because,

9 hundreds divided into 3 equal parts = 3 hundred.
6 tens " " 3 " = 2 tens.
3 units " " 3 " = 1 unit.

Each girl then will have 321.

We write the example in figures thus:

	hun.	tens.	units.
3)	9	6	3
	3	2	1

Example 3.—It is required to divide 3 bags of 100 nuts each equally between 6 monkeys.

Here we have 3 bags of one hundred to divide into 6 parts. This we cannot do without taking the nuts from the bags.

If I have to divide 3*s*. between 6 boys, I can only do it by changing the shillings to sixpences or pence: and so in our example we must change the hundreds to tens. 3 hundreds will give us 30 tens, and 30 tens can be divided into

6 equal parts of 5 tens each; hence each monkey should g
5 tens, viz. 50 nuts.

	hun.	tens.	units.
6)	3	0	0
		5	0

In the same way if we cannot equally divide the tens w
change to units.

Example 4.—Divide 2,618 shillings equally among
men. Set the sum down and proceed thus:

	thou.	hun.	tens.	units.
7)	2	6	1	8
		3	7	4

7 will not divide into 2, we therefore change the
thousands to 20 hundreds, and adding the 6 hundreds w
have 26 hundreds.

7 divided into 26 hundreds gives 3, and 5 hundreds ove
Change the 5 hundreds to 50 tens and add the one ten =
51 tens. 7 divided into 51 tens gives 7 and 2 tens, or twent
over, add the 8 units = 28 units. 7 into 28 gives 4.

In practice we forget all about the thousands, hundrec
tens, &c., and say:

7 into 26 = 3 and 5 over. Set down 3 and carry
Bring down the next figure 1 and place it to right of the
making 51. 7 into 51 = 7 and 2 over. Set down 7 a
carry 2. Bring down the 8 to right hand of the 2, maki
28. 7 into 28 = 4.

EXERCISES.

1. (*a*) Into how many *threes* can 15 be divided? Divide
into *three* equal parts; give the number in each part.

(*b*) Into how many groups of *three* can you divide 585?

2. (*a*) How many times is 5 contained in 25?

(*b*) How many times is 5 contained in 2,015?

3. If sugar is *5d.* per lb., how many lbs. can be bought for 1,090*d.*?

4. 798 oranges were distributed among a number of infants; each had 3 oranges; how many infants were there?

5. How many 6-lb. parcels can be made up of 1,260 lbs. of butter?

6. If you take 3 feet to every step, how many steps will you take in going a mile, which is 5,280 feet?

7. 975 trees were planted in 5 rows; how many trees were planted in each row?

8. How many hours will a man be walking 500 miles, at the rate of 4 miles an hour?

9. In 7 bags of nuts there are 18,874; how many nuts in each bag?

10. How many dozen are there in 10,572 oranges?

11. Divide 10,000 marbles equally among 8 boys; how many will each receive?

12. (*a*) How many fours are there in 2,976?
    (*b*) How many fourpenny-pieces are there in 5,180 pence?

13. To how many boys can you give 7 nuts each out of 8,316?

14. 468 passengers were carried in 9 carriages, an equal number in each carriage; what was the number of passengers in each carriage?

15. Divide 352 acres into 8 fields, putting an equal number of acres in each field.

16. Put 19,005 sheep in 5 pens, and an equal number in each pen.

17. (*a*) What number multiplied by 4 will give 20?
    (*b*) What number multiplied by 8 will give 16,176?

18. (*a*) How many fourpenny-pieces can you get for 12 pence?
    (*b*) How many threepenny-pieces can you get for 1,005 pence?
    (*c*) How many fourpenny-pieces can you get for 1,000 pence?

19. A boy has collected 3,063 fourpenny-pieces; how many shillings are they worth?

20. (*a*) What number must be taken 4 times to make 24?
    (*b*) What number must be taken 4 times to make 62,048?

21. What is the half of 12, the fourth part of 28, the eighth of 936, and the tenth of 1,000?

22. How many plums in the sixth part of a basket containing 1,236?

23. What is the ninth part of a flock of sheep consisting of 2,439?

24. Add the fifth of 2,040 to the sixth of 1,644, and subtract the seventh of 2,604 from the sum.

MISCELLANEOUS EXERCISES.

1. From the sum of 186, 1,039, and 75, take 95, and divide the remainder by 5.

2. If a man earns 3*s*. per day, what will he earn in a year, omitting 52 Sundays and two weeks' holiday?

3. Charles has 92 marbles; Henry has 34 less than Charles, and Robert 25 more than Henry; how many marbles in all have the three boys?

4. I have three cisterns filled with water; the first holds 183 gallons, the second 130 gallons, and the third 23 gallons; the water in the three cisterns will exactly fill 6 casks of equal size; how many gallons does each cask hold?

5. A farmer had 1,002 sheep; he sold 94 and placed the remainder in equal numbers in four fields; how many were put in each field?

6. A father had £13,050; he gave £5,019 to his eldest son, and divided the remainder equally among his remaining 3 children; how much did the younger children each receive?

7. A tailor bought two pieces of cloth; the first measured 84 yards, the second 169; he sold 37 yards from the first piece, and 96 from the second; how many suits of clothes, each taking 5 yards, can he make from the remainder?

8. A woman bought 189 eggs on Monday, 215 on Tuesday, and 190 on Wednesday; on Thursday she sold the whole at 9 for a shilling; how many shillings did she receive?

9. A man borrowed £300, and repaid at different times £29, *£70, and £150;* how much does he still owe, and how many

weeks will it take him to pay the remainder at the rate of £8 per week?

10. A gentleman bought land for £5,019; he sold part of it for £3,106, and the remainder for £2,965; how much did he gain by the transaction?

11. A horse and gig cost £185; the horse was worth £120; what was the value of the gig?

12. What is *five* times the difference between one thousand and three and three thousand and one?

13. A gardener picked 35 quarts of strawberries on Monday, 39 on Tuesday, 50 on Wednesday, and 47 on Friday; how many pence did he receive for the whole at 9*d.* per quart?

14. One bed had 205 tulips in it, another 87, and a third 4 times as many as in the first and second together; how many tulips were there in the three beds?

15. There are 127 boys and 139 girls in a school; how many shillings shall I require to give 6*d.* to each?

16. Find the product of the sum and difference of 804 and 369.

17. Find the total cost in shillings of 89 yards at 17*s.* per yard, and 316 yards at 9*s.* per yard.

18. Add 14, 1,004, 4,005; subtract 214 from their sum, and divide the remainder by 7.

19. One farmer sold his 85 turkeys at 9*s.* each, and another sold his 75 at 10*s.* each; what was the difference in the amounts they received?

20. A woman bought 1,004 eggs at a penny each. She sold 527 of them at a penny each and the remainder at 2*d.* each; how many pennies did she gain by the sale?

21. How many sovereigns will buy 179 cows at £37 each, and 207 more at £27 each?

22. How many pennies per week will be collected from a school of 316 boys and 300 girls, if the boys pay 4*d.* and the girls 3*d.* each per week?

23. Divide £400 between two persons, giving one £50 more than the other.

24. An excursion train carries 1,720 passengers; of these 150 are first-class and 470 are second-class; how many third-class passengers are there?

CHAPTER XVII.

EXAMINATION PAPERS.

STANDARD II.

I.

1. In a packet of pins and needles containing 1,040, 568 were needles; how many pins were there?

2. Multiply 6,048 by 7.

3. I want to buy a house for £900. I have already £369, and a friend has promised to lend me £250 more; how much do I still require?

4. Divide 473,828 by 9.

II.

1. Tom has 39 marbles; when he has 45 more, how many will he still be short of 100?

2. Multiply eight thousand four hundred and ninety-one by five hundred and eight.

3. Take 99 bad oranges from a box containing 489, and sell the remainder at 3 for a penny; how many pennies will be received?

4. How many *sevens* are there in four thousand eight hundred and fifty-eight?

III.

1. How much is 2,019 greater than 1,657?

2. At the commencement of a game Tom and Ned had each 37 marbles; Tom won 19; how many had they each at the end of the game?

3. A boy loses 25 minutes every day from school; how many minutes does he lose in 210 days?

4. A gardener gathered 3,744 apples from 8 trees, and from each tree an equal number; how many did he take from each tree?

IV.

1. My son asked me for 50 marbles; I replied that I had not enough by one and a half dozen; how many had I?

2. A boy earns 5*d.* every morning and 4*d.* every afternoon in delivering newspapers; how much does he earn in a year (omit 52 Sundays) ?

3. A man was born in 1836, and lived till he was 39 years of age; in what year did he die ?

4. Divide 228 score into 19 equal parts.

V.

1. How many sheep have been sold out of a flock of 3,016 to leave 1,989 ?

2. How many pence did the conductor of an omnibus take from 102 passengers who paid 2*d.* each, 94 who paid 3*d.* each, and 67 who paid 4*d.* each ?

3. Which is the greater, and by how much, 8 times 729 or 9 times 657 ?

4. Divide 430,724 by 7.

VI.

1. If I have an income of £500 per year, and spend £1 per day, how much do I save in a year of 365 days ?

2. One man can make 1,500 bricks in a day; how long will it take two men to make 27,000 ?

3. Multiply 4,858 by 8, and prove the correctness of your answer by short division.

4. What is the product of 257 and 257 ?

VII.

1. Add 720, 3,047, 9,671, and 212; divide the sum by 6.

2. From 210 score subtract 15 dozen; find the sixth part of the remainder.

3. If 3 men can make 5 pairs of shoes in a day, how long will they be making 25 pairs, and how many pairs will they make in 17 days ?

4. Which is the greater, and by how much, a fifth of 685, or a sixth of 804 ?

VIII.

1. I want 25 more marbles to enable me to give my two sons 55 marbles each; how many marbles have I ?

2. Multiply seven thousand eight hundred and seven by seven hundred and sixty.

3. A man bought 3 boxes of oranges, each containing 388; divide the whole into 6 equal parts.

4. Find the eighth part of 24,376.

IX.

1. How much is seven times 356 greater than eight times 298?

2. A farmer bought a herd of cows for 1,203 sovereigns, and sold it for 1,300 sovereigns; how many sovereigns did he gain?

3. Multiply 34,790 by 9.

4. I have saved 1,016 crocus bulbs; I want as many as will fill 13 beds, with 90 in each; how many more do I require?

X.

1. There are 1,704 children attending a school; 752 are infants and 521 are girls; how many boys are there?

2. I bought 12 books for 56 pence each, and sold them for 60 pence each; what did I gain?

3. Divide an estate of 10,150 acres into 10 equal parts; what will be the amount of rent to be paid for each part at £4 per acre?

4. Divide 394,263 by 9.

CHAPTER XVIII.

LONG DIVISION.

SECTION I.

Example 1.—Divide 1,645 potatoes into 7 equal lots.

This example we can work by short division, and we commence: 7 into 16 is 2 and 2 over. But how did we arrive at that result? This was the process that went on in the mind—"7 times 2 are 14, and 14 subtracted from 16 leaves 2."

This is easy mental work. But suppose we had to work [illegible]is question, divide 164 potatoes into 47 equal lots; it is not so easy to see at once that 47 goes into 164 three times and leaves 23 over. We should in fact have to say mentally 3 times 47 are 141, and 141 subtracted from 164 leaves 23. It is much easier in this case to put down the working at *length*; and this we do, and call the division *Long Division.*

We will work the example by short and by long division and compare the work.

Figuring by Short Division.	*Figuring by Long Division.*
$\begin{array}{r}7)1645\\ \hline 235\\ \hline\end{array}$	$\begin{array}{r}7)1645(235\\ 14\\ \hline 24\\ 21\\ \hline 35\\ 35\\ \hline\end{array}$

Working by Short Division.	*Working by Long Division.*
(1) 7 into 16 is 2 and 2 over, that is, mentally we say, "7 times 2 are 14 and 14 from 16 leaves 2." Set down 2 and carry 2.	(1) 7 will go into 16 *twice*, put 2 in the *quotient*. 2×7 = 14, put the 14 under the 16, draw a line and subtract, 2 remain.
(2) Bring down the 4 to the 2 we carried, making 24. 7 into 24 is 3 and 3 over, set down 3 and carry 3.	(2) Bring down the 4 to the side of the 2, making 24. 7 will go into 24 *three* times, put 3 in the quotient. 3×7 = 21, put the 21 under the 24, draw a line and subtract, 3 remain.
(3) Bring down the 5 to the side of the 3 we carried, making 35. 7 into 35 is 5, set down the 5.	(3) Bring down the 5 to the side of the 3, making 35. 7 will go into 35 *five* times, put 5 in quotient. 5×7 = 35, and 35 subtracted from 35 leaves nothing.

We see that the work and the result are in each case precisely the same.

Long division only differs from short division in t working being shown at length.

The children should work examples with a divisor of o figure only until they are quite familiar with the process.

We then proceed to examples involving greater dil culties.

Example 2.—Divide £99,820 equally among 35 men.

We proceed as in the preceding example:—

```
35)99820(2852
   70
   ---
   298
   280
   ---
    182
    175
    ---
     70
     70
     ==
```

(1) 35 will not divide into 9, we thei fore take the next figure with the fir viz. 99. 35 divides into 99 twice or tı times; place the 2 in the quotient. T 35 × 2 = 70; place the 70 under the : and subtract; the remainder is 29.

(2) Bring down the 8. 35 will go times into 298; place the 8 in the quotiei Then 35 × 8 = 280; place 280 under t 298 and subtract; 18 remains.

(3) Bring down the 2. 35 will go 5 times into 18: place 5 in quotient. Then 35 × 5 = 175. Subtract 1 from 182 and 7 remains.

(4) Bring down the cipher, 35 will go twice into 7(put 2 in the quotient. 35 × 2 = 70. Subtract and nothir remains.

The difficulty which children experience in working lon division sums is in finding the correct quotient. They should at first assist themselves by writing all the products of the divisor into the nine digits in a column in the margin of the slates.

Thus a glance at the column will show them that 70 is the nearest number in the column to 99, which can be subtracted from 99. 35 therefore goes into 99 twice. Again [illegible]80 is the nearest number in the column

35 × 1 = 3
35 × 2 = 7
35 × 3 = 10
35 × 4 = 14
35 × 5 = 17
35 × 6 = 20
35 × 7 = 24
35 × 8 = 28
35 × 9 = 22

from which 298 can be subtracted, hence 35 goes 8 times into 298, and so on.

From a column of products, such as that given above, the children must be made *thoroughly* to understand that the *trial quotient* is:

(1) *Too large*, if, when multiplied into the divisor it gives a product *greater* than the dividend.

(2) *Too small*, if, after being multiplied into the divisor, and subtracted from the dividend, the *remainder* is *not less than* the divisor. Thus:—

Example 3.—Divide 497 by 65.

(1) Let the trial quotient be 8.

$$65 \times 8 = 520$$

65)497(8
520

But 520 is larger than the dividend, hence the quotient is too large.

(2) Let the trial quotient be 6.

$$65 \times 6 = 390$$

65)497(6
390
107

After subtraction, the remainder is not less than the divisor; hence the quotient is too small.

(3) Let the trial quotient be 7.

$$65 \times 7 = 455$$

65)497(7
455
42

This is not greater than the dividend, and, after subtraction, the remainder, 42, is less than the divisor; hence 7 is the correct quotient.

It may often assist in finding the correct quotient to divide the first figure of the divisor into the first figure or figures of the dividend, and take *one less* than the quotient so obtained. Thus, in the example, $49 \div 6 = 8$, *one less* than 8, viz. 7, is the correct quotient.

Section II.

DIVISION BY FACTORS.

We have seen that the product is the same, whether we multiply by a number or by its factors, and so in division the quotient is the same, whether we divide by the factors o a number or by the number itself.

Example 1.—Divide 8,928 pence equally among 2 women.

By Long Division.

24)8928(372
72
172
168
48
48

By Factors. 24 = 6 × 4.

6)8928
4)1488
372

Example 2.—To how many men can I give 24 penc out of 8,949 pence.

By Long Division.

24)8949(372
72
174
168
69
48
21

By Factors.

6)8949
4)1491 + 3 pence
372 + 3 sixpences.

3 sixpences and 3 pence = 21*d*

It must be noted that in this example in dividing by the factors of 24 we first divide the pence into sixpences, and we have 3 pence over. We next divide the sixpences into florins, and the 3 we get over, is 3 sixpences, that is 18*d.*, and this added to the 3*d.* over after dividing by 6, makes 21*d.* over, as in the long division.

Example 3.—Divide 783 by 35.

35 = 7 × 5

7)783
5)111 + 6 units
22 + 1 seven.

Remainder therefore is one 7 + 6 units = 13.

Example 4.—Divide 575 by 100.

100 = 10 × 10   10)575
10)57 + 5
5 + 7 tens

7 tens + 5 units = 75.
Ans. = 5 and 75 over.

NOTE.—To divide by 10 strike off the first figure to the right for the remainder. To divide by 100 strike off two figures, and so on.

Example 5.—Divide 576 by 20.

20 = 2 × 10
10)576   or   2⟍0)57⟍6
2)57 + 6   28 + 1 ten and 6
28 + 1 ten
Ans. = 28 and 16 over.

Example 6.—Divide 4,372 by 200.

2⟍00)43⟍72
21 + 1 hund. and 72

Ans. = 21 and 172 over.

EXERCISES.

1. (*a*) How many times is 20 contained in 60?
(*b*) How many sovereigns are there in 5,620 shillings?

2. How many companies, each containing 65 men, can be formed with 10,010 men?

3. There are 6,255 onions in a garden, planted in rows of 45 each; how many rows are there?

4. If a half-pint cup holds 81 nuts, how many times can the cup be filled from a bag containing 17,901 nuts?

5. How many *score are there* in a flock of 8,960 sheep?

6. How many quires of paper can be made up out of 1,8 sheets? (24 sheets = one quire.)

7. How many hours are there in 15,620 minutes?

8. How many days of 24 hours each are there in 17,6 hours?

9. To how many boys can you give 156 oranges each out 13,728?

10. How many hours will a train be going 504 miles at t rate of 36 miles an hour?

11. (*a*) One factor of 36 is 4, what is the other?

(*b*) One factor of 5,049 is 187, what is the other?

12. The front wheel of a bicycle in turning once goes over distance of 17 feet; how many times will it turn round in goi 5,287 feet?

13. How many parcels containing 165 lbs. each can be ma up out of 4,125 lbs.?

14. The apple-trees in an orchard yield on the average 156 app each; I gather in all 2,028 apples; how many trees have I?

15. (*a*) What number must be multiplied by 6 to give 42?

(*b*) What number must be multiplied by 905 to make 10,860?

16. (*a*) How often can 4 be subtracted from 12?

(*b*) How often can 65 be subtracted from 2,925?

17. Divide £594 equally among 27 persons.

18. How many times can a six-gallon pail be filled from a ca containing 180 gallons? and how many times from a cistern ho ing 8 of such casks?

19. (*a*) What number must be multiplied by 6 to yield 18?

(*b*) What number of pounds must be multiplied by 21 yield £6,741?

20. A commanding officer invading a country requires an ar of 244,000 men; he has already 93,000 men; how many re ments of 755 each must be sent him to complete the number?

21. Find the ninth part of 27, and the thirty-fifth part of 3,18

22. Which is the greater, and by how much, the thirteenth p of 741, or the seventeenth part of 952?

23. 24,864 persons visit the Crystal Palace in one day and ha to return by rail; if each train conveys 672 passengers, how ma

trains will it require? If the first train starts at 7 o'clock, and the others follow at intervals of 5 minutes, at what time will the last train be despatched?

24. A "man-of-war" captured an enemy's ship, valued at £42,036; the officers received one-third of this sum, and the remainder was divided among the crew of 226 men; what did each man receive?

25. £6,756 are required to pay 563 men 4 weeks' wages; what does each man earn per week?

26. Find the smallest number which must be subtracted from 3756 to make it exactly divisible by 297.

27. Find the smallest number which must be added to 3756 to make it exactly divisible by 297.

28. The divisor is 52, the quotient is 12, and the remainder is 10; find the dividend.

29. The dividend is 624 and the quotient is 12; find the divisor.

30. The dividend is 634, the quotient is 12, and the remainder is 10; find the divisor.

PART III.

FOR JUNIOR SCHOOLS—*continued.*

STANDARDS III. AND IV.

THE COMPOUND RULES.

CHAPTER XIX.

IN commencing the Compound Rules it will be well spend the time of the first lesson or two in recapitulat oral exercises, introducing the various coins* in comn use, and showing their relative value.

1. (1) How many halfpence are there in 1*d.*, 3*d.*, 6*d.*, 2
3½*d.*, 5½*d.*, 9½*d.*, 15½*d.* ?

(2) How many farthings are there in ½*d.*, 1½*d.*, 3½*d.*, 4
3¾*d.*, 6¼*d.*, 9½*d.*, 1*s.* ?

(3) How many sixpences are there in 1*s.*, 2*s.*, 3*s.* 6*d.*, 5*s.*
10*s.*, 11*s.* 6*d.* ?

(4) How many fourpences are there in 1*s.*, 1*s.* 8*d.*, 3*s.*
2*s.* 8*d.*, 4*s.*, 5*s.* ?

(5) How many threepences are there in 1*s.*, 1*s.* 6*d.*, 1*s.*
2*s.*, 2*s.* 6*d.*, 3*s.*, 4*s.*, 5*s.* ?

2. (1) How many shillings are there in 10, 15, 20, 50, 35,
100, &c., sixpences ?

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* The legal coins in use are:—the farthing, halfpenny, and penny *copper;* threepenny-piece, fourpenny-piece, sixpence, shilling, florin, crown, and crown, in *silver;* and the half-sovereign and sovereign in The coins of accounts are *pounds, shillings, pence,* and *farthings.*

(2) How many shillings are there in 18, 24, 36, 48, &c., fourpenny-pieces ?

(3) How many shillings are there in 20, 36, 12, 48, 44, &c., threepenny-pieces ?

(4) How many florins are there in 10*s*., 14*s*., 8*s*., 12*s*., 20*s*., 30*s*., 40*s*. ?

(5) How many half-crowns are there in 5*s*., 10*s*., 7*s*. 6*d*., 15*s*., 17*s*. 6*d*.?

3. (1) Required the number of half-sovereigns in £1, £1 10*s*., £2, £4 10*s*., £6 10*s*., £9, £20 10*s*., &c.

(2) How many sovereigns in 3, 8, 10, 15, 14, 20, 30, 45, 70 half-sovereigns ?

(3) Required the number of half-crowns in 3, 4, 5, 6, 7, 9, 12, &c., half-sovereigns.

(4) How many half-sovereigns in 8, 20, 24, 28, &c., half-crowns ?

(5) How many crowns are there in £1, £1 10*s*., £2, £3 10*s*., 15*s*., 10*s*., &c. ?

4. (1) How many sovereigns are there in 12, 20, 24, 28, &c., crowns ?

(2) How many sixpences are there in half-a-guinea, in half-a-crown, in a crown, and in three half-crowns ?

(3) Required the number of threepenny-pieces in a half-crown, in a florin, a crown, and in a half-sovereign.

(4) How many pence are there in 1*s*., in three fourpenny and five threepenny-pieces, and in two florins ?

(5) How many sixpences in a florin, in half-a-crown, in a crown, in a half-sovereign, and in a sovereign ?

5. (1) Find the value of 15, 18, 14, 20, 30, 24, 35, 100, 200 sixpences.

(2) How many sixpences are there in 1*s*. 6*d*., 2*s*. 6*d*., 3*s*. 6*d*., 10*s*. 6*d*., 15*s*., 17*s*. 6*d*., 13*s*. 6*d*., &c. ?

(3) Find the value of 7, 10, 15, 13, 30, 40, 25, 19, 60, 100 half-sovereigns.

(4) How many half-crowns in £1, and in £5 ?

(5) How many half-sovereigns in 40 crowns ?

NOTATION OF MONEY.

In the next place we have to show how the shillings, pence, and farthings are figured, that is, to teach a new notation.

Place on the table a collection of coins used in a containing, say, *two* sovereigns, *nine* shillings, *eight* and *three* farthings. Bring one scholar from the cl direct him to separate these coins into heaps, placi sovereigns in one heap, the shillings in a second, the in a third, and the farthings in a fourth heap; and se direct him to arrange the heaps so as to have the sov on the left, the shillings next on the right, then the and lastly the farthings.

Draw lines between the heaps of coins, thus dividi table into four spaces. Similarly draw four columns black-board and write the names over, thus:—

Fourth Place.	Third Place.	Second Place.	First Place.
Sovereigns or pounds.	**Shillings.**	**Pence.**	**Farthings.**
2	9	8	3

Now how many sovereigns have we in the sov space on the table? We have *two*, we therefore pl figure 2 in the sovereigns column. We have *nine* sh and we write the figure 9 in the shillings column have also *eight* pence and *three* farthings, and we w figures 8 and 3 in the pence and farthings columns tively.

Here, then, we have in our columns, sovereigns place of thousands, shillings in the place of hundred *in the place of* tens, and farthings in the place of uni

A figure placed in the second column is not now *ten* times the same figure placed in the first column, but only *four* times, because four farthings make one penny.

Again, a figure in the shillings column is *twelve* times the same figure placed in the second or pence column, because twelve pence make a shilling.

Lastly, a figure in the fourth or sovereigns column is *twenty* times the same figure placed in the shillings column, because there are twenty shillings in one sovereign.

In writing in figures the number two thousands, nine hundreds, eight tens, and three units, we do not write the names over; but in writing two sovereigns, nine shillings, eight pence, and three farthings, it is necessary to point out which are sovereigns, shillings, &c., and we do this, not by writing the names in full, but by writing over the sovereigns or pounds column the letter £, over the shillings *s.*, and over the pence *d.*[*]

The farthings are not written as farthings with a letter placed over, but as parts of a penny; we write *one* farthing, or one-fourth of a penny, thus, $\frac{1}{4}$ or ¼; *two* farthings, or one-half of a penny, we write $\frac{1}{2}$ or ½; and *three* farthings or three-fourths of a penny, we write $\frac{3}{4}$ or ¾.

Lastly, we discard the use of the lines, but we always separate the pounds from the shillings, and the shillings from the pence by some mark, usually the double comma, thus „: and the sum of *two* pounds *nine* shillings *eight* pence and *three* farthings is written

£ *s.* *d.*
2 „ 9 „ $8\frac{3}{4}$

which is read as two pounds nine shillings and *eightpence three farthings.*

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[*] These letters are the first letters of the Latin words *libra*, *solidus*, and *denarius*, the Latin names of Roman coins or sums of money.

NOTE.—The children must now be exercised in the numeration and notation of money until they can read any sum from the black-board, or write any sum on their slates from dictation.

Before proceeding to the addition and subtraction of sums of money, the money tables must be thoroughly mastered; and many hours may be well spent in giving oral exercises on these tables.

EXAMPLES OF ORAL EXERCISES TO BE GIVEN ON THE MONEY TABLES.

1. How many pence in 16 farthings? 16 marbles at ½*d*. each? 18 oranges at ¼*d*. each? 24 eggs at 1*d*. each? 30 pears at ¼*d*. each? 48 apples at ¼*d*. each?

2. How many pence in half-a-crown? in a crown? in half-a-sovereign? in a sovereign? in three half-crowns?

3. 60*d*. = 50*d*. = 94*d*. =
72*d*. = 100*d*. = 74*d*. =
65*d*. = 25*d*. = 56*d*. =
37*d*. = 44*d*. = 82*d*. =

4. How many shillings are there in a half-sovereign? in a sovereign? in a £5 note? in a £10 note? in £4? in £8? in £9?

5. 40*s*. = 110*s*. = 90*s*. =
60*s*. = 120*s*. = 50*s*. =
36*s*. = 98*s*. = 134*s*. =
42*s*. = 105*s*. = 172*s*. =

6. Find the value of 19 cards at ½*d*. each; 23 more at 1*d*. each; 30 more at 2*d*. each.

7. 8 books at 6*d*. each? 9 at 7*d*. each? 12 at 5*d*. each? 10 at 8*d*. each? 7 at 8*d*. each?

8. How much will 6 boys earn in a day at 7*d*. each? 9 boys at 6*d*. each? 7 boys at 7*d*. each? 9 boys at 8*d*. each? 7 boys at 11*d*. each? 8 boys at 1*s*. each?

9. Find the value of 7 chairs at 3*s*. each; 11 at 11*s*. each; 9 at 6*s*. each; 12 at 12*s*. each.

10. A carpenter earns 9*s.* per day; how much will he earn in 10 days? in 8 days? in 5 days? in 11 days?

11. What does a mason earn per day when he receives £2 2*s.* for 6 days' work? £3 10*s.* for 7 days? £4 4*s.* for 12 days?

12. A boy earns ¾*d.* per hour; he works 12 hours per day; what will he earn in 6 days and in 12 days? A girl earns 5½*d.* per day; what is that per week and per month?

CHAPTER XX.

ADDITION OF MONEY.

A FEW simple examples should be worked practically, and the figuring shown on the black-board.

Example 1.—I have £1 3*s.* 9¼*d.* in one drawer, 16*s.* 2½*d.* in another, and £1 3*s.* 3½*d.* in a third; how much money have I in the three drawers?

Call up one boy to arrange the coins from the first drawer in their proper spaces on the table, and place them in a horizontal line with the sovereign to the left; next the shillings, and then the pence and farthings. Write the figures which represent this line on the black-board in their proper columns, placing pence under pence, shillings under shillings, &c.

£	*s.*	*d.*
1	3	9¼
	16	2½
1	3	3½
3	3	3¼

Let a second boy arrange the contents of the second drawer in a similar manner, making a horizontal line of coins under the first. Write the figures which represent this sum on the black-board. Call up a third boy to perform a similar process with the money in the third drawer, and write the figures.

Now direct a fourth boy to collect together the farthings—we have *five* farthings. The Teacher will change four of these for one penny. This penny we place in the second space with the other pence. The boy will then collect t pence; we have fifteen pence. Change twelve of these f one shilling. Threepence (besides the farthing) are left, a we place the shilling in the shillings space. The shillin collected give us twenty-three. Change twenty of these f a sovereign, three shillings are left. Transfer the sove eign to the £ space, and collect. Our result or *sum* is thr pounds, three shillings, three pence and one farthing. No turn to the black-board.

Draw a line under the figures and commence to add, collect together, the farthings. The sum is five farthing that is, a penny and a farthing. Put down the farthing the farthings column. Transfer the penny to the pen column, and add the pence. The sum is fifteen pence or o shilling and threepence. Put down the three in the pen column. Transfer the shilling to the shillings column, a add. The sum of the shillings column is twenty-three one pound and three shillings. Set down the three in t shillings column. Transfer the pound to the pounds colum and add.

The total sum is thus £3 3*s.* 3¼*d.*, viz. precisely the sa as we saw when we put the coins together on the table, a collected and changed them.

It will save time and ensure accuracy if the children a taught to add the shillings column as in the next example.

Example 2.—A man earns 16*s.* on Monday, 18*s.* Wednesday, 14*s.* on Thursday, and 17*s.* on Saturday; ho much does he earn in the four days?

7 + 4 + 8 + 6 = 25

Set down 5 and carry 2; in other words, set down 5 units of shillings, and carry 2 *tens* of shillings or two *half-sovereigns.*

2 + 1 + 1 + 1 + 1 = 6, viz. 6 ten-shilling pieces or half-sovereigns; that is, 3 sovereigns. Carry these to the pounds column.

	s.
	16
	18
	14
	17
£3	5

The children should continue to work practically till the process is thoroughly understood; and then examples like the following should be given.

EXERCISES.

1. John bought a pair of trousers for 16*s.* 3*d.*, a hat for 5*s.* 9*d.*, and a pair of boots for 6*s.* 7½*d.*; what did he pay for the whole?

2. I have in my purse 2 sovereigns, 2 half-sovereigns, 2 crowns, 2 half-crowns, 2 florins, 2 shillings, and 2 sixpences; how much have I in all?

3. A lady paid the butcher £5 4*s.* 6*d.*, and the baker £2 19*s.* 7*d.*; she had £3 14*s.* 8*d.* left; how much money had she at first?

4. Here is a draper's bill:—One dress 16*s.* 9*d.*, one jacket 15*s.* 7*d.*, one piece of calico 5*s.* 6½*d.*; find the amount.

5. How much must I pay for the following:—Tea 3*s.* 6½*d.*, sugar 4*s.* 9¼*d.*, bacon £1 3*s.* 6*d.*, eggs 15½*d.*, and raisins 10¾*d.*?

6. What do I pay for the following:—A coat £1 16*s.* 4*d.*, a waistcoat 17*s.* 5*d.*, a pair of trousers £1 2*s.* 9*d.*?

7. A tradesman receives from one customer £2 16*s.* 4*d.*, from another £7 14*s.* 6½*d.*, from a third 12*s.* 6*d.*, and from a fourth £1 0*s.* 0½*d.*; what are his total receipts?

8. How much money must a lady give her servant to pay the following amounts:—To the butcher £6 7*s.* 6*d.*, to the draper £16 14*s.* 10*d.*, and to the grocer £9 15*s.* 11¼*d.*?

9. A young man owes for board and lodging £16 18*s.* 6*d.*, to the tailor £11 19*s.* 7*d.*, and to the shoemaker £5 17*s.* 11*d.*; how much money does he require to pay the whole?

10. A girl paid 17*s.* 11*d.* for dress materials, 5*s.* 7*d.* to the dressmaker, and she had £2 19*s.* 11*d.* left; how much money had she at first?

11. A boy in going to a picnic spends in railway fare 1*s.* 11*d.*, in oranges 5*d.*, in ginger-beer 4½*d.*, in dinner 1*s.* 4*d.*, and he takes home 3*s.* 6*d.*; how much money had he when he started?

12. The secretary of a boys' cricket club has £1 19*s.* 8*d.* in hand; can he purchase and pay for the following:—Bats 19*s.* 6*d.*, balls 7*s.* 11*d.*, wickets 6*s.* 9*d.*, and gloves 5*s.* 6*d.*?

13. A farmer bought four cows for £25 13*s*. 6*d*., £18 10*s*. 9*d*., £27 11*s*. 4*d*., and £19 18*s*., and carried home in his purse 5 sovereigns and 5 half-sovereigns; how much money had he before the purchase?

14. I gave a tailor money to pay for a coat £2 12*s*. 6*d*., for a waistcoat 16*s*. 9*d*., and for a hat 15*s*. 6*d*.; he gave me 15*s*. 3*d*. change; what money did I give him?

15. If I spend £129 17*s*. 9*d*. in a year, save £36 15*s*., and give away £9 16*s*. 4*d*., what is my income?

16. I have 15*s*. left after paying away the sums of £2 9*s*. 4*d*., £1 8*s*. 3*d*., and 11*s*. 1¼*d*.; how much money had I at first?

17. A cabinet-maker charges for a sofa—materials £2 19*s*. 4*d*., labour £1 10*s*. 6*d*., and profit 17*s*. 6*d*.; for how much does he sell it?

18. How much will it cost to clothe a boy if his coat costs 19*s*. 3*d*., his trousers 11*s*. 9*d*., his waistcoat 5*s*. 6*d*., his shirt 3*s*. 9*d*., his stockings, collar, and gloves 7*s*. 8½*d*., his boots 6*s*. 9*d*., and his cap 2*s*. 7*d*.?

19. A merchant receives on Monday £49 17*s*. 6*d*., on Tuesday £75 13*s*. 8*d*., on Wednesday £68 15*s*. 4*d*., on Thursday £56 17*s*. 8*d*., on Friday £65 11*s*. 11*d*., and on Saturday £96 19*s*. 8*d*.; what are his week's receipts?

20. A chest of tea was bought for £6 17*s*. 11*d*., and sold at a profit of £1 19*s*. 10*d*.; for how much was the chest sold?

21. A gentleman left by will to his widow £3,472 19*s*. 11*d*.; to each of his two sons £1,050 10*s*. 6*d*.; to each of his two daughters £596 7*s*. 6*d*., and to his servants £100; how much did he leave in all?

22. Add £376 15*s*. 9¼*d*., £109 16*s*. 11½*d*., £350 13*s*. 4¼*d*., £19 0*s*. 10½*d*., and £1,040 15*s*. 7¾*d*.

23. Find the value of £204 13*s*. 9¼*d*. + £760 15*s*. 5¼*d*, + £606 5*s*. 10½*d*. + 17*s*. 11½*d*. + £104 13*s*. 8¾*d*.

24. Find the sum of £3,090 17*s*., £4,069 17*s*. 5*d*., £9,607 19*s*. 9¾*d*., £657 16*s*. 6¼*d*., £864 15*s*. 5¾*d*., and £1,069 13*s*. 8½*d*.

CHAPTER XXI.

SUBTRACTION OF MONEY.

INTRODUCE the subject of Compound Subtraction by easy oral xamples, to be afterwards worked practically.

Thus:—(1) If I spend 1*s.* 4*d.* out of 2*s.* 3*d.* how much hall I have left?

Place 2 shillings and 3 pence on the table, and ask a boy o pay away the 1*s.* 4*d.* from it, and thus elicit from the class hat we must *change* one of the shillings to enable us to make he payment.

(2) I give *half-a-crown* to pay for a cap at 1*s.* 9*d.* what hange ought I to receive?

In this example we must change the half-crown for a hilling, a sixpence, and 12 pennies.

Having established the principle of *changing* money, give nore difficult examples for practical solution, and to be igured on the black-board.

Example 1.—I have two pounds (viz. sovereigns), twelve shillings, three pennies and two farthings in my drawer; from this I have to pay £1 13*s.* 8$\frac{3}{4}$*d.*; how much money shall I have left?

Place the drawer containing the coins on the table, and call upon one scholar to take out £1 13*s.* 8$\frac{3}{4}$*d.* Of course the boy fails to do this, and the class will see that he cannot do it without *changing* some of the coins.

Direct another boy to assort the coins and place them in their proper spaces on the table, as directed in addition of money. Write the figures which represent the sum of money in hand on the black-board, and under it the amount to be subtracted. Call a

£	*s.*	*d.*	
2	12	3$\frac{1}{2}$	minuend
1	13	8$\frac{3}{4}$	subtrahend
	18	6$\frac{3}{4}$	difference
2	12	3$\frac{1}{2}$	proof.

third boy to do the subtraction on the table, and a fourth to write the figures, which show the working, on the black-board.

First take away three farthings; that is impossible, for we have only two farthings. What is to be done? Many boys will at once suggest that we must change one of the pennies to farthings; do this, and we have six farthings; we take three farthings from six farthings, and three farthings remain. Let the sign for three farthings ($\frac{3}{4}$) be written on the black-board under the line.

Secondly, take away eight pennies from two pennies; we must change one of the shillings to pennies, giving us fourteen pennies; take away eight; six pennies remain; place the figure 8 under the line in the pence column.

Thirdly, take away thirteen shillings from eleven shillings. We can't. Change one sovereign to shillings, this gives us $20 + 11 = 31$ shillings. Thirteen from thirty-one leaves eighteen. Set down the 18 in the shillings column.

Lastly, we have to take away one pound, which we can do as we have one sovereign left on the table.

Now if we put back the money we have taken away it is quite clear that we shall make up the sum we had at first. Our original sum from which we had to subtract is called the *minuend*, the sum to be subtracted is the *subtrahend*, and what we have left after subtraction is the *remainder* or *difference*. Hence if we add the subtrahend to the difference we get the minuend.

This gives us a method of showing or *proving* that our working is correct.

When the children are thoroughly conversant with the changing or "borrowing" they should be directed to subtract from the four farthings or twelve pence or twenty shillings *before* they are added to the farthings, pence, or shillings in the minuend. The principle may be shown in this way:

I have a shilling and four pennies in my purse, I wish to y 9*d.* for a railway-ticket; how shall I pay? I shall nd the booking-clerk the shilling (but not the four penes), he will subtract 9*d.* from the shilling and hand me ck 3*d.*, which, added to the 4*d.* I had before, gives me a mainder of 7*d.* after paying my fare.

Now figure this on the black-board: 9*d.* from *l.* we cannot, we therefore change the shilling to *d.*; 9*d.* from 12*d.* leaves 3*d.*, which added to the . gives 7*d.* as the remainder or difference.

s.	*d.*
1	4
	9
	7

Example 2.—Take 3*s.* 6*d.* from £5. Here we have to ce 6*d.* from nothing, and we have also no llings to change. We must change one the sovereigns to 20 shillings, and then e of the shillings (leaving 19 shillings) to pence. We can now subtract the 6*d.* m 12*d.* and the 3 shillings from the 19 illings, and we have 4 sovereigns 16*s.* and 6*d.* left.

£	*s.*	*d.*
5	0	0
	3	6
4	16	6

EXERCISES.

1. I have £3 2*s.* 9*d.* in my purse, if I buy a dress for £1 11*s.* *d.*, what money shall I have left?

2. I go out with £3 6*s.* 8*d.* in my pocket, when I return I have ly £1 13*s.* 4*d.*; how much have I spent?

3. A grocer buys sugar for £6 19*s.* 4*d.* and sells it for £7 3*s.* 1*d.*; hat are his profits?

4. A company of singers visit a town and at their entertainment ke £40 7*s.* 3*d.*; their expenses are £15 13*s.* 9*d.*; what amount they clear as profit?

5. A man bought a cow for £30 and sold it for £27 16*s.* 6*d.*; did gain or lose, and how much?

6. A farmer bought a horse for £50 12*s.* 6*d.* and sold it for 61 13*s.* 8*d.*; did he gain or lose, and how much?

7. A tenant owed me £50 for rent, he paid me £29 13*s.* 4*d.*; ow much has he still to pay?

8. A boy had a sovereign given him; after spending 7*s.* 5½*d.*, ow much had he left?

9. My taxes last year amounted to £17 10*s.*, this year they amount to £1 11*s.* 6*d.* less; what do I pay this year?

10. I start on a tour with £15 in my pocket; if I spend £13 17*s.* 5*d.*, how much money have I left?

11. How much must I add to £4 13*s.* 6*d.* to make it £5?

12. I paid 3*s.* 6½*d.* for a necktie; what change did I receive out of £5?

13. How much must be added to £318 10*s.* to make it £400 0*s.* 8*d.*?

14. I lost £1 16*s.* 4*d.* in selling a cask of butter which cost £5 3*s.* 8*d.*; for how much did I sell the butter?

15. I gave a tradesman a ten-pound note to pay a bill of £3 17*s.* 5*d.*; what change did he give me?

16. By how much is four guineas greater than £3 14*s.* 8½*d.*?

17. When I have spent £89 19*s.* 11*d.* out of £100, what money have I left?

18. I paid £794 10*s.* 6*d.* for two houses; one of them cost £379 12*s.*; what was the price of the other?

19. A man wishes to buy a house for £500; he has only £375 12*s.* 6*d.*; how much more money does he require?

20. What sum must be subtracted from £10,000 to leave £989 16*s.* 4*d.*?

21. How much must be added to £375 6*s.* 8*d.* to make £400?

22. I paid 10½*d.* for writing paper; how much change did I receive out of £1?

23. How much greater is £836 13*s.* 4*d.* than £786 19*s.* 0½*d.*?

24. A merchant bought goods for £757 15*s.* 7*d.*, and sold them for £904 16*s.* 3½*d.*; what was his gain?

EXERCISES IN ADDITION AND SUBTRACTION OF MONEY.

1. A horse-dealer paid £35 for one horse, £45 10*s.* for a second, and for the third 10 guineas more than for the first; what did he pay for the three horses?

2. A merchant bought wine in France for £75 11*s.* 6*d.*; he paid £19 5*s.* 6*d.* duty, and £3 7*s.* 6*d.* carriage; he sold the whole in England for £143 3*s.* 8*d.*; how much did he gain?

3. A lady paid two bills of £3 5s. 4½d., and £4 17s. 2d. out of a en-pound note; what money had she left?

4. A gentleman bought a horse for £50; it cost him 12s. 6d. ailway carriage, £2 8s. for six weeks' stabling, and £2 14s. for six weeks' hay and corn; he afterwards sold the horse for £70; what was his gain?

5. My father gave me a guinea bidding me give my brother a rown, and 3s. 6d. each to my two sisters; what money had I for myself?

6. A butcher bought two sheep for £4 3s. 6d.; he sold one for £2 15s. 6d. and the other for £2 13s. 9d.; how much did he gain?

7. I paid £13 7s. 5d. for 3 parcels; the first cost £3 17s. 6d., he second £4 15s. 7d.; what was the price of the third?

8. A boy saved £2 6s. 9d. last year, and £4 7s. 6d. this year; how much more money does he want to make £15?

9. A farmer paid £215 6s. 8d. for one flock of sheep, and £300 10s. for another; how much money has he left out of £600?

10. A girl went to a draper's with £5; she bought a dress for 17s. 6d., and a jacket for 16s. 5d.; what sum had she left?

11. A boy had £6 3s. 11d. in the savings bank; he took out £1 5s. 7d. at one time, and £2 14s. 3d. at another; how much has he still in the bank?

12. I have the following bills to pay: the butcher £3 7s. 5½d., the grocer £2 19s. 4d., the baker £1 4s. 11d., the milkman 6s. 11d., and the butterman £1 19s. 9d.; how much money shall I have left out of £12?

13. A house and furniture were bought for £797 10s.; the price of the furniture was £203 12s. 6d., what was the price of the house?

14. What was the cost price of a chest of tea when it sold for £7 3s. 9d., giving a profit of £1 16s. 1d.?

15. How much must be added to £474 3s. 6d. and £219 6s. 5d. to make £943 1s. 11d.?

16. A man has deposited in a bank the following sums of money: £694 10s., £327 15s. 9d., £892 16s. 5d., and £547 6s. 8d.; how much more will he have to deposit before he has £5,000 in the bank?

17. Find the value of £64 17*s.* 5½*d.*+£82 13*s.* 10¼*d.*+£72 5*s.* 6*d.* —£127 16*s.* 11¾*d.*

18. Find the value of £65 17*s.* 3*d.*—£18 3*s.* 11*d.*+£27 3*s.* 4*d.*— £15 3*s.* 4*d.*

CHAPTER XXII.

REDUCTION.*

THE children have already learnt how to change coins of one denomination to another: for instance, pence to shillings and shillings to pence, shillings to pounds and pounds to shillings, &c., but it will perhaps be best to introduce the subject of "Reduction" somewhat as follows:—

I wish to give twelve boys 2*d.* each; I have a florin but no pennies; what must I do to enable me to give the pence? I must *change* the 2*s.* to 24 pennies, and then I can give each boy two.

Instead of the word change, we use the word REDUCE, and I might have said, "I must *reduce* the 2*s.* to 24 pennies."

If I wish to give a halfpenny each to 48 boys out of the florin, I must again change the pennies to halfpennies; here I have still further *reduced.*

Again, I have taken this morning 240 pence as school fees. I don't want to carry so many coppers home with me, will some boy change them for me? What will the boy change the 240 pennies for? For 20 shillings, and the 20 shillings he may again change to one sovereign. This changing is also called Reduction. I may have said to the boy, "Go and get 240 pennies reduced to 20 shillings, and then the 20 shillings to one sovereign."

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* Only such simple reduction is here contemplated as will enable the scholars to answer such questions as:—"Add together 100 shillings, 100 pence, and 100 farthings." "From 1,000 shillings subtract 1,000 farthings." "Take 1,000 pence from £3 7*s.* 6*d.*," which are often given in the examination of Standard III.

Let us work these examples on the black-board. In the first case I have to reduce 2*s.* first to pennies and then to halfpennies.

There are 12 pennies to every shilling, and I must therefore multiply the 2 shillings by twelve, this gives us 24 pence. There are two halfpennies in a penny, and I therefore multiply the 24 pence by 2, and this gives us 48 halfpence.

$$\begin{array}{rl} 2 & \text{shillings} \\ 12 & \\ \hline 24 & \text{pence} \\ 2 & \\ \hline 48 & \text{halfpence.} \\ \hline\hline \end{array}$$

In the second case I have to reduce 240 pence to shillings, and then to pounds. 12 of the pennies make up one shilling, I must therefore divide by 12 to find the number of shillings, and this division gives me 20 shillings.

$$\begin{array}{r@{)}rl} 12 & 240 & \text{pence} \\ \cline{2-2} 20 & 20 & \text{shillings} \\ \cline{2-2} & 1 & \text{pound.} \\ \cline{2-2}\cline{2-2} \end{array}$$

20 shillings go to every pound, I therefore divide by 20 to reduce to pounds, and the result is one pound.

We have already, in subtraction, been working the first kind of reduction, and in addition the second kind of reduction.

For when we borrowed £1 we reduced it to 20 shillings, and when we borrowed 1*s.* we reduced it to 12 pence. And so in addition, the sum of the farthings column we reduced to pence, the sum of the pence column to shillings, and the sum of the shillings column to pounds.

The first of these kinds of reduction is called *Descending* Reduction, because we reduce coins of a *higher* value to coins of a *lower* value. The second is called *Ascending* Reduction, because we reduce coins of a *lower* to coins of a *higher* value.

Example 1. (Descending Reduction.)—Reduce £1
to pence.

1000 pounds
20
20000 shillings
12
Ans. 240000 pence.

Example 2.—Reduce 4*s.* $9\frac{3}{4}$*d.* to farthings.

s.	*d.*
4	$9\frac{3}{4}$
12	
57	pence
4	
Ans. 231	farthings.

In Example 2, after changing the 4*s.* to 48 pence by m
plying by 12, we add the 9*d.*, making 57; then, ha
reduced the 57 pence to 228 farthings, we add in t
farthings, making 231 farthings.

Example 3. (Ascending Reduction.)—Reduce 4(
farthings to pounds.

4)40427 farthings
12)10106 pence and three farthings ove
20)842 shillings and 2*d.* over
42 £ and two shillings over.

Ans. = £42 2*s.* $2\frac{3}{4}$*d.*

Example 4.—From 1,000 farthings subtract 15*s.* 3

4)1000 farthings
12)250 pence
20)20 sh. and 10*d.* over.
= £1 0*s.* 10*d.*

	£	*s.*	*d.*
	1	0	10
		15	$3\frac{1}{2}$
Ans.		5	$6\frac{1}{2}$

EXERCISES.

1. How many pence are there in 5 shillings, in 15 shillings, in 12 shillings, and in 315 shillings?

2. How many shillings are there in 36 pence, in 48 pence, in 96 pence, and in 720 pence?

3. How many shillings are there in £5, in £10, and in £165?

4. How many pounds are equal in value to 20*s.*, to 35*s.*, to 65*s.*, to 140*s.*, and to 3,010 shillings?

5. How many pounds are equal to 156*s.* + 137*s.*?

6. How many shillings are equal to £35 + £217 + £40?

7. How many sixpences are there in 45*s.* + 136*s.* + 250*s.*?

8. How many pence are there in 15 + 14 + 147 + 139 sixpences?

9. How many pence are there in 25 pence + 106 shillings + 17 sixpences?

10. How many shillings in 100 sixpences + 250 sixpences + 730 sixpences?

11. How many shillings are there in 24 sixpences + 132 pence + 85 sovereigns?

12. Find the number of pounds in 220 shillings + 840 sixpences and 240 pence.

13. Reduce 316 shillings and 5 pence to pence.

14. Add together 27, 84, 320, and 165 farthings, and give the answer in pence.

15. How many half-crowns in £1, in £3, in £5, and in £100?

16. Add together 70 shillings, 32 half-crowns, and 132 pence.

17. One bag contains 75 shillings, a second £75, and a third 75 pence; what amount of money is there in the three bags?

18. How many shillings are equal in value to 12 fourpences, and also to 135 fourpences?

19. Subtract 100 threepenny-pieces from 100 fourpenny-pieces.

20. Take 215 pence from 16 half-crowns.

21. What is the difference in value between 360 half-crowns and 50 sovereigns?

22. From 1,000 farthings subtract 100 pence.

23. Take 100 pence from £100.

24. Take 1,000 farthings from £3 0*s.* 11*d.*

25. From 1,000 farthings subtract 16s. 9¾d.

26. How many times can you subtract 3s. 5½d. from 13s. 10

27. How much greater is £150 than 150 farthings?

28. What is the difference between 27 half-crowns and 5 farthings?

29. Which is the greater, and by how much, 1,040 threepen pieces or 1,000 fourpenny-pieces?

30. What remainder will you have after subtracting £1 5s. 9 three times from £5 3s. 1d.?

CHAPTER XXIII.

EXAMINATION PAPERS.

STANDARD III.

I.

1. From £1 17s. 5d. subtract 19s. 5½d.

2. A bag of nuts contains 10,000; how many smaller b each containing 36 can be filled from it, and how many n remain towards filling another bag?

3. I have £316 8s. 4½d. in the bank; how much more mus deposit to make the sum £500?

4. I paid 3s. 4½d. for cheese, 2s. 7d. for tea, 1s. 9d. for sug and 15s. 3¼d. for bacon; what sum have I left out of £5?

II.

1. Take £3 17s. 2¼d. from £5 2s. 3¾d.

2. If a tank holds 3,266 gallons of water, how long wil household be consuming it at the rate of 23 gallons per day?

3. After paying two bills, one of £1 8s. 5d. and another £2 19s. 6d., I have £5 19s. 10d. left in my purse; how m *money* had I at first?

4. A dealer gained £15 17*s.* 9*d.* by selling two cows for £70, what did they cost him ?

III.

1. What is the difference between £471 13*s.* 5*d.* and £470 19*s.* 6½*d.* ?

2. If 1,591 trees are planted in 37 rows, how many trees will there be in each row ? If 9 trees from every row die, how many will be left ?

3. I spent in one day 15*s.* 9*d.* railway fare, 3*s.* 6*d.* for dinner, 1*s.* 9*d.* for tea, 2*s.* 9*d.* cab fare, and bought goods to the amount of £3 18*s.* 4*d.*; what sum of money did I spend, and what remained out of £7 ?

4. Add together £50, 50 crowns, 50 shillings, 50 sixpences, and 50 pence.

IV.

1. Subtract £1,201 17*s.* 2*d.* from £5,004 3*s.* 1*d.*

2. How many baskets of oranges, of 48 each, can be filled from 8 boxes, each containing 240 oranges ?

3. A man earns in four different weeks, £3 1*s.* 8*d.*, £4 9*s.* 3*d.*, £2 16*s.* 8*d.*, and £3 19*s.* 4*d.*; if he spends £3 per week, how much does he save in the four weeks ?

4. What sum should be added to £1 3*s.* 4*d.*+£4 19*s.* 7*d.*+12*s.* 9*d.*+1*s.* 8½*d.* to make it £10 10*s.*

V.

1. Find the value of £47 13*s.* 9¼*d.*+£26 15*s.* 0¾*d.*+£19 0*s.* 6½*d.* less £32 17*s.* 9*d.*

2. Divide 14,352 pence equally among 48 persons.

3. I have saved £39 18*s.* 6*d.*; how much more must I save to make £50 ?

4. A waistcoat cost 15*s.* 4*d.*, a pair of trousers 7*s.* 6*d.* more than the waistcoat, and a coat as much as the waistcoat and trousers together; what was the cost of the suit ?

VI.

1. Take £27 15*s.* 2*d.* from £83 5*s.* 6*d.* three times.

2. Divide a piece of land containing 1,016 acres into 4 equal

parts; then divide each part into 127 gardens of equal size, and say how many acres there will be in each garden.

3. If £3 7*s.* 10*d.* be added to itself three times, by how much will the sum exceed £10?

4. A farmer bought 3 cows for £66; after keeping them a month he sold them for £70; during this month the milk of each cow sold for 3 guineas, and the cost of keep for each was 2 guineas; what did the farmer gain?

VII.

1. Take £27 15*s.* 2*d.* from the sum of £190 16*s.* 3*d.* and £1,000 0*s.* 10¼*d.*

2. Eight boys enter into partnership with their marbles, agreeing to divide equally the marbles they win; they began play with 256, and won 128; what was each boy's share?

3. What is the difference between £6 and 6*s.*, and also between £1,000 and 1,000 shillings?

4. For what must I sell a horse which cost me £85 to make a profit of £10, after keeping it for 15 days at the rate of 2*s.* per day?

VIII.

1. What is the difference between £46 19*s.* 4*d.* added to itself, and £39 2*s.* 5*d.* added to itself?

2. There are 4,840 yards in an acre; divide two acres into 50 equal plots of building land; how many yards will each plot contain?

3. What is the value in shillings of 16 half-crowns + 4 crowns + 20 threepences?

4. A builder undertakes to build a cottage for £250; when it is half finished the builder finds the cost of the half to have been £156. If the second half cost as much as the first, how much will the builder lose by the transaction?

IX.

1. From £100 subtract £89 1*s.* 1¼*d.*

2. A twopenny paper of pins contains 125; how many such papers can be made up out of a packet of 2,125?

3. A man's income in one year was £315, and his expenses

£206 10*s.* 9*d.*; the next year his income was £406 and his expenses £294 18*s.* 9*d.*; how much does the man save in the two years?

4. An apple-tree has 1,392 apples on it, 196 fall off and are sold at 4 a penny, the remainder are sold at 2 a penny; what sum was received for the whole?

X.

1. From 1,000 pence subtract 1,000 farthings.

2. If 35 persons can be conveyed in one carriage, how many carriages will be required to convey 5,040 persons?

3. A grocer bought four casks of sugar for £3 14*s.*, £3 17*s.*, £3 12*s.*, and £3 19*s.* 5*d.* respectively; he sold the whole at a loss of £1 19*s.* 4*d.*; for how much did he sell?

4. What does a pudding cost which has in it eggs to the value of 4¾*d.*, flour 5¼*d.*, raisins 11¾*d.*, currants 8½*d.*, and suet 5½*d.*?

XI.

1. Add the difference between £100 and 100 shillings to the difference between 100 shillings and 100 pence.

2. There are 3,340,000 people in London; how many houses will they require if 20 people live in 3 houses?

3. 12,500 people are admitted to a fête held in a field. If the entrance fee is 6*d.*, how much money is taken? If the expenses are 10 guineas, what are the profits?

4. I gave a boy a sovereign to get change; he lost half-a-crown, a sixpence, and a fourpenny-piece; what did he bring back?

XII.

1. After subtracting £5 11*s.* 11*d.* three times from £17 2*s.* 1½*d.* what is the remainder?

2. An army of 120,250 men is landed in an enemy's country by means of 650 boats each making 5 journeys; how many men were conveyed by each boat?

3. Three houses are bought for £800; one is sold for £120 15*s.* 3*d.*, and a second for £317 9*s.* 6*d.*; for how much must the third be sold so as to avoid any loss?

4. John has in his money-box 2 crowns, 3 half-crowns, 15 shillings, 13 sixpences, and 16 pence; William has in his box 6

half-crowns, 13 shillings, 19 sixpences, 4 threepenny-pieces, and 5 pence; which box contains the most money and by how much?

XIII.

1. Find the value of £147 2*s.* 11½*d.* + £6,047 3*s.* 9¾*d.* + £2,600 3*s.* 11¼*d.* + £27 16*s.* 9½*d.* + £720 12*s.* 5¼*d.* — £5,009 19*s.* 6½*d.*

2. How many 53-lb. parcels can be made out of 7 casks of sugar each containing 371 lbs.?

3. A labourer earns 18*s.* per week for three weeks; he saves 2*s.* 6*d.* the first week, 3*s.* the second week, and 4*s.* 6*d.* the third; what does he spend, and what does he save in the three weeks?

4. A farmer sold a horse for £45, and a cow for £25 10*s.* He bought with the money a cart for £16 10*s.*, and paid a bill of £15 13*s.* 6*d.*; what money did he take home?

XIV.

1. Add £100, 100*s.*, 100*d.*, and 100 farthings, and subtract 10,000 farthings from the sum.

2. Four coal-trains, each consisting of 19 equal-sized trucks, carry 684 tons of coals; how many tons of coals are there in each truck?

3. A woman carrying 304 eggs to market broke 48; how much did she gain if she bought the whole at 1*d.* each, and sold what remained at 2*d.* each?

4. I have 12*s.* 6*d.* left after paying the following bills: Butcher, 18*s.* 9*d.*, grocer £2 5*s.*, baker £1 10*s.* 5¾*d.*, fishmonger 8*s.* 5*d.*, and the milkman 5*s.* 9*d.*; what money did I take with me?

CHAPTER XXIV.

MULTIPLICATION.

Multiplication of money, like simple multiplication, is simply a short method of adding equal quantities.

Example 1.—A man earns £1 6*s.* 8*d.* per day, how much does he earn in a week?

By Addition.

£	s.	d.
1	6	8
1	6	8
1	6	8
1	6	8
1	6	8
1	6	8
£8	0	0

By Multiplication.

£	s.	d.
1	6	8
		6
£8	0	0

The multiplier is always placed under the pence column, and as in simple multiplication we commence to multiply the figure on the right-hand side.

8*d.* × 6 = 48*d.* = 4*s.* Set down 0 in pence column, and carry 4 to be presently added to the shillings column.

6*s.* × 6 = 36*s.* 36*s.* + 4*s.* = 40*s.* = £2. Set down 0 in the shillings column, and carry 2 to be presently added to the £'s column.

£1 × 6 = £6. £6 + £2 = £8. Set down 8 in the £'s column.

Example 2.—A grocer bought nine chests of tea at £3 16*s.* 8½*d.* per chest; what did he pay for the whole?

£	s.	d.
3	16	8½
		9
34	10	4½

2 far. × 9 = 18 far. = 4½*d.* Set down ½*d.* and carry 4.

8*d.* × 9 = 72*d.* 72*d.* + 4*d.* = 76*d.* = 6*s.* and 4*d.* Set down 4*d.* in pence column and carry 6*s.*

16*s.* × 9 = 144*s.* 144*s.* + 6*s.* = 150*s.* 150*s.* = £7 + 10*s.* Set down 10*s.* in the shillings column and carry £7.

£3 × 9 = £27. £27 + £7 = £34. Set down £34 in the £'s column.

Example 3.—What is the value of 54 tons o at £1 17*s.* 5*d.* per ton?

It is not easy to multiply by 54 in one line, we th multiply by the factors of 54, viz. 6 and 9.

	£	*s.*	*d.*		
54 = 6 × 9	1	17	5	value of 1 ton	
			6		
	11	4	6	„	6 tons
			9		
	101	0	6	„	54 tons.

The value of each product should *always* be written side.

Example 4.—If one sheep is worth £2 7*s.* 9*d.*, w the value of 79 sheep?

In this example we have to multiply by 79, but 79 factors. We must therefore multiply first by a n near to 79 which can be factored, and then find the v the remainder. Thus 8 × 9 = 72. So we find the v 72 sheep and then of the remaining 7 sheep, and add, th

£	*s.*	*d.*			
2	7	9	val. of 1 sheep		
		8			
19	2	0	„	8	„
		9			
171	18	0	„	72	„

£	*s.*	*d.*		
2	7	9	val. of 1	
		7		
16	14	3	„	7

£	*s.*	*d.*			
171	18	0	val. of	72	sheep
16	14	3	„	7	„
188	12	3	„	79	„

rtens the number of figures to be written if we by 7 without making a separate sum. Thus :—

£	s.	d.		
2	7	9	× 7	
		8		
19	2	0	value of 8 sheep	
		9		
171	18	0	„	72 „
16	14	3	„	7 „
188	12	3	„	79 „

ple 5.—What is the value of 432 sacks of wheat ck is worth £2 *7s.* 9½*d.*?

is example we have to multiply by 432. Now × 9 × 8, and we may multiply by these factors in n, thus :—

£	s.	d.		
2	7	9½	value of 1 sack	
		6		
14	6	9	„	6 sacks
		9		
129	0	9	„	54 „
		8		
1032	6	0	„	432 „

not easy, however, to find the factors of high , and the following method will be found more nt when the multiplier exceeds 150.

$$432 = 400 + 30 + 2$$

e find the value of these separately, and then add ucts together.

1st.—400 sacks at £2 7*s.* 9½*d.*

$$400 = 10 \times 10 \times 4$$

£	*s.*	*d.*	
2	7	9½	
		10	
23	17	11	value of 10 sacks
		10	
238	19	2	„ 100 „
		4	
955	16	8	„ 400 „

2nd.—30 sacks at £2 7*s.* 9½*d.*

£	*s.*	*d.*	
2	7	9½	
		10	
23	17	11	value of 10 sacks
		3	
71	13	9	„ 30 „

3rd.—2 sacks at £2 7*s.* 9½*d.*

£	*s.*	*d.*	
2	7	9½	
		2	
4	15	7	value of 2 sacks

£	*s.*	*d.*	
955	16	8	value of 400 sacks
71	13	9	„ 30 „
4	15	7	„ 2 „
1032	6	0	„ 432 „

It will be seen that in finding the value of 400 sacks, an the value of 30 sacks, we in each case first found the val 10 sacks, that is, we found the value of 10 sacks twice c *By placing* the multipliers for 30 and 2 at the side o tiplicand we avoid this, and considerably shorten the ber of figures to be written. Thus:—

£	s.	d.		
2	7	9½	(2)	value of 1 sack
		10		
23	17	11	(3)	„ 10 sacks
		10		
238	19	2		„ 100 „
		4		
955	16	8		„ 400 „
71	13	9		„ 30 „
4	15	7		„ 2 „
1032	6	0		„ 432 „

OTE 1.—The teacher should strive to give the children an intelligent knowledge of the processes above described, and practice should be given in short methods of obtaining the required products, as a test both of the intelligence of the scholars and the accuracy of their work.

Example 6.—Multiply £4 3*s.* 7½*d.* by 399.

It is much easier in this example to multiply by 400 and subtract one, than to multiply by 399.

1st Method (multiplying by 399).

£	s.	d.		
4	3	7½	(9)	
		10		
41	16	3	(9)	10 times the multiplicand
		10		
418	2	6		100 „ „
		3		
1254	7	6		300 „ „
376	6	3		90 „ „
37	12	7½		9 „ „
1668	6	4½		*399* „ „

2nd Method (multiplying by 400 and subtracting or

£	s.	d.	
4	3	7½	(1)
		10	
41	16	3	10 times the multiplicand
		10	
418	2	6	100 ,, ,,
		4	
1672	10	0	400 ,, ,,
4	3	7½	1 ,, ,,
1668	6	4½	399 ,, ,,

Note 2.—In multiplying by 12 call the pence shill set down in the pence column the pence derived the farthings (if any) and carry the shillings.

Thus :—

£	s.	d.
	10	9¼
		12
6	9	3

12 farthings = 3*d.* Set down the 3*d.* in the column and carry 9.

Of course the reason for this is that 9 × 12*d.* is the sa 12*d.* × 9 or 1*s.* × 9 = 9*s.*

Note 3.—In multiplying the shillings by 10 call the lings half-sovereigns, and reduce to sovereign dividing by 2.

Thus :—

£	s.	d.
	13	7½
		10
6	16	3

0 farthings = 5*d.*
d. × 10 = 70*d.* 70*d.* + 5*d.* = 75*d.* = 6*s.* 3*d.* Set down 3*d.*
carry 6*s.* 13 half-sovereigns = £6 10*s.*, and £6 10*s.* +
= £6 16*s.* Set down 16*s.* and carry £6 to the £'s
umn.

TE 4.—In multiplying the shillings by any number except 10 proceed thus :—

£	*s.*	*d.*
	18	8
		8
7	9	4

d. × 8 = 64*d.* = 5*s.* 4*d.* Set down 4*d.* and carry 5*s.*
× 8 = 64*s.*, 64*s.* + 5*s.* = 69*s.* Set down 9*s.* and carry
viz. 6 half-sovereigns; 1 × 8 = 8, viz. 8 half-sovereigns; and
half-sovereigns + 8 half-sovereigns = 14 half-sovereigns,
£7.

EXERCISES IN MULTIPLICATION OF MONEY.

1. Find the cost of 7 tons of coals at £1 6*s.* 6*d.* per ton.
2. If I spend £245 10*s.* 6*d.* in a year, how much shall I spend 6 years ?
3. If a boy earns 1*s.* 6*d.* a day, what does he earn in a week ?
4. What will be the cost of 4 dinners at 3*s.* 9*d.* each ?
5. If 1 lb. of tea cost 2*s.* 11*d.*, what will 9 lbs. cost ?
6. A lb. of butter costs 1*s.* 11½*d.*, what will 11 lbs. cost ?
7. Find the value of 16 yards of calico at 5½*d.* per yard.
8. Find the value of 9 boxes of raisins when each box cost £2 19*s.* 6½*d.*
9. What is the cost of 12 sacks of potatoes at 16*s.* 7*d.* per sack ?
10. Cabbages are 1½*d.* each ; what is that for 4 dozen ?
11. What is the price of 8 yards of ribbon at 1*s.* 7½*d.* per yard ?
12. Find the value of 12 sheep at £2 5*s.* 10¾*d.* each.
13. A man earns £1 10*s.* 6*d.* in a week; what does he earn in 11 weeks ?

14. What will 8 bushels of beans cost at 7*s.* 6½*d.* per bushel?

15. What money will pay 18 men's weekly wages at 18*s.* 6 a week?

16. What money does a master require to pay 7 men for 6 da at 3*s.* 9*d.* a day?

17. A farmer sold 84 sheep at £2 1*s.* 6*d.* each; what money d he receive?

18. Find the rent of a house and garden for 39 weeks at 13*s.* 6 per week.

19. What will a man earn in 97 days if he gets 4*s.* 6½*d.* a day

20. Find the cost of 27,000 cubic feet of gas at 4*s.* 8*d.* p thousand.

21. Find the value of 7 dozen of wine at 3*s.* 6*d.* per bottle.

22. A gentleman bought 250 acres of land at £76 8*s.* 6*d.* p acre; find the total cost.

23. What will be the cost of 215 gallons of wine at £1 13*s.* 6½ per gallon?

24. Calculate the amount of wages earned by a man in 17 weeks at £2 9*s.* 10*d.* per week.

25. A tradesman makes a profit of £12 19*s.* 3*d.* a week; wh will he gain in a year? (52 weeks.)

26. What must I pay a butcher for 37 lbs. of beef at 10¾ per lb.?

27. I spend daily 12*s.* 9*d.*; what will this amount to in a year (365 days.)

28. What will be my income for a year at £1 3*s.* 9*d.* pe day? (313 days.)

29. How much rent has a farmer paid in 19 years, who ha paid £156 12*s.* 9*d.* a year?

30. What will the fees of 56 pupils amount to in a year, at th rate of 29*s.* 6*d.* each quarter?

31. Find the cost of 17 dozen oranges at 1½*d.* each.

32. A boy earns 1*s.* 4*d.* a day, and his father 5 times as much what amount do they together earn in a year? (313 days.)

33. Find the cost of material to make 5 suits of clothes, each suit takes 5 yards at 17*s.* 3*d.* per yard.

34. What must I pay for a library of 356 books, if each boo *costs, on* an average, 3*s.* 7*d.*?

35. If 3 thousand feet of gas are consumed in a day, what will this cost in a year at 3*s*. 7*d*. per thousand feet ?

36. Find the value of 25 dozen cocoa-nuts at 4*s*. 5*d*. a dozen.

37. What will 16 dozen cocoa-nuts cost at 3¾*d*. each ?

38. Calculate the cost of 615 lbs. of tea at 3*s*. 7*d*. per lb.

39. A man earns £270 a year; he pays 9*d*. in the £ as taxes; by how much is his income lessened ?

40. What should a farmer receive for 319 cows, each being worth £31 18*s*. 6*d*. ?

41. Find how much a boy earns in 313 days who gets 7½*d*. a day ?

42. What sum of money is 569 times as great as £2 19*s*. 7*d*. ?

43. What will be the cost of 350 pairs of kid gloves at 2*s*. 9*d*. a pair ?

44. What must I pay a mechanic for working 237 hours at 6½*d*. per hour ?

45. What will a farmer receive for 7 score sheep at £2 12*s*. 3*d*. each ?

46. How much will a draper charge for 137 yards of calico at 4¼*d*. per yard ?

47. Find the wages of 36 men for 5 days at 2*s*. 5*d*. each per day.

48. What money shall I require to pay for 17 railway shares at £83 11*s*. 6*d*. per share ?

49. What will be the income-tax on £156 at 5*d*. in the £ ?

50. If I give away 8*d*. out of every £ of my yearly income, and I have £250 a year, what amount do I give away ?

CHAPTER XXV.

DIVISION.

DIVISION of money, like simple division, is a short method of working subtraction.

Example 1.—To how many boys can I give 4*d.* out of 1*s.* 4*d.*?

By Subtraction.

$$\begin{array}{rrl} s. & d. & \\ 1 & 4 & \\ & 4 & \text{1st boy} \\ \hline 1 & 0 & \text{remainder} \\ & 4 & \text{2nd boy} \\ \hline & 8 & \text{remainder} \\ & 4 & \text{3rd boy} \\ \hline & 4 & \text{remainder} \\ & 4 & \text{4th boy} \\ \hline\hline \end{array}$$

Ans. 4 boys.

By Division.

$$\begin{array}{r} 4d.\overline{)16d.} \\ \underline{\underline{4}} \end{array}$$

We cannot divide 4*d.* into 1*s.* without reducing to pence. $12d. + 4d. = 16d.$ $16d. \div 4d. = 4$.

NOTE.—We can divide 16*d.* by 4*d.*, because this expression means, into how many times 4*d.* can we divide 16*d.* But we cannot multiply 16*d.* by 4*d.*, because that would mean 16*d.* taken 4*d.* times, which is absurd.

Example 2.—Divide £11 16*s.* $6\frac{3}{4}d.$ equally among 5 persons.

$$\begin{array}{rrrr} & \pounds & s. & d. \\ 5) & 11 & 16 & 6\frac{3}{4} \\ \hline & 2 & 7 & 3\frac{3}{4} \\ \hline\hline \end{array}$$

$\pounds 11 \div 5 = \pounds 2$ and £1 over: set down £2 and carry £1.

$\pounds 1 = 20s.$, and $20s. + 16s. = 36s.$

$36s. \div 5 = 7s.$ and 1*s.* over: set down 7*s.* and carry 1*s.*

$1s. = 12d.$, and $12d. + 6d. = 18d.$

$18d. \div 5 = 3d.$ and 3*d.* over: set down 3*d.* and carry 3*d.*

$3d. = 12$ farthings, and 12 farthings + 3 farthings = 15 farthings.

15 farthings ÷ 5 = 3 farthings $= \frac{3}{4}d.$: set down $\frac{3}{4}d.$

Example 3.—A man earns £67 4*s.* 9*d.* in 36 weeks, how much does he earn per week?

In this example we have to divide by 36; the factors of

36 are 6 and 6, or 12 and 3; and we may divide by these factors as in simple division, thus:—

$$\begin{array}{rrrr} & \pounds & s. & d. \\ 6) & 67 & 4 & 9 \\ \hline 6) & 11 & 4 & 1\frac{1}{2} \\ \hline & 1 & 17 & 4\frac{1}{4} \\ \hline\hline \end{array}$$

Example 4.—A grocer bought 20 boxes of soap for £2 6*s*. 2*d*.; how much was that per box?

$20 = 4 \times 5$ or 10×2.

$$\begin{array}{rrrl} & \pounds & s. & d. \\ 10) & 2 & 6 & 2 \\ \hline 2) & & 4 & 7\frac{1}{4} + 6 \text{ farthings} \\ \hline & & 2 & 3\frac{1}{2} + 1 \text{ ten farthings over.} \\ \hline\hline \end{array}$$

$\therefore$ remainder $= 10 + 6 = 16$ farthings.

Example 5.—Work the preceding example by long division:—

The process is precisely the same as in short division, except that in long division we show the reduction, &c., in figures.

20 will not go into £2 without reduction; we therefore reduce to shillings by multiplying by 20; also add the 6*s*. of the dividend = 46*s*. Proceed as in simple division; 20 goes into 46 *twice*. 2 × 20 = 40, 40 from 46 leaves 6, viz. 6*s*. Reduce the 6*s*. to pence by multiplying by 12 = 72*d*. Add the 2*d*. of the dividend = 74*d*. 20 goes into 74 *three* times. 3 × 20 = 60; 60 from 74

$$\begin{array}{l} \qquad\ \ \pounds \quad s. \quad d. \quad s. \quad d. \\ 20)2 \quad 6 \quad 2(2 \quad 3\frac{1}{2} \\ \qquad \underline{20} \\ \qquad 46(2 \text{ shillings} \\ \qquad \underline{40} \\ \qquad\ \ 6 \\ \qquad \underline{12} \\ \qquad 74(3 \text{ pence} \\ \qquad \underline{60} \\ \qquad 14 \\ \qquad\ \ \underline{4} \\ \qquad 56(\frac{1}{2} \text{ penny} \\ \qquad \underline{40} \\ \qquad 16 \text{ farthings over.} \end{array}$$

leaves 14, viz. 14*d.* Reduce the 14*d.* to farthings = 56 farthings. 20 goes into 56 *twice.* 2 × 20 = 40; 40 from 56 leaves 16, viz. 16 farthings, as a remainder.

To prove the accuracy of the working, multiply the quotient by the divisor and add the remainder; this should give the dividend thus:—

£	*s.*	*d.*	
	2	3½	× 20
		10	
1	2	11	
		2	
2	5	10	
		4	add the remainder, viz. 16 far. or 4*d.*
2	6	2	= dividend.

Example 6.—How many times is 3*s.* 6*d.* contained in 10*s.* 6*d.*? We may work this example by subtraction, thus:—

s.	*d.*	
10	6	
3	6	1
7	0	
3	6	2
3	6	
3	6	3

We see that 3*s.* 6*d.* is contained in 10*s.* 6*d.* *three* times.

Suppose the question to be "How many times is 3*s.* 6*d.* contained in 5 guineas?" We can also find the answer to this question by successive subtractions to the number of 30, but as this may be a very long process, we prefer to work by division. But in this and like cases before we can divide, we must reduce the two sums to the same coins or denominations. In this case we may reduce to sixpences or to pence, *thus:—*

To sixpences.

s. *d.*
3 6
2
7 sixpences

£ *s.* *d.*
5 5 0
20
105
2
210 sixpences

7)210
30 times.

Or thus, to pence :—

s. *d.*
3 6
12
42 pence

£ *s.* *d.*
5 5 0
20
105 shillings
12
1260 pence

d. *d.*
42)1260(30 times
126
0

EXERCISES IN DIVISION OF MONEY.

1. Find the half of 8*s.* 6*d.*, 7*s.* 6*d.*, 15*s.*, 8*s.* 4*d.*, 17*s.* 6*d.*, £1 2*s.* 6*d.*, and £1 7*s.* 6*d.*

2. Find the quarter of 2*s.* 6*d.*, 3*s.* 9*d.*, 5*s.*, 7*s.* 6*d.*, 15*s.*, 17*s.* 6*d.*, £1 15*s.* 6*d.*, and £1 17*s.* 6*d.*

3. Find the third of 6*s.* 9*d.*, 17*s.* 4½*d.*, £1, £1 7*s.* 6*d.*, £1 12*s.*, and £2 8*s.* 9¾*d.*

4. Divide £11 8*s.* 3*d.* equally among 4 men.

5. If 11 persons rent a piece of ground for £32 14*s.*, what is each person's share?

6. A whole piece of cloth costs £39 8*s.* 4*d.*; if the piece makes 15 suits of clothes, what will the cloth of each suit cost?

7. My income is £156 12*s.* 3*d.* per annum; how much is that per quarter of a year?

8. Divide £5 equally among 8 persons.

9. Divide 5 guineas equally among 9 persons.

10. If 12 pairs of socks cost £1 15*s.* 6*d.*, what is the price of 1 pair?

11. I bought a cheese at 11½*d.* per lb.; it cost me 9*s.* 7*d.*; how many lbs. did the cheese weigh?

12. A merchant finds his gains in 18 years to be £8,016 10*s.*; what did he gain on an average each year?

13. If 2 months' wages amount to £20 16*s.* 8*d.*, what is that per day, reckoning 28 days to a month?

14. A grocer gains 2 guineas by selling 56 lbs. of tea; what is his gain per lb.?

15. Three dozen shirts cost 9 guineas; what is the price of each shirt?

16. (*a*) Divide 18 pence between A and B, giving to A twice as much as B.

(*b*) Divide £210 18*s.* 6*d.* between A and B, giving to A twice as much as B.

17. (*a*) Divide 21 pence between A and B, giving A half as much as B.

(*b*) Divide £276 16*s.* 3¾*d.* between A and B, giving A half as much as B.

18. (*a*) Divide 3*s.* between a boy and a girl, giving the boy twice as much as the girl.

(*b*) Divide £519 4*s.* 6¾*d.* between a man and a woman, giving the man twice the share of the woman.

19. A farmer sold 215 geese for £34 18*s.* 9*d.*; what was the price of one?

20. For what should a farmer sell me one duck, when he asks £4 5*s.* for 17?

21. A gross of silver buttons cost a tailor £25 4*s.*; what was the price of one?

22. A grocer gained 14 shillings by selling 112 lbs. of sugar; what was his gain on every lb.?

23. A cask of wine, containing 236 gallons, was sold by a wine-merchant at a loss of £1 19*s.* 4*d.*; what did he lose on every gallon?

24. A sack of flour containing 280 lbs. costs £2 12*s.* 6*d.*; what is that per lb.?

25. What do I give a lb. for meat when 156 lbs. cost £5 10*s.* 6*d.*?

26. (*a*) What sum of money multiplied by 8 will make 3*s.*?
(*b*) What sum of money multiplied by 16 will make £1,396 14*s.* 8*d.*?

27. Divide £100,000 equally among 250 persons.

28. What sum of money multiplied by 69 will be equal to £9,018 2*s.* 6*d.*?

29. What sum of money must a boy earn in a week to make £39 a year (52 weeks)?

30. How much must I save per week to purchase a bicycle worth £9 15*s.* in half a year (26 weeks)?

31. If I pay a debt of £369 18*s.* 1¼*d.* in 15 months, an equal part each month, how much do I pay each time?

32. What sum multiplied by 139 will make £350 19*s.* 6*d.*?

33. If I pay £2 for 480 flower-pots, what is one worth?

34. What is the price of cheese per lb. when a ton costs £37 6*s.* 8*d.*?

35. How many yards of calico at 5¼*d.* per yard can I buy for £1 9*s.* 4*d.*?

36. How long shall I be in saving 24 guineas at the rate of 3*s.* 6*d.* per week?

37. If I have property which brings me in £312 a year (52 weeks), what is that in a fortnight?

38. How many times is 3*s.* 6¼*d.* contained in £12 13*s.* 6*d.*?

39. Divide £29 10*s.* by £1 4*s.* 7*d.*

40. A farmer bought a number of sheep at £2 3*s.* 6*d.* each; the amount paid for the whole was £32 12*s.* 6*d.*; what number did he purchase?

MISCELLANEOUS EXERCISES.

1. A man earns 2*s*. 6*d*. and his son 1*s*. 4*d*. per day; what do they earn together in a year (313 days)?

2. A earns 19*s*. 6*d*. per week and B £1 13*s*. 4*d*.; how much does B earn more than A in a year?

3. A man's income is £350 per year; how much may he spend in each week so as to save £90 a year?

4. A farmer sold 75 quarters of wheat at 45*s*. per quarter, and bought 85 sheep at £1 10*s*. 6*d*. each; what money did he take home?

5. A boy earns 9½*d*. per day; what will his wages amount to in 313 days?

6. To the sum of 3 half-crowns and 25 sixpences add three times their difference.

7. The bread of a household costs 15*s*. 2*d*. each week; if the price is two loaves for 7*d*., how many loaves are consumed? If the bread be increased in price ½*d*. per loaf, what will be the amount of the "bread bill," presuming that the same number of loaves are taken?

8. A man's income is £250; he spends 2 guineas per week; what does he save in a year?

9. A grocer bought tea at 3*s*. 6*d*. per lb. and sold at 4*s*. 3*d*. per lb.; what did he gain on 4 chests, each weighing 28 lbs.?

10. At a concert 150 persons were present in the 2*s*. 6*d*. seats, 350 in the 1*s*. seats, and 208 in the 6*d*. seats; how much money was taken?

11. A dealer bought 39 horses at £32 10*s*. each; they cost him £8 19*s*. 9*d*. for their keep and other expenses, and he sold them at a profit of £50 on the whole; find the selling price of each horse.

12. A tailor bought 12 pieces of cloth of 50 yards each at 3*s*. 4½*d*. per yard, and sold it again to gain £1 10*s*. 6*d*. on each piece; for how much did he sell each piece, and what were his profits on the whole?

13. A penny newspaper has a weekly sale of 3,655 copies; how many copies are sold in a year, and what is the income derived therefrom?

14. A wine-merchant bought 30 dozen of wine at 40*s*. per dozen; what is the retail price per bottle to gain £6 on the whole?

15. A mother sent her daughter to pay a bill, and gave her £2; she brought back the change 13*s.* 6½*d.*, which was 1*s.* 4½*d.* short; what was the amount of the bill?

16. There are 600 people in a concert-room; 150 paid 2*s.* 6*d.* each, 200 paid 1*s.* 6*d.* each, and the remainder paid 6*d.* each; how much money was taken altogether?

17. What does a boy receive for selling 3,168 newspapers, his commission being 6*d.* for every 3 dozen sold?

18. A labourer earns £1 5*s.* per week; if his rent and taxes amount to £9 a year, and he saves £4 per year, what does he spend per week?

19. A father and son together earn £130 a year; if the father earns four times as much as the son, what are the weekly wages of each?

20. A bricklayer, working 10 hours per day, earns 9*d.* per hour and his labourer 6½*d.*; how much more does the former earn than the latter in 313 days?

21. A tradesman gained by his business in the first 6 months of the year £50 0*s.* 9*d.* per month, in the next 2 months £35 16*s.* 7*d.* per month, in the next 3 months £17 16*s.* per month, and lost £18 2*s.* 10½*d.* in the last month; what was his net gain in the year?

CHAPTER XXVI.

REDUCTION—continued.

The scholars should now be exercised in Reduction of Money involving more difficult processes. This practice will serve as an introduction to the reduction of "weights and measures." The tables of weights and measures must be committed to memory, and whilst the children are working exercises involving a knowledge of these tables a few minutes at the commencement of each lesson should be devoted to oral practice.

ORAL EXERCISES IN WEIGHTS AND MEASURES.

1. How many ozs. in ½ lb. of sugar? in ¾ of a lb.? Beef is 16*d.* per lb., what is that per oz.? Cheese is 8*d.* per lb., what is that per oz.? Bacon is 1*s.* per lb., what is that per oz.?

2. How many 4-lb. loaves will make a cwt.? If coal is £1 10*s.* per ton, what is that per qr.-cwt.? A bushel of potatoes weighs 56 lbs.; what is the weight of a peck? and of a gallon?

3. An oz. of tea costs 1½*d.*; what is that per lb.? at 2½*d.* an oz., how much per lb.? at 4*s.* per lb., how much per oz.?

4. If a gold chain is worth £3 per oz., what is it worth per lb.? What is the value of one dwt. of the same chain?

5. If a man takes a yard at one step, how many steps will he take to the mile? Into how many inches is a "two-foot" rule divided? How many such rules placed end to end will make a fathom?

6. A boy has to walk 1 m. and 2 fur. to school each day; supposing he goes home to the mid-day meal and returns in the afternoon, how many miles does he walk in a day?

7. Bought cloth at 3*s.* per yard; what is that per in.? at 2*s.* 8*d.* per yard; what is that per nail?

8. I bought 3 nails of red silk, 2 nails of white, and 3 nails of black; what part of a yard did I buy in all?

9. 2*s.* per gallon is how much per pint? £2 per qr. is how much per peck?

10. How many sheets in a quire of paper? how many quires in a ream? how many sheets in a ream?

11. A square table is 3 ft. in the side; how many square feet does it contain? A slate is 11 in. long and 8 in. broad; how many square inches does it contain?

12. A boy attends school 16 minutes late every morning; how much time does he lose in a week? how much in a fortnight? and how much in a month?

In the reduction, addition, subtraction, multiplication, and division of weights and measures, no new principles are involved, and no new processes are employed, other than those used in money sums.

EXERCISES IN REDUCTION.

(For the Black-board.)

Example 1.—Reduce 73,254 lbs. to tons.

28 { 7)73254 lbs.
   { 4)10464 and 6 lbs. over
4)2616 qr.-cwts.
2ˌ0)65ˌ4 cwts.
32 tons and 14 cwts. over.

Ans. 32 tons 14 cwts. 0 qr. 6 lbs.

	tons	cwts.	qr.	lbs.	
Example 2.—Reduce	32	14	0	6	to tons.

tons	cwts.	qr.	lbs.
32	14	0	6

20
654 cwts.
4
2616 qr.-cwts.
28
20934
5232
73254 lbs.

Ans. 73,254 lbs.

Example 3.—Reduce 14,132 feet to miles.

3)14132
4710 yds. and 2 ft. over.
2
11)9420
4ˌ0)85ˌ6 po. and 4 half-yds. or 2 yds. over
8)21 fur. and 16 poles over
2 m. and 5 fur. over.

	m.	fur.	po.	yds.	ft.
Ans.	2	5	16	2	2

Example 4.—Reduce 2 m. 5 fur. 16 po. 2 yds. 2 ft. to feet.

m.	fur.	po.	yds.	ft.
2	5	16	2	2

8
21 furlongs
40
856 poles
5½
4282
428
4710 yards
3
14132 feet.

Ans. 14,132 :

In Example 3, the attention of the scholars must be ticularly directed to the division by 5½. 5½ yds. or 11 l yds. make a pole or perch. We reduce the yards to l yards by multiplying by 2, and then divide by 11 : of co the remainder will be half-yards. In Example 4, point how we multiply by ½.

EXERCISES IN REDUCTION OF MONEY AND WEIG AND MEASURES.

1. If eggs are 6*d*. per dozen, how many can be bough £7 13*s*. 6*d*.?

2. I asked at the bank to have a cheque for 5 guineas cas one half in half-crowns and the other in threepenny-pieces ; many coins of each did I get?

3. How many pears at ½*d*. each can be bought for 1½ guine

4. Reduce 60,000 ozs. to cwts.

5. Find the difference between 10,000 farthings and 10 gui

6. Reduce 15,000 feet to miles and yards.

7. How many more crowns are there in £1,000 than sovereigns in £1,500?

8. Reduce 3 lbs. 9 ozs. 0 dwt. 22 grs. to grains.

9. How many threepences are there in 82 crowns 4*s*. 9*d*. ?

10. What is the price of 10,305 nails at ¼*d*. each ?

11. How many £'s will pay for 340 articles at £1 1*s*. each ?

12. How many lbs. are there in 1,000 tons ?

13. How many sheets in 6 reams of paper, and what are they worth at ½*d*. per sheet ?

14. How many times does a clock strike in a week ?

15. How many seconds are there from nine o'clock on Monday morning till six o'clock on Wednesday evening ?

16. A fruiterer sold 210 pecks of apples at 2*d*. per quart; required the amount.

17. If a boy saves 2*d*. per day, how long will he be in saving £13 17*s*. 4*d*. ?

18. A gold chain weighs 5,760 grains; express its weight in lbs.

19. How many florins will pay a bill of £25 18*s*. ?

20. How many pints are there in 1,000 quarters of wheat ?

21. In how many years will a man earn £1,000 at the rate of 5*s*. per day, supposing he works 301 days in a year ?

22. How many ¼ lbs. of sugar can be made up out of a hogshead weighing 5 cwts. 2 qrs. 12 lbs. ?

23. How many oranges can be bought for £10 15*s*. at half-a-crown a dozen ?

24. How many times can you fill a pint measure out of 13 cans of milk, each holding 9 gallons ?

25. A man in walking takes 30 steps of 30 in. each in 15 seconds; how far will he walk at the same rate in 6 hours ?

MISCELLANEOUS EXERCISES.

1. By what number must you multiply 97 to make it 38,800 ?

2. Take 14 pence from 14 sovereigns; how much can you give away of the remainder and have 10 guineas left ?

3. How many lbs. are there in 5 cwts. 2 lbs. ?

4. How many ½-lb. packets can be made out of a cask of sugar weighing 3 cwts. 2 qrs. 15 lbs. ?

5. Take 100 farthings from £100; how much must you take from the remainder to leave 90 guineas?

6. When the income-tax is 1*d*. in the £, what will a gentleman pay in income-tax whose income is £1,000 a year?

7. How many 3-oz. parcels can be made out of a parcel of tea weighing 15 lbs. 9 ozs., allowing 6 ozs. to be wasted in weighing?

8. Take 1,000 farthings from £1,000, and then divide the remainder equally among 4 persons.

9. Into how many boxes, each containing 3 cwts., can you pack 5 tons 5 cwts. of cheese?

10. Divide £1,000 among three persons, giving to two of them equal shares, and the third £84 more than each of the others.

11. What number of miles has a man walked in 20 days, if he walked 156 furlongs each day?

12. Two numbers multiplied together give a product of 4,116, and one of the numbers is 196; what is the other number?

13. How many yards are there in 3 fur. 27 pls. 2 yds.?

14. The product of three numbers is 600; two of the numbers are 5 and 12; what is the other?

15. How many steps of 2 ft. 8 in. does a man take in walking a distance of 2 miles?

16. What number of yards multiplied by 8 will make a mile?

17. Divide £360 between A and B, so that A shall have £10 less than half.

18. How many times does a wheel of a coach, measuring 16 feet 6 inches round, turn in a distance of 12 miles 2 fur. 1 pl.?

19. A has £150 and B has 300 fourpenny-pieces less than A; how much has B?

20. How many hours has a boy lived who is 13 yrs. 5 dys. 2 hrs. old?

21. A toll-collector at a halfpenny bridge took in one year enough halfpence to make £360 14*s*.; how many people must have passed over?

22. Into how many periods o 5 minutes each can you divide 13 hrs. 15 min.?

23. How many lbs. of beef, at 10*d*. per lb., can be bought with *180 half-crowns?*

24. A man sets out on a tour for 70 days, beginning April 13th; on what date does it end?

25. If you pay ¾*d.* a mile for railway travelling, what distance must you have travelled over when the ticket cost £2 6*s.* 9*d.*?

26. If a wheel turns round twenty times in a minute, how many times will it turn round from 20 minutes past 3 o'clock to 5 minutes past 5 o'clock?

27. What does a bookseller gain by selling 850 books at 1*s.* 7*d.* each, which cost him 1*s.* 2¾*d.* each?

28. Some school-boys played cricket for 1 hr. 45 min.; if each boy has 15 minutes at batting, how many boys are there?

29. The quarter of a sum of money is £3 16*s.* 8*d.*; what is the whole? what is ¾ of it?

30. My butcher's bill in two months amounted to £8 3*s.* 4*d.*; how many lbs. of meat must I have purchased at 8*d.* per lb.?

31. How many sheets of paper do I get by buying 3 quires? how many quires can 5,816 sheets be made up into? how many more sheets would be required to make just another quire?

32. If you earn £2 15*s.* 6*d.* a week, how much may you spend weekly if you save £39 a year?

33. From a peck of beans how many times can I fill a pint cup, allowing 2 pints to be wasted in measuring?

34. Suppose you have a quart cup only, and want to measure 3 bushels of beans, how many times must you fill it?

35. An apprentice earns in his first year 5*s.* a week; his earnings are increased 2*s.* 6*d.* per week each year; what does he earn in three years?

36. A seedsman sold in one season 1,365 pints of peas; how many bushels has he sold? what quantity is wanted to make up another bushel?

37. If I consume in a year 12 tons of coal at 17*s.* 6*d.* per ton, 20,000 feet of gas at 5*s.* 3*d.* a thousand, and 28 lbs. of candles at 5¾*d.* per lb., what are my yearly expenses for lighting and heating?

38. How many quarters of an acre are there in 856 acres of land?

39. If you divide 242 square perches into two gardens, how many yards will each contain?

40. If a house and garden occupy 20 perches, and the house covers 130 sq. yds.; how many sq. yds. does the garden contain?

41. A playground occupies one rood of land; how many sq. yds. are in it?

42. How many oranges at 1*s.* 9*d.* a dozen are equal in value to 18 boxes of figs at 3*s.* 6*d.* per box?

43. Into how many plots, each containing 40 sq. yds., can an acre be divided?

44. If I pay my butcher £29 18*s.* 6*d.*, my baker half as much again as the butcher, and the grocer as much as the butcher and baker together, how much shall I have left out of £150?

45. A cubic foot of water weighs 1,000 ozs.; what is the weight in lbs. of 3 cubic yards? how many cwt. will these lbs. equal?

46. A woman bought eggs at the rate of 15 for a shilling, and sold them at the rate of 12 for a shilling; what did she gain on the sale of 360?

47. What would a grocer receive by selling ¼ of a ton of currants at 6*d.* per lb.; how much would he gain on the whole if he gave only 4*d.* a lb. for them?

48. A butcher sells a sheep for £3 10*s.* 6*d.* at the rate of 6*d.* per lb.; what did it weigh?

49. How many boxes of matches at 1*s.* 3*d.* a doz. can be bought for 16*s.* 3*d.*?

50. A grocer buys 50 lbs. of tea for £6 5*s.*, and retails it at 3*s.* 4*d.* per lb.; what does he gain per lb.? and what is his profit on the whole?

51. How many half-crown books will be equal in value to 360 books at 4*d.* each?

52. A bookseller takes off 2*d.* in the shilling from the published price of books; what does he charge for a 5*s.* book, 10*s.* book, and a guinea book respectively?

CHAPTER XXVII.

EXAMINATION PAPERS.

STANDARD IV.

I.

1. It is desired to raise £50 by a penny subscription; if 5,384 subscribers have paid, how many more subscribers do I want?

2. Find the number of inches in a mile.

3. If 150 tables are bought for £103 15*s*., what must be the selling price per table so as to gain £20 on the whole?

4. If a father earns 3*s*. 6*d*. per day and his son 1*s*. 2*d*. per day, how many days will they together be earning £23 6*s*. 8*d*.?

II.

1. Find the value of 120 tons of coals at £1 5*s*. 9*d*. per ton.

2. What will 120,000 bricks cost at 32*s*. per thousand?

3. If 60 pairs of boots are bought for £38 10*s*., and sold at 16*s*. 6*d*. per pair, what is the gain on the whole?

4. How many seconds are there from 8 o'clock A.M. till 5 o'clock P.M.?

III.

1. Multiply the difference between £50 and 21 guineas by 69.

2. Which will cost the more, and by how much, 25 lbs. of tea at 3*s*. per lb., or 47 lbs. of coffee at 1*s*. 8*d*. per lb.?

3. A draper bought 6 doz. dresses for 100 guineas and sold them at a loss of £15; what was the selling price per dress?

4. A boy in walking covers 1 ft. 10 in. at each step; how many steps does he take in walking a mile?

IV.

1. Divide the sum of £1,000 and 1,000 guineas by 42.

2. How many packets of paper, of 5 quires each, can be made out of 40 reams?

3. A coal-dealer bought coals at 25*s*. per ton; at what price per cwt. must he sell them to gain 5*s*. per ton?

4. What is the value of 2 acres 3 roods 15 poles of land at 5*s*. 6*d*. per pole?

V.

1. Pay the following out of 10*s*. and find what change you ought to receive: 14 ozs. of tea at 3½*d*. per oz., 5¼ lbs. of sugar at 4*d*., 2 lbs. of coffee at 1*s*. 8½*d*., and half a pound of butter at 1*s*. 6*d*. per lb.

2. A man has 2,540 shillings, 2,540 pence, and 2,540 farthings, to divide equally among 6,731 boys; what amount will each boy receive?

3. A grocer bought a gross of marmalade for 3 guineas, and sold it at 6½*d*. per jar; what did he gain on each pot? and how much on the whole?

4. Reduce 4 tons 3 cwts. 1 qr. 10 lbs. 4 ozs. to ozs.

VI.

1. Take 10¼*d*. from £10, and find how much the remainder is less than a thousand halfpence.

2. Will a man gain or lose, and how much, by purchasing 15 lbs. of tea for £1 7*s*. 6*d*., and selling it at 2*s*. 6½*d*. per lb.?

3. Divide £99 19*s*. 6*d*. by 150, and state the number of farthings required so as to make an equal division into 151 parts.

4. If I burn 2 cwts. 3 qrs. of coal per week, how long will 6 tons 3 cwts. 3 qrs. last me?

VII.

1. Take 13 guineas and 4 half-crowns from £1,000, and then find the number of fourpences in the remainder.

2. A woman bought 10*s*. worth of eggs at 7 for 6*d*., and sold them at 5 for 6*d*.; how much did she gain?

3. A girl earns ¾*d*. per hour; if she works 10 hours per day, how much will she earn in 24 weeks?

4. How many 7½-lb. parcels can be made out of 1 cwt. 2 qrs. 12 lbs.?

VIII.

1. Multiply £740 13*s*. 5*d*. by 99.

2. How many silver spoons each weighing 4 ozs. 5 dwts. can be made out of 1,020 dwts. of silver?

3. I spent 13*s*. 9*d*. per day when on a holiday tour; my total expenditure was £7 11*s*. 3*d*.; how many days did my tour last?

4. A milliner bought 38 bonnets for £39 18*s*.; she sold them to gain £7 12*s*. on the whole; what was the selling price per bonnet?

IX.

1. I want 5½*d*. more to make £100; how many pence have I?

2. If I save £5 10*s*. per month, in how many months shall I save £154?

3. Which will cost the more, and by how much, 20 lbs. of tea at 2*d*. per oz. or 25 lbs. at 1½*d*. per oz.?

4. A gold chain weighs 3 ozs. 12 dwts. 5 grains; what will it cost at 2*d*. per grain?

X.

1. How many subscriptions of 1*s*. 4*d*. each do I want to make £1,050?

2. I bought eggs at the rate of 13 for a shilling, and received 3*s*. change out of 3 crowns; how many eggs did I buy?

3. How many perches are there in 1,760 yards?

4. If 1 oz. of tea costs 2*d*., what is the value of 1 cwt.?

XI.

1. Take 15 half-crowns from £100, and divide the remainder equally among 157 persons.

2. Reduce 14,065 lbs. to tons.

3. A grocer bought 365 lbs. of tea at 2*s*. per lb.; 40 lbs. were found to be damaged, and had to be sold at 1*s*. 2*d*. per lb.; the remainder was sold at 3*s*. per lb.; find the gain or loss on the whole.

4. A man has 10 guineas, £10, 10 half-crowns, 10 sixpences, and 10 pence, to be equally divided among 529 persons; what will each receive?

XII.

1. If a pint measure holds 95 beans, how many beans will fill a peck measure?

2. If 1 yard of cloth costs 19*s*. 11*d*., what will 170 yards cost?

3. I handed a stationer a £5 note to pay for 5 reams of paper at 6*s*. per ream, and 5 thousand envelopes at 8*s*. per thousand; he tradesman allowed me 2*d*. in the shilling for ready money; what change did he give me?

4. Divide £84 15*s*. 6*d*. between John and William, giving John alf as much as William.

XIII.

1. An estate containing 432 acres is to be sold in building plots; the plots in one half of the estate are to contain 96 poles each, and in the other half 48 poles each; find the number of plots.

2. A man saves £2 5*s.* 6*d.* a week, and spends £3 4*s.* 6*d.*; in addition to this he pays £18 8*s.* 4*d.* for taxes; what is his yearly income? (Reckon 52 weeks to the year.)

3. Divide £290 between A, B, and C, so that A and B shall have equal shares, but C £40 more than either.

4. A farmer bought 300 sheep for £1,050, and sold them for £1,500; what did he gain per sheep?

XIV.

1. How much must you add to, or subtract from, 156 five-pound notes to make a sum of money equal to one-fourth part of 1,000 guineas?

2. How many times is 1*s.* 4½*d.* contained in £5 3*s.* 1½*d.*?

3. Mix 1 cwt. of sugar at 5*d.* per lb. with 96 lbs. at 4½*d.*, and find at what price per lb. the mixture must be sold to gain £1 1*s.* 4*d.* on the whole.

4. Six fields of 9 acres each are rented at 3*d.* per pole; find the amount of the rent.

XV.

1. A draper bought 21 dresses at £3 17*s.* 6*d.* each, and sold them for £4 8*s.* 11½*d.* each; what profit did he make on the whole?

2. How many times can you subtract 5*s.* 2*d.* from £72 6*s.* 8*d.*?

3. What quantity of sugar at £1 8*s.* per cwt. will be worth £18 4*s.*?

4. A train 80 yards long passes a signal post in 3 seconds; how many miles an hour is it going?

XVI.

1. How many lbs. of sugar at 4*d.* per lb. can be bought for a sovereign, a crown, a half-crown, a florin, and a shilling?

2. How many times is 69 farthings contained in £72 19*s.* 0¾*d.*?

3. A grocer mixes 8 lbs. of tea at 3*s.* 6*d.* a lb. with 16 lbs. at 2*s.* 6*d.*; at what price should he sell the mixture so as not to gain or lose by the sale?

4. I bought 400 eggs at 20 for a shilling, and sold them for 25*s.*; how many did I sell for a shilling, and what did I gain on the whole?

PART IV.

FOR SENIOR SCHOOLS.

STANDARD V.

CHAPTER XXVIII.

BILLS OF PARCELS.

INTRODUCE the subject of bills of parcels by writing on the black-board three or four little examples with which the children have now become familiarised, thus:—

1. Find the value of 14 lbs. of beef at 1*s.* 2*d.* per lb. = 16*s.* 4*d.*
2. Also of 18 lbs. of mutton at 1*s.* 1*d.* per lb. = 19*s.* 6*d.*
3. Also of 10 lbs. of veal at 1*s.* 3*d.* per lb. = 12*s.* 6*d.*

Let the scholars work these examples, and then add the three separate amounts together to find the total amount, viz. £2 8*s.* 4*d.*

Now if we go to the butcher's shop and make these purchases: the butcher will make out his account of the transaction thus:—

LONDON, *June* 6*th*, 1877.

MR. W. JONES,

Bought of JOHN THOMAS.

		£	*s.*	*d.*
June 1st	14 lbs. of beef at 1*s.* 2*d.* . .		16	4
„ 2nd	18 lbs. of mutton at 1*s.* 1*d.* .		19	6
„ 5th	10 lbs. of veal at 1*s.* 3*d.* . .		12	6
		2	8	4

Let us go to the grocer and make the following purchases —3 lbs. of sugar at 5½*d.* per lb., 4 lbs. of rice at 2½*d.* per lb., 2½ lbs. of tea at 2*s.* 9*d.* per lb., and 1½ lb. of coffee at 1*s.* 4*d.* per lb.; we shall find on asking for the bill that the grocer has written it somewhat thus:—

LONDON, *June* 20*th*, 1877.

MR. J. BUTTON,

Bought of G. CHEESEMAN.

		£	*s.*	*d.*
June 20th, 1877.	3 lbs. loaf at 5½*d.* . .		1	4½
	4 ,, rice ,, 2½*d.* . .			10
	2½ ,, tea ,, 2*s.* 9*d.* . .		6	10½
	1½ ,, coffee ,, 1*s.* 4*d.* . .		2	0
			11	1

And so if we make purchases of other tradesmen their accounts will be rendered as "Bills," very much like the butcher's and the grocer's.

It is of great importance that we should be able to test the accuracy or inaccuracy of accounts or bills, and to do this we must work the little multiplication sums either mentally or on paper.

Most of the kinds of food we eat are sold at so much per lb. weight. It is easy enough for the grocer to weigh exactly a lb. of tea or sugar or coffee, but it is not so easy for the butcher to cut off a lb. of steak, or the cheeseman a lb. of cheese. It is pretty certain in the latter cases there will be some ozs. too much or too little. Bacon, cheese, and all kinds of meat vary in price from 4*d.* to 1*s.* 6*d.* a lb., and we should be able to calculate readily the price of the odd ozs. at these rates per lb.

16 ozs. = 1 lb.

Therefore:— 16*d.* per lb. = 1*d.* per oz.

16 ÷ 2 = 8*d.* ,, = ½*d.* ,,

16 ÷ 4 = 4*d.* ,, = ¼*d.* ,,

8 + 4 = 12*d.* ,, = ¾*d.* ,,

12*d*. ÷ 2 = 6*d*. per lb. = ¾*d*. for 2 ozs.
6*d*. × 3 = 1*s*. 6*d*. „ = 2¼*d*. „
6*d*. + 4*d*. = 10*d*. „ = 1¼*d*. „
8*d*. + 6*d*. = 14*d*. „ = 1¾*d*. „

Intermediate prices per lb. do not give exact prices per oz., and we must calculate to the nearest value we can.

EXERCISES.

1. Calculate the price of—

(1) 11 ozs. of butter at 1*s*. 4*d*. per lb.
(2) 13 „ suet „ 1*s*. „
(3) 3 „ cheese „ 1*s*. 2*d*. „
(4) 5 „ beef „ 1*s*. „
(5) 10 „ cheese „ 10*d*. „
(6) 13 „ butter „ 1*s*. 6*d*. „

2. Find the value of—

(1) Leg of lamb 6 lbs. 10 ozs. at 1*s*. 2*d*.
(2) Joint of beef 12 „ 9 „ 1*s*.
(3) Loin of mutton 9 „ 0 „ 10*d*.
(4) Cutlets 4 „ 2 „ 1*s*. 2*d*.
(5) Loin of veal 6 „ 8 „ 10*d*.
(6) Shoulder of mutton 10 „ 4 „ 1*s*.

3. Write out the following bills in their proper forms, and calculate their amounts :—

(1) 12 lbs. of figs at 8*d*. per lb.; 7 lbs. of raisins at 5*d*. per lb.; 14 lbs. of rice at 2½*d*. per lb.; and 9 lbs. of treacle at 3¼*d*. per lb.

(2) 15 lbs. of cherries at 7½*d*. per lb.; 23 lbs. of currants at 3*d*. per lb.; 9 quarts of strawberries at 14½*d*. per quart; and 19 lbs. of raspberries at 11½*d*. per lb.

(3) 2 doz. pairs of stockings at 2*s*. 8*d*. per pair; 3 pairs of gloves at 2*s*. 9¼*d*. per pair; and 11 yards of silk at 8*s*. 3½*d*. per yard.

(4) 3 doz. boxes of matches at 6½*d*. per doz.; ½ a chest c tea weighing 48 lbs. at 2*s*. 4½*d*. per lb.; 1½ doz. lbs of candles at 10¼*d*. per lb.; and 13 lbs. of soap at 4½*d* per lb.

4. Find the amount of the following accounts :—

(1) 4¼ yards of cloth at 1*s*. 4*d*. per yard.
2½ doz. yards of calico at 5¾*d*. per yard.
16 yards of flannel at 1*s*. 2¾*d*. per yard.
4¼ yards of silk at 9*s*. 6*d*. per yard.

(2) 17 yards of Brussels carpet at 4*s*. 6*d*. per yard.
13½ yards of chintz at 11*d*. per yard.
15 yards of gimp at 1*s*. 3*d*. per yard.
19 yards of damask at 4*s*. 6½*d*. per yard.

(3) 3 bushels of potatoes at 2*s*. 9½*d*. per bushel.
8 pecks of apples at 1*s*. 8½*d*. per peck.
7 lbs. of "kidney beans" at 3¾*d*. per lb.
4 heads of celery at 5¼*d*. each.

(4) 18 plates at 9½*d*. each.
3 dozen tumblers at 2¾*d*. each.
4½ dozen small plates at 2¾*d*. each.
1½ dozen wine-glasses at 1*s*. 4*d*. each.

(5) 2 pairs of gloves at 1*s*. 2*d*. per pair.
8 dozen yards of braid at 1*s*. 3½*d*. per doz.
3 dozen yards of unbleached calico at 4½*d*. per yard.
7 dozen pieces of tape at 1½*d*. per piece.

(6) 3 tins of biscuits, weighing 6¼ lbs. each, at 4*d*. per lb.
4½ dozen tarts at 9*d*. per doz.
3½ dozen plain buns at ¾*d*. each.
1¼ lb. of sweets at 8*d*. per lb.

5. A lady took £5 in her purse to pay the following bills; wha change had she after paying the first, and how much after payin the second?

(1) 10¼ lbs. of salmon at 1*s*. 7*d*. per lb.
3½ dozen oysters at 2*s*. 3*d*. per dozen.
4 rabbits at 1*s*. 6½*d*. each.
3 pairs of soles at 1*s*. 9½*d*. per pair.

(2) 13 lbs. of flour at 2¾*d.* per lb.
18 loaves at 5¼*d.* each.
12 small loaves at 2¾*d.* each.
81 buns at 9*d.* per dozen.

6. The following is a leaf from my butcher's book; examine it, nd correct if necessary:—

		£	*s.*	*d.*
1877.	Brought forward	2	11	6½
Aug. 1st	Leg of lamb, 9¼ lbs. at 11*d.* per lb.		8	5¾
„ 3rd	10 lbs. 3 ozs. beef at 14*d.* . .		12	0¼
„ 4th	9 ozs. suet at 1*s.*			6¼
„ 6th	5 lbs. 6 ozs. veal at 10*d.* . .		4	10
„ 8th	2 chops at 8¼*d.* each . . .		1	4½
„ 10th	Leg of mutton, 9 lbs. at 11*d.* .		8	3
„ 13th	7 ozs. suet at 1*s.* a lb. . . .			11
„ 15th	5 lbs. 4 ozs. pork at 10*d.* . .		4	4½
„ 16th	20 ozs. steak at 14*d.* a lb. . .		1	5
	Total	4	14	6¾

7. A schoolmaster bought 2 gross of pieces of chalk at 1¼*d.* per dozen; 25 reading-books at 1*s.* 6½*d.* each; 35 reading-books at 9½*d.* each; and one dozen boxes of pens at 7½*d.* per box: allowing 2*d.* in the shilling for cash, what was the amount to be paid?

8. Test the correctness or otherwise of the following accounts:

		£	*s.*	*d.*
(1) 1½ chest of tea (25 lbs. to a chest) at 2*s.* 3*d.* per lb.	=	4	4	4½
A box of soap, weighing 1½ cwt., at 2½*d.* per lb.	=	1	10	0
1¼ cwt. "golden syrup" at 2¼*d.* per lb.	=	1	6	3
		£7	0	7½

		s.	*d.*
(2) 40 lbs. soda at 11*s.* 8*d.* per cwt.	=	4	2
14 lbs. soap at £3 2*s.* 8*d.* per cwt.	=	4	1
8 dozen eggs at 1¾*d.* each	=	1	2
		9	5

		£	s.	d.
(3)	3½ tons of coals at 19s. 6d. per ton . . =	2	18	3
	2¼ ,, ,, £1 1s. 6d. per ton . =	2	8	3¼
	1½ chaldron of coke at 15s. 9d. per chaldron . . . . =	1	3	7½
		£6	10	2

CHAPTER XXIX.

SIMPLE PRACTICE.

Example 1. (*Oral.*)—What is the value of 24 lbs. of candles at 9*d.* per lb.?

The answer to this question will be readily given by most of the scholars. Ask for the method of working. Some will, no doubt, have multiplied 24 by 9, and reduced to shillings; others will have said 24 lbs. at 6*d.* = 12*s.*, and 24 at 3*d.* = 6*s.*; and 12*s.* + 6*s.* = 18*s.*

The latter method—viz. by *partial payments*—will most probably be the one used in *practice* by the tradesman, as the shortest and simplest.

Example 2. (*Oral.*)—Find the cost of 4 tons of coals at £1 15*s.* per ton.

Working this also by "partial payments":—

	£
4 tons at £1 per ton =	4
4 ,, 10*s.* ,, =	2
4 ,, 5*s.* ,, =	1
4 ,, £1 15*s.* ,,	£7

Example 3. (*For the black-board.*)—Find the value of 9 *tons* of coals at £1 17*s.* 6*d.* per ton.

We will first find the price of the coals by our old method of multiplication:—

£	s.	d.		
1	17	6	value of	1 ton
		10		
18	15	0	„	10 tons
		10		
187	10	0	„	100 tons
		4		
750	0	0	„	400 tons
1	17	6	„	1 ton
748	2	6	„	399 tons

Let us now find the value of the coals as in the oral examples just given.

Suppose the price to be £1 per ton, then the 399 tons will cost £399.

Now take the price at 10*s.* per ton; this will be one-half of the price at £1 per ton, or £399 ÷ 2 = £199 10*s.*

Next take the price at 5*s.* per ton; this will be half the price at 10*s.*, or £199 10*s.* ÷ 2 = £99 15*s.*

Lastly take the price at 2*s.* 6*d.* per ton; this will be half the price at 5*s.* per ton, or £99 15*s.* ÷ 2 = £49 17*s.* 6*d.*

Collecting all these prices together, we get—

					£	s.	d.
399 tons	at	£1	per ton	=	399	0	0
399	„	10*s.*	„	=	199	10	0
399	„	5*s.*	„	=	99	15	0
399	„	2*s.* 6*d.*	„	=	49	17	6
And 399	„	£1 17*s.* 6*d.*	„	=	£748	2	6

For convenience we arrange the work thus :—

	£	*s.*	*d.*			
10*s.* = $\frac{1}{2}$ of £	399	0	0	value at	£1	per ton
5*s.* = $\frac{1}{2}$ of 10*s.*	199	10	0	„	10*s.*	„
2*s.* 6*d.* = $\frac{1}{2}$ of 5*s.*	99	15	0	„	5*s.*	„
	49	17	6	„	2*s.* 6*d.*	„
	748	2	6	value at	£1 17*s.* 6*d.*	„

We see from this example that just as multiplication is a short method of adding equal quantities, so PRACTICE (as the process shown above is called) is a short method of working multiplication.

To help the scholars to work long multiplication sums by this short method, they should be well exercised with oral examples on—

(1.) The *even* or *aliquot* parts of a £, a shilling, &c. ; and

(2.) The splitting up or division of amounts into aliquot parts, so as to help them to work the example with the shortest and easiest process.

(1.) The aliquot parts of £1 are—

10*s.* =	$\frac{1}{2}$	2*s.* 6*d.* =	$\frac{1}{8}$
6*s.* 8*d.* =	$\frac{1}{3}$	2*s.* =	$\frac{1}{10}$
5*s.* =	$\frac{1}{4}$	1*s.* 8*d.* =	$\frac{1}{12}$
4*s.* =	$\frac{1}{5}$	1*s.* 3*d.* =	$\frac{1}{16}$
3*s.* 4*d.* =	$\frac{1}{6}$	1*s.* =	$\frac{1}{20}$

The aliquot parts of a shilling are—

6*d.* =	$\frac{1}{2}$	2*d.* =	$\frac{1}{6}$
4*d.* =	$\frac{1}{3}$	1$\frac{1}{2}$*d.* =	$\frac{1}{8}$
3*d.* =	$\frac{1}{4}$	1*d.* =	$\frac{1}{12}$

(2.) Much more practice should be given to the best selection of aliquot parts from given amounts, than to the tables of aliquot parts. Numerous examples should be worked on the black-board, the different aliquot parts being elicited *from the scholars.*

Example 4.—Find the cost of 400 dresses at 16*s.* 7½*d.* each.

First Method.

6*s.* 8*d.* = ⅓ of £1	400			cost at £1	
6*s.* 8*d.* = ⅓ „ £1	£133	6*s.*	8*d.*	value at	6*s.* 8*d.*
2*s.* 6*d.* = ⅛ „ £1	133	6	8	„	6*s.* 8*d.*
6*d.* = ⅕ „ 2*s.* 6*d.*	50	0	0	„	2*s.* 6*d.*
3*d.* = ½ „ 6*d.*	10	0	0	„	6*d.*
½*d.* = ⅙ „ 3*d.*	5	0	0	„	3*d.*
		16	8	„	½*d.*
	£332	10*s.*	0*d.*	„	16*s.* 7½*d.*

Second Method.

10*s.* = ½ of £1	400		cost at £1	
5*s.* = ½ „ 10*s.*	£200		value at	10*s.*
1*s.* = ⅕ „ 5*s.*	100		„	5*s.*
6*d.* = ½ „ 1*s.*	20		„	1*s.*
1½*d.* = ¼ „ 6*d.*	10		„	6*d.*
	2	10*s.*	„	1½*d.*
	£332	10*s.*	„	16*s.* 7½*d.*

On comparing the two methods of taking the aliquot parts, it will be seen at once that the second is much to be preferred to the first for shortness and simplicity.

Example 5.—Find the value of 360 pairs of muslin curtains at £1 17*s.* 6*d.* per pair.

First Method.

	£		
10*s.* = ½ of £1	360	value at £1	
5*s.* = ½ „ 10*s.*	180	„	10*s.*
2*s.* 6*d.* = ½ „ 5*s.*	90	„	5*s.*
	45	„	2*s.* 6*d.*
	675	„	£1 17*s.* 6*d.*

Second Method.

	£		
2s. 6d. = $\frac{1}{8}$ of £1	360	value at £1	
	2		
	720	,,	£2
	45	,,	2s. 6d.
	675	,,	£1 17s. 6d.

The second method—by subtraction—is valuable only a a test of accuracy of working, and of the intelligence of th scholars.

NOTE.—Children should always be encouraged to work thei examples wherever possible by more than one method.

Example 6.—If one man earns £1 19s. 11d. per week how much will 769 men earn at the same rate?

First Method.

	£	s.	d.			
10s. = $\frac{1}{2}$ of £1	769	0	0	wages, 769 men, at	£1	per wk
5s. = $\frac{1}{2}$,, 10s.	384	10	0	,,	10s.	,,
4s. = $\frac{1}{5}$,, £1	192	5	0	,,	5s.	,,
8d. = $\frac{1}{6}$,, 4s.	153	16	0	,,	4s.	,,
2d. = $\frac{1}{4}$,, 8d.	25	12	8	,,	8d.	,,
1d. = $\frac{1}{2}$,, 2d.	6	8	2	,,	2d.	,,
	3	4	1	,,	1d.	,,
	£1,534	15	11		£1 19s. 11d.	,,

Second Method.

	£769 wages of 769 men at		£1	per week.
	2			
12)769 pence	1538	,,	£2.	,,
20)64 1	3 4 1	,,	1d.	,,
£3 4s. 1d.	£1534 15s. 11d.	,,	£1 19s. 11d.	,,

In this example the second method is preferable for shortness and simplicity.

Example 7.—A boy earns 7½*d.* per day, how much will hat amount to in 313 days?

		s.		
6*d.*	= ½ of 1*s.*	313	=	amount at 1*s.* per day
1½*d.*	= ¼ ,, 6*d.*	156 6*d.*		
		39 1½*d.*		
	2,0)	19,5 7½*d.*		
		£9 15*s.* 7½*d.*		

Note.—The teacher must rely upon a large number of varied examples worked on the black-board to instil into the minds of the children the different processes and expedients employed in working examples by practice.

EXERCISES IN SIMPLE PRACTICE.

1. Find by practice the value of the following:—

(1) 250 articles at ½*d.*, ¼*d.*, ¾*d.*, and 1*d.*
(2) 1074 ,, 6*d.*, 4*d.*, 3*d.*, 2*d.*, and 1*d.*
(3) 286 ,, 1*d.*, 2*d.*, 2½*d.*, and 3½*d.*
(4) 384 ,, 1¼*d.*, 1½*d.*, 1¾*d.*, 2*d.*, and 2¼*d.*
(5) 189 ,, 2¾*d.*, 3¼*d.*, 3¾*d.*, 4¼*d.*, and 4¾*d.*
(6) 216 ,, 5*d.*, 6*d.*, 7*d.*, 8*d.*, and 9*d.*
(7) 159 ,, 10*d.*, 11*d.*, 5¼*d.*, 5½*d.*, and 5¾*d.*
(8) 268 ,, 6½*d.*, 7½*d.*, 8½*d.*, 9½*d.*, and 10½*d.*
(9) 389 ,, 6¼*d.*, 7¼*d.*, 8¼*d.*, 9¼*d.*, and 10¼*d.*
(10) 921 ,, 6¾*d.*, 7¾*d.*, 8¾*d.*, 9¾*d.*, and 10¾*d.*
(11) 496 ,, 11*d.*, 11¼*d.*, 11½*d.*, 11¾*d.*, and 8¾*d.*
(12) 182 ,, 10*s.*, 6*s.* 8*d.*, 5*s.*, 4*s.*, and 3*s.* 4*d.*
(13) 382 ,, 4*s.*, 3*s.* 4*d.*, 2*s.* 6*d.*, 1*s.* 8*d.*, and 2*s.*
(14) 485 ,, 3*s.* 4*d.*, 1*s.* 8*d.*, 2*s.*, 4*s.*, and 6*s.* 8*d.*
(15) 375 ,, 6*s.* 8*d.*, 10*s.*, 3*s.* 4*d.*, 5*s.*, and 1*s.* 8*d.*
(16) 396 ,, 6*s.*, 8*s.*, 7*s.*, 9*s.*, and 3*s.*
(17) 492 ,, 15*s.*, 14*s.*, 13*s.* 4*d.*, 12*s.* 6*d.*, and 16*s.* 8*d.*

(18) 875 articles at 12*s.*, 8*s.* 4*d.*, 7*s.* 6*d.*, 11*s.* 8*d.*, and 7*s.*
(19) 596 ,, 7*s.* 4*d.*, 6*s.* 6*d.*, 10*s.* 8*d.*, 6*s.*, and 5*s.*
(20) 871 ,, 9*s.* 2*d.*, 8*s.* 8*d.*, 7*s.* 6*d.*, 11*s.* 8*d.*, 12*s.*
(21) 815 ,, 9*s.*, 15*s.*, 12*s.*, 7*s.* 4*d.*, and 9*s.* 2*d.*
(22) 789 ,, 7*s.*, 14*s.*, 8*s.* 4*d.*, 6*s.* 6*d.*, and 8*s.* 8*d.*
(23) 892 ,, 6*s.*, 18*s.* 4*d.*, 7*s.* 6*d.*, 10*s.* 8*d.*, and 7*s.*
(24) 369 ,, 6*s.* 6*d.*, 8*s.*, 12*s.* 6*d.*, 11*s.* 8*d.*, and 9*s.*

2. Calculate the value of the following :—

(1) 186 articles at 1*s.* 1½*d.*, and at 1*s.* 4½*d.*
(2) 390 ,, 8*s.* 6½*d.*, ,, 2*s.* 9¼*d.*
(3) 189 ,, 4*s.* 8¾*d.*, ,, 5*s.* 10½*d.*
(4) 297 ,, 6*s.* 7½*d.*, ,, 8*s.* 8¼*d.*
(5) 187 ,, 8*s.* 6½*d.*, ,, 9*s.* 11½*d.*
(6) 330 ,, 10*s.* 1½*d.*, ,, 10*s.* 6¾*d.*
(7) 275 ,, 7*s.* 6¾*d.*, ,, 7*s.* 11¾*d.*
(8) 368 ,, 9*s.* 4¾*d.*, ,, 7*s.* 6¾*d.*
(9) 299 ,, 9*s.* 10*d.*, ,, 6*s.* 8¼*d.*
(10) 336 ,, 11*s.* 7¾*d.*, ,, 12*s.* 3½*d.*
(11) 596 ,, 18*s.* 5¾*d.*, ,, 16*s.* 3¾*d.*
(12) 556 ,, 18*s.* 5¾*d.*, ,, 18*s.* 9¼*d.*

3. Find the amounts of the following :—

(1) 230 articles at £2 3*s.* 4*d.*, and at £6 5*s.* 6*d.*
(2) 1396 ,, £1 5*s.* 10*d.*, ,, £2 7*s.* 9*d.*
(3) 2304 ,, £6 7*s.* 11*d.*, ,, £5 13*s.* 2*d.*
(4) 2916 ,, £15 5*s.* 10*d.*, ,, £1 17*s.* 5*d.*
(5) 360 ,, £9 11*s.* 9½*d.*, ,, £5 19*s.* 4½*d.*
(6) 732 ,, £15 2*s.* 3¼*d.*, ,, £6 18*s.* 5½*d.*
(7) 1004 ,, £2 15*s.* 7½*d.*, ,, £3 13*s.* 1½*d.*
(8) 4444 ,, £1 17*s.* 9¾*d.*, ,, £3 16*s.* 5¾*d.*
(9) 1368 ,, £3 19*s.* 11*d.*, ,, £1 18*s.* 10½*d.*
(10) 540 ,, £18 18*s.* 8*d.*, ,, £9 9*s.* 9*d.*
(11) 4060 ,, £7 17*s.* 7*d.*, ,, £6 17*s.* 11¾*d.*
(12) 672 ,, £2 13*s.* 9¾*d.*, ,, £1 19*s.* 11¾*d.*

4. What will a coal-merchant pay for 1,066 tons of coals 19*s.* 10*d.* per ton?

5. Find the value of 764 yards of broadcloth at 6*s.* 9¾*d.* *yard.*

6. Required the cost of 972 bushels of wheat at 8*s.* 5½*d.* per bushel.

7. A man earns 5*s.* 6½*d.* per day; how much does he earn in 320 days?

8. Find by practice the cost of 1½ cwt. of beef at 11½*d.* per lb.

9. A bankrupt pays 6*s.* 9*d.* in the £; what will he pay on a debt of £750?

10. What is the amount paid for 637 books at 17*s.* 6½*d.* each?

11. Find the cost of 96 yards of velvet at £1 5*s.* 4*d.* per yard.

12. A merchant invests 1,000 guineas; if he makes a profit of 3*s.* 9*d.* on each £, what is the total amount of his profit?

13. What is the amount of 526 subscriptions of 2½ guineas each?

14. What will £1 15*s.* 10*d.* per week amount to in 15 years?

15. The average cost of 256 tables is £1 2*s.* 11*d.*; find the total cost.

16. Find the value of 300 cwts. of cheese at 5½ guineas per cwt.

17. Calculate the value of 1,045 lbs. of gold at £3 17*s.* 9*d.* per oz.

18. What is the rent of 344¾ acres of land at £1 15*s.* 7*d.* per acre?

CHAPTER XXX.

COMPOUND PRACTICE.

Example 1. (*Oral.*)—Find the cost of 6 lbs. 8 ozs. of butter at 1*s.* 6*d.* per lb.

The method most likely to be adopted by the children in working this example mentally, will be—

		s.	*d.*
6 lbs. at 1*s.* 6*d.*	=	9	0
and 8 ozs. or ½ lb.	=		9
	Total	9	9

Show the process on the black-board, thus:—

	s.	d.	
8 ozs. = ½ lb.	1	6	value of 1 lb.
		6	
	9	0	value of 6 lbs.
		9	„ 8 ozs.
	9	9	value of 6 lbs. 8 ozs.

Example 2. (*Black-board.*)—What is the cost of 1 cwt. 1 qr. 7 lbs. of potatoes at £1 per cwt.?

	£	s.	d.	
1 qr. = ¼ cwt.	1	0	0	= cost of 1 cwt.
7 lbs. = ¼ qr.		5	0	= „ 1 qr.
		1	3	= „ 7 lbs.
	1	6	3	= cost of 1 cwt. 1 qr. 7 lbs.

Example 3.—4 tons 6 cwts. 2 qrs. 14 lbs. at £10 10*s.* per ton.

	£	s.	d.		
5 cwts. = ¼ ton	10	10	0	= cost of 1 ton	
			4		
	42	0	0	= „ 4 tons	
1 cwt. = ⅕ of 5 cwts.	2	12	6	= „ 5 cwts.	} = 6 cwts.
2 qrs. = ½ cwt.		10	6	= „ 1 cwt.	
14 lbs. = ¼ of 2 qrs.		5	3	= „ 2 qrs.	
		1	3¾	= „ 14 lbs.	
	45	9	6¾	= cost of 4 tons 6 cwts. 2 qrs. 14 lbs.	

EXERCISES IN COMPOUND PRACTICE.

1. 5 cwts. 1 qr. 14 lbs. at £1 10*s*. 4*d*. per cwt.
2. 3 cwts. 2 qrs. 14 lbs. at £2 1*s*. 6*d*. per cwt.
3. 7 cwts. 3 qrs. 14 lbs. at £1 14*s*. 6*d*. per cwt.
4. 2 cwts. 3 qrs. 7 lbs. at £2 15*s*. 6*d*. per cwt.
5. 7 cwts. 3 qrs. 20 lbs. at £3 18*s*. 6*d*. per cwt.
6. 3 tons 12 cwts. 1 qr. at £1 10*s*. 6*d*. per ton.
7. 17 tons 3 cwts. 2 qrs. at £20 10*s*. 6*d*. per ton.
8. 4 tons 2 cwts. 3 qrs. at £0 8*s*. 4*d*. per cwt.
9. 3 tons 1 cwt. 1 qr. at £4 13*s*. 9*d*. per ton.
10. 7 tons 15 cwts. 3 qrs. at £1 6*s*. 4*d*. per cwt.
11. 3 qrs. 16 lbs. 4 ozs. at 16*s*. 4*d*. per qr.
12. 2 qrs. 18 lbs. 9 ozs. at 13*s*. 7½*d*. per qr.
13. 3 qrs. 12 lbs. 6 ozs. at 12*s*. 9*d*. per lb.
14. 2 qrs. 17 lbs. 4 ozs. at 6*s*. 9*d*. per qr.
15. 13 lbs. 5 ozs. 5 dwts. at 4*s*. 9*d*. per lb.
16. 17 lbs. 2 ozs. 10 dwts. at 3*s*. 9*d*. per oz.
17. 16 lbs. 2 ozs. 19 dwts. at 16*s*. 4½*d*. per lb.
18. 2 gals. 3 qts. 1 pt. at £1 10*s*. 6*d*. per gal.
19. 4 gals. 2 qts. at £2 5*s*. per gal.
20. 3 gals. 1 qt. 1 pt. at £1 8*s*. per gal.
21. 17 gals. 3 qts. at 15*s*. per gal.
22. 13 gals. 1 qt. 1 pt. at 10*s*. 6*d*. per gal.
23. 25 gals. 3 qts. 1 pt. at 12*s*. 6*d*. per gal.
24. 17 yds. 3 qrs. 2 nls. at 36*s*. per yd.
25. 3 yds. 1 qr. 3 nls. at 17*s*. 6*d*. per yd.
26. 87 yds. 2 qrs. 2 nls. at 13*s*. 10*d*. per yd.
27. 3 qrs. 3 nls. at 80*s*. 6*d*. per yd.
28. 83 yds. 2 qrs. 1 nl. at 2*s*. 6*d*. per qr.
29. 2 furs. 25 poles at £29 18*s*. 6*d*. per fur.
30. 7 furs. 15 poles 2¾ yds. at 10*s*. 6*d*. per pole.
31. 13 perches 2¾ yds. at 13*s*. 6*d*. per fur.
32. 3 acres 2 roods 20 perches at £4 10*s*. per acre.
33. 17 acres 3 roods 15 perches at £5 15*s*. 6*d*. per acre.
34. 29 acres 1 rood 35 perches at £1 10*s*. 4*d*. per acre.
35. 26 acres 1 rood 36 poles at £13 6*s*. 8*d*. per acre.

36. 2 acres 3 roods 7 poles at 11*s*. per rood.
37. 57 acres 1 rood 12 poles at £5 13*s*. per acre.
38. 7 acres 3 roods 13 poles at 18*s*. 6*d*. per rood.
39. 12 bus. 2 pks. 1 gal. at 14*s*. per bus.
40. 7 bus. 3 pks. 1 gal. at 7*s*. 6*d*. per pk.
41. 10 bus. 1 pk. 1 gal. at 13*s*. 6*d*. per bus.
42. 317 yds. 3 qrs. 3 nls. at 1*s*. 6*d*. per qr.
43. 35 weeks 4 days 12 hours at £1 10*s*. per week.
44. 8 weeks 6 days 8 hours at 14*s*. per day.
45. 9 reams 6 quires at £8 6*s*. per ream.

MISCELLANEOUS EXERCISES IN PRACTICE.

NOTE.—The scholars should be encouraged to work the following examples by more than one method.

1. When wheat is sold at 42*s*. 6*d*. per qr., what does a farmer get for 185 qrs. 6 bus. 3 pks. ?

2. What does a grocer pay for 12 cwts. 2 qrs. 17 lbs. of sugar at 26*s*. 6*d*. per cwt. ?

3. For how much does a grocer sell 12 cwts. 2 qrs. 17 lbs. of sugar at 3½*d*. per lb. ?

4. What should a coal-merchant charge for 12 tons 16 cwts. 3 qrs. of coals, when they are sold at 14*s*. 6*d*. a ton ?

5. What is the selling price of 25 cwts. 2 qrs. 18 lbs. of tea, being the quantity sold by a grocer in a year, at 3*s*. 4½*d*. per lb. ?

6. Find by practice the income-tax on £314 6*s*. 8*d*. at 1*s*. 3*d*. in the £.

7. What sum of money will be required to pay the wages of 35 workmen for 50 days at 3*s*. 6*d*. each per day ?

8. What is a bankrupt worth when he owes £396 10*s*. 6*d*. and can pay only 5*s*. 6*d*. in the £ ?

9. What would a silversmith charge you for a silver match-box, weighing 2 ozs. 3 dwts. 4 grs., at 15*s*. 6*d*. per oz. ?

10. How much does a merchant pay in duty on 486 cwts. 3 qrs. of sugar at 5*s*. 6*d*. per cwt. ?

11. What should I pay for constructing a fence, 2 furlongs 13 *perches 4 yards* long, at the rate of 1*s*. 6¾*d*. per yard ?

12. What is the expense of constructing a railway 115 miles 3 furs. long at £5,000 per mile?

13. Find the value of 132¾ cwts. of old iron at ¾*d.* per lb.

14. If gold be valued at £3 15*s.* 6*d.* per oz., find the worth of 8 lbs. 7 ozs. 14½ dwts.

15. Find the dividend on £305 12*s.* 6*d.* at 3*s.* 6*d.* in the £.

16. What will be the exact wages for 7 weeks 5 days at £1 1*s.* per week?

17. If hops are worth £2 19*s.* 6*d.* per cwt., what should be paid for 15 cwts. 3 qrs. 15 lbs.?

18. Find the cost of 18 lbs. 6 dwts. of silver at 7*s.* 6*d.* per oz.

19. If a builder charges £310 3*s.* 4*d.* for erecting a wall a mile long, what would be his account for enclosing a park 25 miles 2 furs. 110 yds. round, at the same rate?

20. Find the dividend on £817 16*s.* 8*d.* at 15*s.* 6*d.* in the pound.

21. A tradesman owes his creditors £8,050, and pays 6*s.* 9*d.* in the £; how much do all his creditors lose?

22. What does a person pay in income-tax on his yearly income of £368 15*s.* when the tax is levied at 4*d.* in the £?

23. Find what 3 cwts. 1 qr. 11 lbs. of plum-cake will cost at 6*s.* 3*d.* per stone; and what will the same quantity of plain bread cost at 1*s.* 7*d.* per stone?

24. What should a farmer pay for the mowing of his fields, measuring 180 acres 3 roods 15 perches, at 15*s.* 6*d.* per acre?

25. A grocer bought sugar, weighing 9 cwts. 3 qrs. 15 lbs. at 8¼*d.* per lb., and the same quantity at 3¾*d.* per lb.; what was the difference in the amount on the two invoices?

26. What will it cost to drain 560 acres of land at 13*s.* 6*d.* per acre; and to sow it at 5*s.* 6*d.* per acre?

27. What wages will be due to a servant for 13 weeks 3½ days at 1*s.* 8*d.* per week?

28. If the rent of a house is £65 per annum, what should I have to pay for 2 months 3 weeks 8 days' residence, reckoning 4 weeks to the month?

CHAPTER XXXI.

MISCELLANEOUS EXERCISES.

(On the preceding rules.)

TYPICAL examples to be worked on the black-board by the teacher, assisted by scholars.

Example 1.—Divide 10 gs., £10, 10 half-sovereigns, 10 crowns, 10 half-crowns, 10 shillings, and 10 sixpences equally among 72 persons.

		£	*s.*	*d.*
10 guineas	=	10	10	0
10 £	=	10	0	0
10 half-sovs.	=	5	0	0
10 crowns	=	2	10	0
10 half-crowns	=	1	5	0
10 shillings	=		10	0
10 sixpences	=		5	0
Total to be divided among 72 persons	=	30	0	0

```
      £   s.   d.
  72)30   0    0(8s. 4d.
     20
    ----
    600(8
    576
    ----
     24
     12
    ----
    288(4
    288
    ====
```

Ans. 8*s.* 4*d.*

Example 2.—£28 9*s.* 3*d.* is the exact sum of an equal number of *pounds*, *shillings*, and *pence*; what is the number of each?

This question means that we have to take away a sovereign, a shilling, and a penny, and again a sovereign, a shilling, and a penny, and so on until nothing remains. Of course we shall have an equal number of pounds, shillings, and pence. In other words we have to find how many times a £, a shilling, and a penny, together, are contained in £28 9*s*. 3*d*.

d.
£1 + 1*s*. + 1*d*. = 253
£28 9*s*. 3*d*. = 6831

```
253)6831(27
     506
     ----
     1771
     1771
     ====
```

253*d*. is contained 27 times in 6831*d*., *i.e.* £1, 1*s*., and 1*d*. are contained each 27 times in £28 9*s*. 3*d*.

Ans. 27 of each coin.

Example 3.—I wish to divide £10 among four persons so that for every 8*d*. I give the first, I give the second 6*d*., the third 4*d*., and the fourth 2*d*.; what is the total amount received by each?

If I make one distribution according to the terms of the question, I distribute 8*d*. + 6*d*. + 4*d*. + 2*d*. = 20 pence.

The question then is how many times can I do this, or how many times is 20*d*. contained in £10.

£10 = 2400*d*.

```
2,0)240,0
    -----
     120
     ===
```

I can distribute 8*d*., 6*d*., 4*d*., and 2*d*. 120 times so that

					£
the	1st will get	8*d*. × 120	=	960*d*.	= 4
„	2nd „	6*d*. × 120	=	720*d*.	= 3
„	3rd „	4*d*. × 120	=	480*d*.	= 2
„	4th „	2*d*. × 120	=	240*d*.	= 1
					£10

Ans. £4, £3, £2, £1.

Example 4.—(*a*) If one yard of cloth costs 5*s.*; what will 15 yds. cost? (*b*) If 15 yds. cost £3 15*s.*, what will 1 yard cost? (*c*) At the same rate, what will 7 yds. cost? (*d*) If 7 yds. cost £1 15*s.*, what will 15 yds. cost?

(*a*) 1 yard costs 5*s.*
∴ 15 yds. cost 5*s.* × 15 = £3 15*s.*

(*b*) 15 yds. cost £3 15*s.*
∴ 1 yd. costs £3 15*s.* ÷ 15 = 5*s.*

(*c*) 1 yd. costs 5*s.*
∴ 7 yds. cost 5*s.* × 7 = £1 15*s.*

(*d*) 7 yds. cost £1 15*s.*
∴ 1 yd. costs £1 15*s.* ÷ 7 = 5*s.*
and ∴ 15 yds. cost 5*s.* × 15 = £3 15*s.*

or £1 15*s.* ÷ 7 × 15
or £1 15*s.* × 15 ÷ 7

	£	*s.*	*d.*
	1	15	0
			15
7)	26	5	0
	3	15	0

NOTE.—The last part (*d*) of Example 4 is an ordinary exercise in "Simple Proportion." The method adopted above merely involves multiplication and division. It is sometimes called the *unity* or *unitary* method, because we first compare numbers with unity (unit); and sometimes "First principles," because we depend on the simplest elementary processes to solve our problems.

In solving problems of this type it is important to lead the scholars to see in the first place *what is required*, and then that *something is given* of the like nature. For instance, in Ex. 4 (*d*), we are required to *find* the value of 15 yards, and we are *given* the fact that 7 yards cost £1 15*s*. It is the data given which must always form the first line of the working, and it is this line which pupils find most difficult to construct. Practice only can give the required facility, but the following rule will materially assist.

In the first line, place that term last which is of like kind to the required answer.

Example 5.—(*a*) If 15 oranges can be bought for 3¾*d*., what will 37 cost? (*b*) If I pay 3¾*d*. for 15 oranges, how many can I buy for 9¼*d*.?

(*a*) 15 oranges cost 3¾*d*.
∴ 1 orange costs 3¾*d*. ÷ 15
and ∴ 37 oranges cost 3¾*d*. ÷ 15 × 37

$$\begin{array}{rrl}
 & & 3\frac{3}{4}d. \times 87 \\
 & & 6 \\ \cline{2-3}
 & 1s. & 10\frac{1}{2}d. \\
 & & 6 \\ \cline{2-3}
 & 11s. & 3d. \\
 & & 3\frac{3}{4}d. \\ \cline{2-3}
15\Big\{ & 5)\overline{11s.} & 6\frac{3}{4}d. \\ \cline{2-3}
 & 3)2s. & 3\frac{3}{4}d. \\ \cline{2-3}
 & & 9\frac{1}{4}d. \\ \hline\hline
\end{array}$$

Ans. 37 oranges can be bought for 9¼d.

(*b*) 3¾d. or 15 farthings will purchase 15 oranges.
∴ 1 farthing will purchase (15 ÷ 15) oranges.
and ∴ 9¼d., or 37 farthings, will purchase 15 ÷ 15 × 37
Ans. = 37 oranges.

The scholars shoud be instructed (except in such a very simple example as the one just given), *first* to multiply and then to divide.

Example 6.—(*a*) When the income-tax is 3*d*. in the £, what is the tax on an income of £320? (*b*) My income-tax amounts to £4 on an income of £320; what is the amount of the tax in the £? (*c*) My income-tax at 3*d.* in the £ amounts to £4; what is my income? (*d*) When I have paid my income-tax of 3*d.* in the £, I have £316 left; what is my income?

(*a*) £1 yields a tax of 3*d.*
∴ £320 yield a tax of 3*d.* × 320 = £4.

(*b*) £320 yield a tax of £4
∴ £1 yields a tax of £4 ÷ 320
and £4 or 960*d.* ÷ 320 = 3*d.*

(*c*) Every 3*d.* I pay denotes an income of £1
∴ ,, 1*s.* ,, ,, ,, £4
∴ ,, 20*s.* ,, ,, ,, £80
∴ ,, £4 ,, ,, ,, £320.

(*d*) After paying 3*d.* tax out of each £ of my income I shall have left 19*s.* 7*d.* or 237*d.*, that is,

237*d.* is left out of £1
∴ 1*d.* ,, ,, £1 ÷ 237
∴ 240*d.* ,, ,, £1 ÷ 237 × 240
or £1 ,, ,, £1 ÷ 237 × 240
∴ £316 ,, ,, £1 ÷ 237 × 240 × 316.

316
240
12640
632
237)75840(320
711
474
474
0

Ans. £320 is the income.

Example 7.—(*a*) If 21 bushels of oats will feed 35 horses for a week, how many bushels are required to feed 25 horses? (*b*) If 35 horses consume 21 bushels in a week, how many horses will consume 15 bushels?

(*a*) 35 horses require 21 bushels,
∴ 1 horse will require (21 ÷ 35) bushels, and
∴ 25 horses will require (21 ÷ 35 × 25) bushels.
Ans. = 15 bushels.

(*b*) 21 bushels serve 35 horses,
∴ 1 bushel will serve (35 ÷ 21) horses,
∴ 15 bushels will serve (35 ÷ 21 × 15) horses.
Ans. = 25 horses.

Example 8.—If it takes 7½ hours to travel 150 miles, how long will it take to travel 20 miles?

150 miles are gone over in 7½ hours, or 15 half-hours,
∴ 1 mile is gone over in (15 ÷ 150) half-hours;
∴ 20 miles are gone over in (15 ÷ 150 × 20) half-hours.
Ans. = 2 half-hours, or 1 hour.

Example 9.—(*a*) A commercial traveller receives a commission of £2 10*s*. on every £100 worth of goods he sells; what is his commission on the sale of £750 worth of goods? (*b*) If the commission is £2 10*s*. per cent., required the value of goods sold to realise £18 15*s*. (*c*) If £18 15*s*. be realised on the sale of £750 worth of goods, what is the commission per cent.?

(*a*) £100 realises £2 10*s*.
∴ £1 will realise £2 10*s*. ÷ 100
∴ £750 " " £2 10*s*. ÷ 100 × 750.
Ans. = £18 15*s*.

(*b*) £2 10*s*. = 50*s*.
£18 15*s*. = 375*s*.

50*s*. is the commission on £100 worth of goods,
∴ 1*s*. " " (£100 ÷ 50) worth of goods,
∴ 375*s*. " " (£100 ÷ 50 × 375) worth of goods.
Ans. = £750 worth of goods.

(*c*) £750 give a commission of £18 15*s.*
∴ £1 gives „ „ £18 15*s.* ÷ 750
∴ £100 give „ „ £18 15*s.* ÷ 750 × 100.
Ans. = £2 10*s.*

Example 10.—(*a*) A garrison, consisting of 3,264 men, has provisions for 243 days; how long will the provisions last if the garrison be increased by 1,632 men? (*b*) A garrison of 4,896 men has provisions for 162 days; how many men must be sent away so that the provisions may last 243 days?

(*a*) 3264 men have food for 243 days,
∴ 1 man will have food for 243 days × 3264
∴ 4896 men will have food for 243 days × 3264 ÷ 4896.
Ans. = 162 days.

(*b*) We have
162 days' provisions for 4896 men,
∴ 1 day's „ „ (4896 × 162) men,
∴ 243 days' „ „ (4896 × 162 ÷ 243) men.
= 3264 men.

That is, we must send away 4896 — 3264 = 1632 men.

NOTE.—Example 10 contains two exercises in what is sometimes called *inverse proportion.* The attention of the children should be directed to the difference in the working of these and the preceding examples.

Example 11.—A house, rated at £90, pays £4 10*s.* for the poor-rate; what is the amount of the poor-rate on a house rated at £240?

£90 gives £4 10*s.* as poor-rate,
∴ £1 * „ £4 10*s.* ÷ 90 as poor-rate,
∴ £240 „ £4 10*s.* ÷ 90 × 240 as poor-rate.
Ans. = £12.

Example 12.—(*a*) A bankrupt owes £3,500; he can pay 5*s.* 6*d.* in the £; what are his effects? (*b*) A bankrupt owes £3,500; his effects are worth £962 10*s.*; how much can he pay in the £? (*c*) A bankrupt pays 5*s.* 6*d.* in the £; his effects are worth £962 10*s.*; what is the amount of his debts?

---

* This gives the amount in the £, viz. 1*s.*

(*a*) For every £ the bankrupt pays 5*s.* 6*d.*
∴ for £3,500 „ „ 5*s.* 6*d.* × 3,500
= £962 10*s.*

(*b*) £3,500 produces £962 10*s.*
∴ £1 „ £962 10*s.* ÷ 3,500
= 5*s.* 6*d.*

(*c*) 5*s.* 6*d.* represents £1 of debt, or
11 sixpences „ £1 „
1 sixpence „ (£1 ÷ 11) of debt,
∴ £962 10*s.*, or 38,500 sixpences, represents (£1 ÷ 11 × 38,500) of debt.

Ans. = £3,500 debt.

Example 13.—A banker lent a man £300, to be paid back at the end of 3 years, on condition that he also paid £5 for the use of every £100 per year; what was the total amount to be paid?

£ £
100 for 1 year gives 5 for the use
300 „ 1 year „ 15 „
300 „ 3 years „ 45 „

Therefore the total to be paid is £300 + £45 = 345.

NOTE.—In this example the money lent is called the PRINCIPAL, the money paid for the use of the money lent is the INTEREST. The interest is said to be so much *per cent. per annum,* viz. so much per £100 per year.

MISCELLANEOUS EXERCISES.

1. If 4 lbs. of sugar cost 1*s.* 8*d.*, what will 20 lbs. cost?

2. If 5 horses can be bought for £288 15*s.*, what must be given for 45 at the same rate?

3. 6 yards of cloth cost me £2 15*s.* 6*d.*; what shall I have to pay for 42 yards more at the same price?

4. The price of 7 lbs. of tea is £1 1*s.* 3¼*d.*; what will a chest weighing 63 lbs. cost?

5. A truck load of coal weighing 7 tons cost me £6 6*s.* 7*d.*; what should I charge my neighbour for 4 tons of it?

6. 12 bushels of potatoes are worth £1 16*s.* 6*d.*; what will 8 sacks of 4 bushels each cost?

7. 17 lbs. of bacon are bought for 12*s.* 9*d.*; what will 23 lbs. cost at the same price?

8. 15 yards of calico cost me 3*s.* 9*d.*; what should I give for 3 pieces, each measuring 56 yards?

9. A man earns in 6 weeks £36 18*s.* 9*d.*; what does he earn in 37 weeks?

10. When 3 cwts. of sugar cost 36*s.* 2*d.*, what should be given for 272 cwts.?

11. If I buy eggs at the rate of 8 for 1*s.*, what must I give for 15 score?

12. I bought black currants at the rate of 5 lbs. for 1*s.* 0½*d.*; what will 39 lbs. cost me?

13. A farmer had 315 sheep in his field; he sold 120 of them for £300; what will he get for the remainder at the same rate?

14. Divide £16 10*s.* between John, Henry, and Thomas, giving John 10*s.* to every 7*s.* to Henry, and to every 5*s.* to Thomas; what will each receive?

15. Plums are bought at the rate of 5 quarts for 2*s.* 8½*d.*; what will 316 quarts cost?

16. If 15*s.* are given for 8 lbs. of butter, how much must be given for 58 lbs.?

17. If 18 lbs. of figs cost 13*s.* 6*d.*, what will ½ cwt. cost?

18. 16 quarts of strawberries are bought for 9*s.* 4*d.*; what will 10 gals. 2 qts. cost?

19. A bushel of beans can be bought for 7*s.* 2*d.*; what will 18 pks. cost?

20. 15 lbs. of soap cost 3*s.* 10½*d.*; find the price of ½ cwt.

21. 28 lbs. of sugar are bought for 8*s.* 2*d.*; what is the value of *1½ cwt.?*

22. When sugar is bought for 23*s*. 4*d*. a cwt., what is the value of 17 lbs.?

23. A chest of tea weighing 35 lbs. is bought for £3 18*s*. 9*d*.; what is the value of another chest weighing 76 lbs., at the same price?

24. How many eggs at 15 for 11¼*d*. can I get for 2*s*. 6*d*.?

25. If for 15 acres a farmer pays a rent of £67 10*s*., what will his whole farm of 450 acres cost him?

26. 560 boys and girls attend a certain school, and there are 3 boys to every 2 girls; how many girls are there?

27. 18 oranges out of a box of 1 gross are sold for 2*s*. 3*d*.; for how much will the remainder be sold at the same rate?

28. I bought 5 acres 3 roods of land for £575; for what should I sell 3 roods of it at the same rate; and for what should I sell them to gain £20 on the whole?

29. What will 260 lbs. of beef cost, if 18 lbs. cost 11*s*. 3*d*.?

30. How many lbs. of tea at 2*s*. 6*d*. per lb. in addition to 180 lbs. of tea at 1*s*. 8*d*. per lb. must be given in lieu of 5 chests, each weighing 56 lbs., at 3*s*. 4*d*. per lb.?

31. Make out the following bill:—3½ lbs. of arrowroot at 1*s*. 8½*d*. per lb., 25¼ lbs. rice at 3*d*. per lb., ¾ lb. carraway seed at 10*d*. per lb., 4 nutmegs at 3*s*. 6*d*. per doz.

32. Find the number of grains in a lb. troy, and calculate the value of 1 lb. of gold at the rate of 3*s*. 6*d*. for 5 dwts.

33. There are 7,000 grs. in a lb. avoirdupois; how many are there in an oz.? Find the difference in grains between an oz. avoirdupois and an oz. troy.

34. When a lb. of gold is worth £48, what should a sovereign weigh?

35. If 2 men, who do as much work as 5 boys, can reap a field in 30 days, in what time can 10 boys do it working alone?

36. A parcel is carried 125 miles for 7*s*. 6*d*.; how far can the same be carried for 1*s*. 6*d*.?

37. What is the value of 5 cwts. of soap at 3¼*d*. per lb., and 5 gross of black-lead at 3*s*. 4*d*. per packet? How much more or less than £15 will the whole cost?

38. If you lose 15*s*. by selling £5 worth of goods, calculate the loss on £100 worth?

39. If you gain 17*s.* 6*d.* by selling goods which cost £5, what will be your gain on goods bought for £85?

40. If the loss on £100 be £5, find the selling price; then find the selling price on goods which cost £35, and which were sold at a loss of 5 per cent.

41. What is the cost of 12 cwts. 3 qrs. 21 lbs. of sugar at 38*s.* 6*d.* per cwt., and what should the whole be sold for to gain 2*d.* on every 7 lbs.?

42. How many yards of cloth at 3*s.* 4*d.* per yard should be given for 5 pieces, each measuring 120 yards at 2*s.* 6*d.* per yard?

43. A bankrupt has in money £550 to pay £1,000; what will a creditor lose to whom he owes £250?

44. Pay 3*s.* 6*d.* in the £ on liabilities amounting to £360, and say how much your creditors will receive.

45. Pay 8*s.* 4*d.* in the £ on debts to the amount of £1,500, and say how much the creditors will lose.

46. In a piece of copper ore, weighing 5 lbs., there is found 1¼ lb. of pure copper; what weight of copper may be extracted from 5 tons of this ore, and what would be its value, when extracted, at £4 per cwt.?

47. The rents of a person's property amount to £2,500, and the land-tax is 6½*d.* in the £; what will be the net income?

48. Find by practice the dividend on £360 10*s.* 6*d.* at 17*s.* 9*d.* in the £.

49. How much tea, sold in 12-lb. parcels at 30*s.* each, can be bought for £15 15*s.*?

50. The tax on an income of £586 is £18 6*s.* 3*d.*; what is that in the £, and what will the tax amount to in the following year at the same rate when the income is increased by £154?

51. If 5 men and 3 boys reap a field in 18 days, how long will it take 2 men and 6 boys to do it, allowing that 3 boys do as much work as 2 men?

52. When the income-tax was 2*d.* in the £, I paid £12 10*s.* upon my income; what shall I have to pay upon the same income when the tax is 3*d.* in the £? also find my income.

53. When coffee is 1*s.* 1½*d.* per lb., I buy 3½ cwts.; how much *can* I buy with the same money, when it is 1*s.* 8¼*d.* per lb.?

54. To gain 3 per cent. I sell an article for 15*s.*; for what must I sell it to gain 5 per cent.?

55. If £350 put out to interest will gain £12 10*s.*, what principal will gain £17 in the same time?

56. If £25 put out to interest gain £3 in a given time, what principal will be required to gain £7 in half the time?

57. If in 2 years I can make £25, by putting out a certain sum of money to interest, for how many years should the same sum be at interest to gain £45?

58. If 6 men can erect part of a fence in 15 days, how many will be required to do twice the quantity in 45 days?

59. Find, by practice, the dividend on £280 16*s.* 8*d.* at 10*s.* 9*d.* in the £.

60. Calculate the value of 3 roods 16 perches at 80 guineas per acre.

61. There is enough grass in a field to graze 150 oxen for 25 days; how many oxen might graze for 125 days on the same quantity of grass?

CHAPTER XXXII.*

ELEMENTARY IDEAS OF A FRACTION, ETC.

It is impossible to work examples in *proportion* intelligently, without first understanding the nature of a *fraction*, for the ideas of the former are evolved from the latter.

It will therefore greatly facilitate after progress if the children are now taught thoroughly the nature of a *fraction*, of *ratio*, of *proportion*, and of an *equation*.

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* If the teacher is content to teach his scholars to work examples in proportion by the unity method only, the work of Standard V. may stop here; but as six months will probably be ample time to accomplish so much, we strongly advise that this and the following chapter be carefully studied both as an introduction to the work of Standard VI., and for a better understanding of problems in proportion.

I.—A FRACTION.

Take an apple and cut it into *two equal parts.*

Each of these parts is called *one-half.*

Cut each half into *two equal parts*; or, which is the same thing, cut the whole into *four equal parts.*

Each of these parts is called *one-fourth,* or a quarter.

A farthing, viz. *one-fourth* of a penny, is written thus, $\frac{1}{4}$. One halfpenny is written thus, $\frac{1}{2}$; but as a halfpenny is two farthings, or *two-fourths* of a penny, we may write it thus, $\frac{2}{4}$; and three farthings, or *three-fourths* of a penny is written $\frac{3}{4}$.

And so of the apple

one quarter, or one-fourth, is written $\frac{1}{4}$
one-half, or two-fourths, is written $\frac{2}{4}$
three-quarters, or three-fourths, is writtten $\frac{3}{4}$

Each of these parts, $\frac{1}{4}$, $\frac{2}{4}$, $\frac{3}{4}$, is said to be a FRACTION or *even part* of the apple.

Let us take one of these fractions, say $\frac{3}{4}$, and look at it a little more closely.

(1) We first *divided* the apple into four equal parts. The sign $\div$ indicates *division,* and we use this sign in the fraction to indicate division, but we omit the dots. The *line* then between the 3 and the 4 indicates *division.*

(2) We divided the apple into *four* equal parts, and we place the figure 4 *below* the line to show this particular division.

(3) We took *three* of those parts to make the *three-fourths,* and to show the number taken we place the figure 3 *above* the line.

A fraction then consists of three parts.

(*a*) The line to show division.
(*b*) A figure *below* the line, to show the number of equal parts into which the whole has been divided.

(*c*) A figure *above* the line to show how many of the equal parts have been taken.

Since the figure below the line *points out* or *denominates* the number of equal parts into which the whole has been divided, we call it the DENOMINATOR. And since the figure above the line shows the *number* of parts taken to form the particular fraction, we call it the numberator, or NUMERATOR. And since the line indicates division, we say a fraction is a numerator divided by a denominator, thus:

$$\frac{\text{Numerator}}{\text{Denominator}}$$

Suppose we divide the apple into *eight* equal parts; how many fractions can we make?

The denominator = 8.

The numerator may be 1, 2, 3, 4, 5, 6, 7, or 8.

And the fractions will be—

$$\frac{1}{8},\ \frac{2}{8},\ \frac{3}{8},\ \frac{4}{8},\ \frac{5}{8},\ \frac{6}{8},\ \frac{7}{8},\ \frac{8}{8}$$

In the last fraction we have taken the whole of the parts, that is the whole apple, and we see that when the numerator and denominator are equal the fraction is a whole number.

Let us now divide a second apple of similar size into eight equal parts; and let us take 3 of these parts as well as the 8 from our first apple; how shall we write this as a fraction?

Our whole *one*, or unit, is still divided into 8 equal parts; hence 8 will be our denominator; and, since we have taken 11 of these parts, our numerator will be 11, and the fraction $\frac{11}{8}$.

But this is plainly a very different sort of fraction from $\frac{3}{8}$, for instance. It is not, properly speaking, a part of an apple, but one whole apple, and part of another. It is therefore called an *improper fraction*. But $\frac{3}{8}$ and $\frac{1}{2}$ and $\frac{1}{4}$ being all parts of the one apple are called proper fractions.

We may write $\frac{11}{8}$ as $1\frac{3}{8}$, viz. one whole apple and $\frac{3}{8}$ of another.

Definitions :—

(1) A FRACTION is one or more of the equal parts of a whole.

(2) A PROPER FRACTION is one which has the numerator *less* than the denominator, as $\frac{3}{8}$.

(3) An IMPROPER FRACTION is one which has the numerator *not less* than the denominator, as $\frac{11}{8}$.

(4) A MIXED FRACTION is one which is made up of a *whole number and a fraction*, as $1\frac{3}{8}$.

MIXED AND IMPROPER FRACTIONS.

We may change an improper fraction into a mixed fraction and a mixed fraction to an improper fraction.

Thus (1) $\frac{11}{8} = \frac{8}{8} + \frac{3}{8} = 1\frac{3}{8}$

„ (2) $\frac{15}{6} = \frac{6}{6} + \frac{6}{6} + \frac{3}{6} = 2\frac{3}{6} = 2\frac{1}{2}$

„ (3) $\frac{17}{4} = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = 4\frac{1}{4}$.

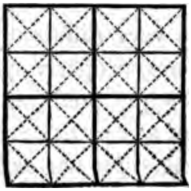
In (1) we found first the number of eights in 11.

In (2) the number of sixes in 15.

In (3) the number of fours in 17, and these gave whole numbers. Hence (1) *To reduce an improper fraction to a mixed fraction, divide the numerator by the denominator.* (2) *To reduce a mixed fraction to an improper fraction, multiply the whole number by the denominator of the fraction, and add the numerator to make the new numerator.*

LOWEST TERMS.

Draw a square on the black-board; divide it into 4 equal squares, and these again each into 4 equal squares.



From this it is easy to elicit from the scholars that—

(1) $\frac{1}{4} = \frac{4}{16}$ (2) $\frac{1}{2} = \frac{8}{16}$

(3) $\frac{3}{4} = \frac{12}{16}$ (4) $\frac{4}{4} = \frac{16}{16}$

If we again divide each small square into 2 equal triangles and then into 4, that—

(1) $\frac{1}{4} = \frac{4}{16} = \frac{8}{32} = \frac{16}{64}$

(2) $\frac{1}{2} = \frac{8}{16} = \frac{16}{32} = \frac{32}{64}$

(3) $\frac{3}{4} = \frac{12}{16} = \frac{24}{32} = \frac{48}{64}$

From this we see that to multiply (and hence also to divide) the numerator and denominator of a fraction by the same number does not alter its value.

Thus in (3) we obtain $\frac{12}{16}$ from $\frac{3}{4}$ by multiplying numerator and denominator by 4; and then $\frac{24}{32}$ from $\frac{12}{16}$ by multiplying numerator and denominator by 2, and so on.

Again (3), we can get $\frac{3}{4}$ from $\frac{48}{64}$ by dividing numerator and denominator by 4, and then again by 4.

Thus :—

$$\frac{48 \div 4}{64 \div 4} = \frac{12}{16}$$

$$\frac{12 \div 4}{16 \div 4} = \frac{3}{4}$$

This last process is called reducing to lowest terms. In a future lesson we shall call it "cancelling."

ORAL EXERCISES.

1. Divide an apple into 8 equal parts; give Mary 2 parts and John 3 parts; what fraction of the whole will you have given away, and what fraction will you have left?

2. I want to divide an orange, giving John and William equal portions, and Jane as much as John and William together; how shall I divide the orange, and what fraction of the whole shall I give to each?

3. A post is driven $\frac{1}{3}$ of its length into the ground, and there are 6 feet above the ground; what is the length of the post?

4. My garden measures 60 feet in length; what is the length of $\frac{1}{2}$, $\frac{2}{3}$, and $\frac{3}{4}$, respectively?

5. $\frac{2}{3}$ of my week's wages is 18*s*.; what do I earn per week?

6. I am 16 years old, and $\frac{2}{5}$ of the age of my father; what is my father's age?

7. My father's age is 65; I am $\frac{2}{5}$ of my father's age; how old am I?

8. How much is—
   (1) $\frac{2}{3}$ of 1*s*.?
   (2) $\frac{3}{4}$ of 1*s*. 4*d*.?
   (3) $\frac{2}{9}$ of 27*s*.?
   (4) $\frac{2}{7}$ of 21*s*.?
   (5) $\frac{4}{5}$ of 2*s*. 6*d*.?
   (6) $\frac{3}{8}$ of 30*s*.?

9. What is the difference between—
   (1) $\frac{1}{3}$ of 1*s*. and $\frac{1}{2}$ of 6*d*.?
   (2) $\frac{2}{5}$ of 1*s*. 3*d*. and $\frac{3}{4}$ of 8*d*.?
   (3) $\frac{4}{7}$ of 2*s*. 4*d*. and $\frac{5}{9}$ of 2*s*. 3*d*.?
   (4) $\frac{3}{8}$ of £2 and $\frac{3}{7}$ of 2 guineas?
   (5) $\frac{7}{10}$ of 5*d*. and $\frac{5}{12}$ of 6*d*.?
   (6) $\frac{5}{6}$ of 3 half-crowns and $\frac{6}{7}$ of 7*s*.?

10. How much is $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, $\frac{4}{5}$, and $\frac{5}{6}$ of an hour respectively?

11. Which is the greater, and by how much, $\frac{2}{3}$ of a foot or $\frac{5}{18}$ of a yard?

12. What is the price of $\frac{3}{8}$ of a lb. of butter at 2*s*. per lb.? and of $\frac{2}{5}$ at 1*s*. 8*d*. per lb.?

13. How many cwts. in $\frac{3}{4}$ added to $\frac{4}{5}$ of a ton?

14. What is the cost of $2\frac{1}{3}$ yds. of ribbon at 4*d*. per yd.?

15. $\frac{2}{7}$ of the contents of a cistern are emptied in 30 minutes; how long will it take to empty the cistern?

SLATE OR PAPER EXERCISES.

1. Find the half of £2 16*s*. 9*d*., and the third of £1 13*s*. $6\frac{3}{4}$*d*.

2. How much is $\frac{2}{3}$ of 17*s*. 9*d*.? Multiply £4 13*s*. 6*d*. by $\frac{3}{4}$.

3. How many lbs. are there in $\frac{4}{7}$ of a ton?

4. $\frac{4}{5}$ of a house is worth £567; what is the value of the house?

5. Multiply £17 13*s*. 9*d*. by $6\frac{3}{4}$.

6. Which is the greater, and by how much, $\frac{3}{7}$ of 20 guineas or $\frac{2}{3}$ of £20?

7. My income is £434 10*s*. If I spend $\frac{3}{5}$ of it, how much do I save?

8. In copying an example I wrote $\frac{3}{5}$ of £3 instead of $\frac{3}{8}$ of £3; what was the error in the amount?

9. Add together $\frac{2}{7}$ of a guinea, $\frac{4}{5}$ of a £, and $\frac{2}{3}$ of 1*s*.

10. What is the difference between $\frac{5}{12}$ of a guinea and $\frac{5}{12}$ of a £?

11. If $\frac{2}{3}$ of an estate is worth £18,000, what is the value of the whole?

12. $\frac{3}{5}$ of an estate is sold for 1,000 guineas; what is the value of $\frac{1}{3}$ of the estate?

13. If $\frac{5}{7}$ of a ship's cargo is worth 1,000 guineas, what is the value of $\frac{3}{5}$ of it?

14. Add together $\frac{2}{5}$ of a crown, $\frac{3}{5}$ of half-a-sovereign, $\frac{1}{5}$ of a sovereign, and $\frac{4}{5}$ of a £5 note.

15. Subtract $\frac{6}{7}$ of £25 4*s*. from $\frac{7}{9}$ of £23 8*s*.

16. Divide $\frac{5}{6}$ of a guinea, $\frac{7}{12}$ of a £, and $\frac{3}{8}$ of a florin equally among 359 persons.

17. Reduce the following mixed to improper fractions—

$$2\frac{3}{4},\ 5\frac{6}{7},\ 4\frac{3}{5},\ 11\frac{1}{3},\ 19\frac{6}{7}.$$

18. Reduce the following improper fractions to whole numbers and proper fractions:—

$$\frac{11}{4},\ \frac{41}{7},\ \frac{23}{5},\ \frac{34}{3},\ \frac{139}{7}.$$

19. Reduce to their lowest terms—

$$\frac{16}{20},\ \frac{3}{15},\ \frac{25}{35},\ \frac{24}{48},\ \frac{17}{119}.$$

20. Draw a square, and divide it—

(1) To show that if $\frac{5}{8}$ are taken from it $\frac{3}{8}$ are left.

(2) To show that $\frac{2}{8} + \frac{6}{8}$ is the same as $\frac{1}{4} + \frac{3}{4}$, and that it makes up the whole.

(3) To show that $\frac{1}{4}$ of $\frac{1}{4}$ is $\frac{1}{16}$ of the whole.

II.—RATIO.

Place on the table a group of 12 pennies and another of 4 pennies, and ask the children to compare these two sums as to their quantity or magnitude.

We shall probably get two answers,—

(1) 12*d.* is 8*d.* more than 4*d.*
(2) 12*d.* is *three times* as great as 4*d.*

The first answer is perfectly correct, it shows a certain relation between the sums; but it is with the second answer that we now propose to deal.

" 12*d.* is 3 times 4*d.*"

We obtained the 3 by dividing 12*d.* by 4*d.*; viz. the first sum by the second.

The number 3 is said to show what relation 12*d.* bears to 4*d.*

If then we ask the question, "What relation is there between 12*d.* and 4*d.*?" the answer will be *three;* and we obtain the *three* by dividing 12*d.* by 4*d.*; *the first by the second.*

What relation is there between—

(1)	8*d.* and 2*d.*?	*Ans.* 4.
(2)	9*d.* and 3*d.*?	,, 3.
(3)	16*d.* and 2*d.*?	,, 8.
(4)	4*d.* and 8*d.*?	,, $\frac{1}{2}$.
(5)	2*d.* and 8*d.*?	,, $\frac{1}{4}$.
(6)	3*d.* and 9*d.*?	,, $\frac{1}{3}$.

Next place 8 marbles and 2 pennies on the table, and ask "What relation is there between these groups?" We may get the answer *four.* But another question, "Can 2*d.* be contained *four* times in 8 marbles?" will show the impossibility of the comparison.

relation can exist only between things of the same kind. ere can be no relation between pounds and yards, or tween schools and stones.

We have then to remember,

1. That when we compare two numbers, we compare the st with the second by dividing the first by the second, d the quotient is the relation or RATIO between the mbers.

2. That we can have a ratio only between numbers which nd for things of a like kind.

The teacher should fix these two ideas in the minds of the holars by a few oral questions, thus:—

What is the ratio of—

12 lbs. to 4 lbs.? *Ans.* 3.
12 lbs. to 2 yds.? Impossible.
18 yds. to 2 yds.? *Ans.* 9.
3 feet to 6 feet? *Ans.* $\frac{1}{2}$.
5*s.* to 15*s.*? *Ans.* $\frac{1}{3}$.

We have been talking of the *ratio of one number to other.* It would be inconvenient to often write so long an pression when we are dealing with ratios, and so we adopt symbol, just as we do for addition and multiplication, for a action, &c.

To find the ratio of 12 to 4, we divide the first number by e second, and so we may write $12 \div 4$, or, as a fraction, $\frac{12}{4}$, ut in practice we express the ratio thus, $12:4$.

We saw that in a fraction we used the sign for division mitting the dots. In ratio we adopt the dots and omit the ne.

It must be strictly borne in mind that $12 \div 4$, $\frac{12}{4}$, or* $2:4$ all have precisely the same meaning.

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* The first term of the ratio is called the *antecedent*, and the second the *onsequent.*

It is not usual actually to perform the division and find the ratio: we merely indicate it thus—

The ratio of—

(1) 9 to 6 = 9 : 6 or $\frac{9}{6}$ or 9 ÷ 6.
(2) 6 to 9 = 6 : 9 or $\frac{6}{9}$ or 6 ÷ 9.
(3) 1 to 5 = 1 : 5 or $\frac{1}{5}$ or 1 ÷ 5.
(4) 3 to 1 = 3 : 1 or $\frac{3}{1}$ or 3 ÷ 1.

Note.—We cannot have a ratio between yards and feet, or between shillings and pence, but we can reduce the yards to feet, or the feet to yards; and the shillings to pence or the pence to shillings, and then we can express the ratio.

ORAL EXERCISES.

1. Express the ratios of the following:—
   (1) 18 cows to 12 cows.
   (2) 19 horses to 37 horses.
   (3) 12*s*. to 13*s*.
   (4) £1 to £4.
   (5) 5 to 9.
   (6) $\frac{2}{3}$ to $\frac{3}{4}$.
2. Express as fractions the ratios of the following:—
   (1) 3 lbs. to 5 lbs.
   (2) 25 yds. to 10 yds.
   (3) 1 foot to 9 feet.
   (4) 3 : 7.
   (5) 7 : 3.
   (6) 21 : 19.
3. Express the ratios of—
   (1) 12*s*. 6*d*. to 18*s*. 9*d*.
   (2) 4 tons to 12 cwts.
   (3) £1 15*s*. to 11*d*.
   (4) 13 ozs. to 15 lbs. (Troy).
   (5) 13 ozs. to 15 lbs. (Avoir.).
   (6) £1 13*s*. 9½*d*. to £2 15*s*. 7½*d*.

III.—PROPORTION. EQUATION.

$$\begin{array}{rcl} 9 \div 3 & = & 3 \\ 18 \div 6 & = & 3 \end{array}$$

Hence we may write—

$$\begin{array}{llcl} (1) & 9 \div 3 & = & 18 \div 6 \\ (2) \text{ or} & \frac{9}{3} & = & \frac{18}{6} \\ (3) \text{ or} & 9:3 & = & 18:6 \end{array}$$

That is, the ratio of 9 to 3 is equal to the ratio of 18 to 6, and we have indicated this by placing the sign of equality (=) between them. But here again in practice it is not customary to use the sign of equality, but in place thereof four dots, thus ::

So that we show *equal ratios* by writing thus—

$$9:3 :: 18:6$$

We read this "9 has the same relation to 3 as 18 has to 6," or shortly,

"9 is to 3 as 18 is to 6."

The numbers in the ratio are called *terms*.

	1st term.		2nd term.		3rd term.		4th term.
(*a*)	9	:	3	::	18	:	6

	1st term.		2nd term.		3rd term.		4th term.
(*b*) or	3	:	9	::	6	:	18

It will be seen from these two expressions of equal ratios that: The *first* term is as many times greater or less than the *second* as the *third* term is greater or less than the *fourth*.

In (*a*) the first term is three times greater than the second, and so the third is three times greater than the fourth.

In (*b*) the first term is three times less than the second, and so the third term is three times less than the fourth.

Definition :—Four terms are said to be in Proportion, *when the first term is as many times greater or less than the second as the third is greater or less than the fourth.*

The terms at the ends, viz. the 1st and 4th, are called *extremes;* the middle terms, viz. 2nd and 3rd, are called *means.*

When four numbers are in proportion, the product of the extremes is equal to the product of the means.

Illustrate this by examples, thus :—

$$\begin{aligned}(1)\ 4:3::8:6 \quad \text{but} \quad 4\times 6 &= 24\\ \text{and} \quad 3\times 8 &= 24\end{aligned}$$

$$\text{that is } \overset{\text{extremes.}}{4\times 6} = \overset{\text{means.}}{3\times 8} \quad (a)$$

$$\begin{aligned}(2)\ 1:3::6:18 \quad \text{but} \quad 18\times 1 &= 18\\ \text{and} \quad 6\times 3 &= 18\end{aligned}$$

$$\text{that is } \overset{\text{extremes.}}{18\times 1} = \overset{\text{means.}}{6\times 3} \quad (b)$$

$$\begin{aligned}(3)\ 9:3::18:6 \quad \text{but} \quad 6\times 9 &= 54\\ \text{and} \quad 3\times 18 &= 54\end{aligned}$$

$$\text{that is } \overset{\text{extremes.}}{6\times 9} = \overset{\text{means.}}{3\times 18} \quad (c)$$

The expressions (*a*) (*b*) (*c*) are called Equations, because we have a number on one side of the sign of equality exactly *equal to* the number on the other.

Given any three terms of a proportion and we can always find the fourth.

Write on the black-board the following four proportions:—

$$\begin{aligned}&(1)\ 2:4::4:8\\ &(2)\ 6:3::12:6\\ &(3)\ 6:4::3:2\\ &(4)\ 3:5::9:15\end{aligned}$$

Suppose in the first of these proportions we have to find the 1st term. Put x to stand for the term.

Then $x : 4 :: 4 : 8$

The product of the means $= 16$

$\therefore$ „ „ „ extremes $= 16$

But one factor of the product of the extremes, viz. 16, is 8, therefore the other must be $\frac{16}{8}$ or 2.

Secondly, to find the 2nd term in the 2nd proportion—

$6 : x :: 12 : 6$

The product of the extremes $= 36$

$\therefore$ „ „ means $= 36$

And one factor of the product of the means, viz. 36, is 12, therefore the other is $\frac{36}{12}$ or 3.

Thirdly, to find the 3rd term in the 3rd proportion given—

$6 : 4 :: x : 2$

The product of the extremes $= 12$

$\therefore$ „ „ „ means $= 12$

And one factor of the product of the means, viz. 12, is 4, therefore the other is $\frac{12}{4} = 3$.

Lastly, to find the 4th term of the 4th proportion—

$3 : 5 :: 9 : x$

The product of the means $= 45$

$\therefore$ „ „ „ extremes $= 45$

And one factor of 45 is 3, therefore the other is $\frac{45}{3} = 15$.

We may write the working of the four examples thus :—

(1) $x : 4 :: 4 : 8$

extremes. means.

$$8x = 16$$

$$x = \frac{16}{8} = 2$$

$$(2)\quad 6 : x :: 12 :: 6$$

means. extremes.

$$12\,x = 36$$

$$x = \frac{36}{12} = 3$$

$$(3)\quad 6 : 4 :: x : 2$$

means. extremes.

$$4\,x = 12$$

$$x = \frac{12}{4} = 3$$

$$(4)\quad 3 : 5 :: 9 : x$$

extremes. means.

$$3\,x = 45$$

$$x = \frac{45}{3} = 15$$

EXERCISES IN RATIO AND PROPORTION.

1. What is the ratio of 6*s*. to 9*s*.? of 14 lbs. to 6 lbs.? of 1 yard to 21 yards?

2. Find the missing term in each of the following proportions:—

   (1) 6 : 4 : :   : 12.
   (2) 5 : 9 : : 90 :
   (3) 2 :   : : 1 : 19.
   (4)   : 7 : : 3 : 2.

3. To what number has 15 the same ratio as 10 has to 50?

4. 18 has the same ratio to 39 as 3 has to a certain number; find that number.

5. Find a fourth proportional to 15, 27, and 600.

6. Complete the following proportions:—

   (1) 4 : 9 : : 8 :
   (2) 9 : 15 : : 27 :
   (3) 100 : 15 : : 400 :

Find the number which bears the same ratio to 8 which 7 to 4.

A father's age has the same proportion to his son's age as 9 ; if the father's age is 54, how old is the son?

One girl earns 6*d.* whilst another earns 5*d.*; when the second arned 190*d.*, how much will the first have earned?

CHAPTER XXXIII.

IPLE PROPORTION, OR RULE OF THREE.

rule by which the principles of proportion are put into tice is called the Rule of Three; because in all the tions proposed three terms of a proportion are given, and required to find the fourth.

xample 1.—If 4 oranges cost 3*d.*, what will 9 oranges ?

e have in this example three terms given, of which two alike, and the third is similar to the required answer.

ake the like terms, viz. 4 oranges and 9 oranges, and ess the ratio,

$$9 : 4 \quad \text{or} \quad 4 : 9$$

et x stand for the number of pence in the answer. What now have to do is to arrange the other two terms, viz. and x *d.* so as to make the same ratio as

$$9 : 4 \quad \text{or} \quad 4 : 9$$

o do this we have to ask the question, "Which is the ter, 3*d.* or x *d.*?" This is *the* question of every example roportion.

d. must be more than 3*d.*, because 3*d.* is the price of anges, and x *d.* is the price of 9 oranges.

So that—

$$\begin{array}{rl} (1) & 9 : 4 = x : 3 \\ \text{or } (2) & 4 : 9 = 3 : x \\ \text{or } (3) & x : 3 = 9 : 4 \\ \text{or } (4) & 3 : x = 4 : 9 \end{array}$$

In each of these proportions we have

$$4\,x = 27$$

$$\therefore\ x = \frac{27}{4} = 6\tfrac{3}{4}d.$$

Although it is quite immaterial in which place we put the unknown term, it is more convenient in practice to place it in the 4th place, as in No. (2) above.

Note.—In the proportion—

$$\overset{\text{oranges.}}{4} : \overset{\text{oranges.}}{9} :: 3d. : 6\tfrac{1}{2}d.$$

We cannot multiply 9 oranges by 3*d.* The fact is, ratios are always abstract numbers, and we deal with the numbers apart from the objects they represent.

Example 2.—If 9 books cost 15*s.*, what will 5 cost?

As for the future the unknown term is to occupy the 4th place in the proportion, we *first* express the ratio between the unknown term and the term of like kind in the question, thus— 15*s.* : x

Secondly, which is the larger of these two terms? 15*s.* is the larger because this is the price of 9 books, and the price of 5 books must be less.

Then—

$$\begin{array}{rl} & 9 : 5 = 15 : x \\ \text{or} & 9 : 5 :: 15 : x \\ \text{and} & 9\,x = 75 \end{array}$$

$$x = \frac{75}{9} = 8s.\ 4d.$$

From these examples the teacher will deduce the following *rules*—

1. Put x in the 4th place to stand for the required answer.

2. In the 3rd place put the term of the question which is of the like kind to the required answer.

3. Inquire which is the greater term, the 3rd or the 4th, and then arrange the 1st and 2nd terms accordingly. If the 4th term be greater than the 3rd, the 2nd term must be greater than the 1st; but if the 4th term be less than the 3rd then the 2nd term must be less than the 1st.

4. Reduce (if necessary) the 1st and 2nd terms to the same denomination.

5. Multiply the 2nd and 3rd terms together and divide by the first.

EXAMPLES FOR THE BLACK-BOARD.

Example 3.—If a trench 55 yds. long be dug by 25 men in a certain time, how many men will it require to dig a similar trench 110 yds. long in the same time?

It will require more men, hence the 2nd term must be the greater.

$$\overset{\text{yds.}}{55} : \overset{\text{yds.}}{110} :: \overset{\text{men.}}{25} : \overset{\text{men.}}{x}$$

$$x = \frac{110 \times 25}{55} = \frac{2750}{55}$$

We thus get an improper fraction, and, as we have seen, we may divide the numerator and denominator of a fraction by the same number without altering its value.

Divide by 11 and then by 5.

$$\frac{2750}{55} = \frac{250}{5} = \frac{50}{1} = 50$$

It is better to reduce the fraction before multiplying the factors of the numerator, but in this case we must draw a line through the number divided, and write the quotient above or below the original number thus :—

$$x = \frac{110 \times 25}{55}$$

(1) Divide by 11—

$$\frac{\overset{10}{\cancel{110}} \times 25}{\underset{5}{\cancel{55}}}$$

(2) Divide by 5—

$$\frac{\overset{10}{\cancel{110}} \times \overset{5}{\cancel{25}}}{\underset{\underset{1}{\cancel{5}}}{\cancel{55}}} = 50$$

This process is called *cancelling.*

Example 4.—If 40 men can build a wall in 28 days, how many men will it require to build it in 7 days?

It will take more men to build the wall in 7 days than in 28 days.

Hence— $\overset{\text{dys.}}{7} : \overset{\text{dys.}}{28} :: \overset{\text{men.}}{40} : \overset{\text{men.}}{x}$

$$x = \frac{\overset{4}{\cancel{28}} \times 40}{\underset{1}{\cancel{7}}} = 160$$

Example 5.—A ship's crew is provisioned with br for 18 weeks, at the rate of 2 lbs. per man per day; on morning of starting, the proposed voyage is changed another which will occupy 27 weeks; what must now b daily allowance per man?

The allowance will be less.

Hence— $\overset{\text{wks.}}{27} : \overset{\text{wks.}}{18} :: \overset{\text{ozs.}}{32} : \overset{\text{ozs.}}{x}$

$$x = \frac{\overset{2}{\cancel{18}} \times 32}{\underset{3}{\cancel{27}}} = \frac{64}{3} = 21\tfrac{1}{3} \text{ ozs. per day.}$$

Example 6.—A man has £393 6*s*. 8*d*. after paying his income-tax at the rate of 4*d*. in the £; what is his income?

After paying the tax he has 19*s*. 8*d*. out of every £.

$$\begin{array}{llll} \text{Hence—} & 19s.\ 8d. : & \text{£}393\ 6s.\ 8d. :: & \text{£}1 : \text{£}x \\ & \quad 12 & \quad\ \ 20 & \\ & \overline{236d.} & \overline{7866} & \\ & & \quad\ \ 12 & \\ & & \overline{94400d.} & \end{array}$$

$$x = \frac{94400}{236} = 400$$

Income is £400.

Example 7.—If a garrison of 1,200 men have provisions to last 18 weeks, how long will the provisions last if the garrison be increased by 600 men?

The total number to be provisioned will be 1,800, and the provisions will last this number a less time.

Hence— $\overset{\text{men.}}{1800} : \overset{\text{men.}}{1200} :: \overset{\text{wks.}}{18} : \overset{\text{wks.}}{x}$

$$x = \frac{12\cancel{00} \times \cancel{18}}{\cancel{1800}} = 12 \text{ weeks.}$$

Example 8.—A railway train starts at 6 o'clock in the morning, and travels at the rate of 30 miles an hour; a second train starts at 9 o'clock, and travels at 40 miles an hour; how far will the trains have gone when the second overtakes the first?

The first will have travelled 90 miles when the second starts. The question then becomes,—the second train gains 10 miles an hour, how long will it be in gaining 90 miles?

Hence—

$$\begin{array}{ccccccc} \text{miles.} & & \text{miles.} & & \text{hour.} & & \text{hours.} \\ 10 & : & 90 & :: & 1 & : & x \end{array}$$

$$\therefore \quad x = \frac{90 \times 1}{10} = 9 \text{ hours.}$$

$\therefore$ The distance travelled will be $9 \times 40 = 360$ miles.
or $12 \times 30 = 360$ „

Example 9.—If 4 cows *or* 6 horses can eat the produce of a field in 39 days, in how many days will 6 cows *and* 4 horses eat it up?

From the question—

4 cows eat as much as 6 horses
$\therefore$ 2 „ „ 3 „
and 6 „ „ 9 „
$\therefore$ 6 cows + 4 horses eat as much as 13 horses

The question now becomes—

If 6 horses can eat the produce in 39 days, how long will it take 13 horses to eat it?

It will take them a shorter time.

Hence—

$$\begin{array}{ccccccc} \text{horses.} & & \text{horses.} & & \text{days.} & & \text{days.} \\ 13 & : & 6 & :: & 39 & : & x \end{array}$$

$$x = \frac{6 \times \overset{3}{\cancel{39}}}{\cancel{13}} = 18 \text{ days.}$$

Example 10.—If 2,000 men have provisions for 95 days, and if after 15 days 400 men are sent away, how long will the remaining provisions serve the number left?

When the 400 men are sent away 1,600 men will be left with provisions for 2,000 for 80 days. This amount will serve 1,600 longer than 2,000.

$$\text{Hence—}\quad \begin{array}{ccccccc} \text{men.} & & \text{men.} & & \text{days.} & & \text{days.} \\ 1600 & : & 2000 & :: & 80 & : & x \end{array}$$

$$x = \frac{\overset{100}{\cancel{2000}} \times \cancel{80}}{\underset{20}{\cancel{1600}}} = 100 \text{ days.}$$

Example 11.—A garrison of 1,000 men was victualled for 30 days; after 10 days it was reinforced, and then the provisions were exhausted in 5 days; of how many men did the reinforcement consist?

After 10 days, provisions for 1,000 men for 20 days were left. These will serve a larger number of men for 5 days.

$$\text{Hence—}\quad \begin{array}{ccccccc} \text{days.} & & \text{days.} & & \text{men.} & & \text{men.} \\ 5 & : & 20 & :: & 1000 & : & x \end{array}$$

$$x = \frac{1000 \times \overset{4}{\cancel{20}}}{\cancel{5}} = 4000 \text{ men.}$$

Therefore the reinforcement was 4,000 − 1,000 = 3,000 men.

Example 12.—Find the interest on £1,113 6*s.* 8*d.* for 6 years at 4½ per cent.

We must first find the interest for one year and multiply this amount by 6.

$$\begin{array}{ccccccc} \text{Prin.} & & \text{Prin.} & & \text{Int.} & & \text{Int.} \\ 100 & : & £1113\ \ 6\ \ 8 & :: & 4\tfrac{1}{2} & : & x \end{array}$$

$$x = \frac{£1113\ \ 6\ \ 8 \times 4\tfrac{1}{2}}{100}$$

$$\begin{array}{rrrr} & 1113 & 6 & 8 \\ & & & 4\tfrac{1}{2} \\ \hline & 4453 & 6 & 8 \\ & 556 & 13 & 4 \\ \hline 10) & 5010 & 0 & 0 \\ \hline 10) & 501 & & \\ \hline & £50 & 2s. & \end{array} = \text{Interest for 1 year.}$$

£50 2*s.* × 6 = £300 12*s.* = Interest for 6 years.

Example 13.—In what time will £350 amount to £406 at 4 per cent.?

Here the interest is £406 – £350 = £56, that is, £350 gains £56.

But the interest on £350 for one year = £14,

$$\therefore \text{£}14 : \text{£}56 :: 1 \text{ year} : x \text{ years}.$$

$$x = \frac{56}{14} = 4 \text{ years}.$$

Example 14.—At what rate per cent. will £350 amount to £406 in 4 years?

$$\text{Whole interest} = \text{£}406 - \text{£}350 = \text{£}56$$

$$\therefore \text{Interest for 1 year} = \frac{56}{4} = \text{£}14$$

$$\text{and} \quad \overset{\text{Prin.}}{350} : \overset{\text{Prin.}}{100} :: \overset{\text{Int.}}{14} : \overset{\text{Int.}}{x}$$

$$x = \frac{\overset{2}{\cancel{100}} \times \overset{2}{\cancel{14}}}{\underset{7}{\cancel{350}}} = 4 \text{ per cent.}$$

EXERCISES IN SIMPLE PROPORTION OR RULE OF THREE.*

1. If oranges are bought at the rate of 9 for 6*d.*, what will 3 doz. cost?

2. What will be charged for 39 bottles of wine at 39*s.* per doz.?

3. If a servant's wages for 12 months be £18 18*s.*, what is that for 7 months? and what for 9 months?

4. When paper is sold at the rate of 3 quires for 7¾*d.*, what is the value of 2 reams at the same rate?

5. 4 ozs. of tea cost 5¾*d.*; what is the value of 4 lbs. at the same price?

6. A cask of butter weighing 73 lbs. costs £5, and 4*s.* 6*d.* for

---

* The Scholars should work the problems in this chapter by the unity method as well as by proportion.

ge; for what would 25 lbs. of it be sold to gain 1¼*d*. . ?

If the carriage of 25 cwts. of goods a certain distance costs *s*. 3*d*., what will the carriage of 13 cwts. cost for the same ice ?

If 15 lbs. of tea can be bought for 38*s*., what will 7 ozs. be ?

When wheat is 39*s*. per quarter, a loaf of a certain weight cost what will the same loaf cost when wheat rises to 57*s*. per er ?

If 17 bushels of potatoes cost £2 19*s*. 6*d*., what will be the of 38 bushels ?

A piece of cloth measuring 58 yards is bought for .2*s*. 6*d*.; what is the value of 29 yards of it, supposing it to ld at a profit of 6*d*. per yard ?

My yearly salary is £250 10*s*. 6*d*.; what should I receive year 10 months ?

If a gardener charges me £2 18*s*. 6*d*. for 13 days' work, should he be paid for 3 days ?

I paid £4 10*s*. for 6 sacks of potatoes; what is the worth of cks ?

. Find the cost of 21⅓ dozen eggs at 16 for 1*s*.

. If 16 men can build a house in 156 days, how many could finished in 32 days ?

. If 20 men finish a piece of work in 13 days, how many men l be required to do it in 52 days ?

. A servant's wages for 3 months amount to £8 2*s*. 6*d*.; what l be her wages for 2½ years ?

. If I lend a friend £150 for 6 months, for how long should d me £50 in return ?

. If a rod 6 feet long, standing upright, casts a shadow of 7 what is the length of an upright pole which casts a shadow of et ?

. If 15 men can reap a field in 3 days, how long would 45 be doing it ?

. What would it cost to clothe a company of 78 men if it £17 10*s*. for every 7 men ?

. How far can I travel for 5 guineas if 18 miles cost 1*s*. 9*d*. ?

24. How much land can I rent for £210, if 2 acres can be rented for £3?

25. A bankrupt pays a dividend of 12*s.* 6*d.* in the £; what does a creditor receive for a debt of £350?

26. What wages should a servant receive who is engaged at £15 a year, if she commences work on the 1st of August and leaves again on the 12th of October? [Include first and last days.]

27. A man pays in income-tax £4 18*s.* 8*d.*; what must his yearly income be, if the tax is levied at 4*d.* in the £?

28. When wheat is 60*s.* a quarter a 6*d.* loaf weighs 9 lbs.; what will it weigh when wheat is 45*s.* per quarter?

29. If 12 men can do a piece of work in 15 days, in what time will one man do it? and in what time will 5 do it?

30. Three gallons of water weigh 30 lbs.; what quantity is there in 1½ cwt.?

31. What is the cost of 20 tons of coals, if 2 tons 10 cwts. cost 37*s.* 6*d.*?

32. By working 4 hours a day a workman can complete a piece of work in 17 days; in what time can he do it by working 12 hours a day?

33. When the poor-rate is levied at 1*s.* 6*d.* in the £, what is the assessment of a property which pays a rate of £2 17*s.*?

34. A bankrupt pays £150, and this is at the rate of 3*s.* 6*d.* in the £; what are his liabilities?

35. A piece of work has to be completed in 16 hours; if 10 men can do it in 3 days 4 hours, how many men will be sufficient to do it in the 16 hours? [12 hours to a day.]

36. If a shopkeeper makes 3½*d.* profit out of every shilling received, what are his profits per year on a weekly taking of £47 10*s.* 6*d.*?

37. A railway train passes over 16 miles 1,540 yards in 22½ minutes; what is its rate per hour?

38. A crew of 46 men had provisions sufficient for 28 days; at the end of a week 4 of them died; for how long ought the provisions to last the remainder?

39. If the interest on £100 is £3 15*s.* for one year, what is the interest on £640 for the same period?

40. If the interest on £372 10*s.* be £14 16*s.*, what is the interest on £165 at the same rate?

CHAPTER XXXIV.

EXAMINATION PAPERS.

STANDARD V.

I.

f 13 lbs. of sugar cost 4*s.* 4*d.*, how many lbs. may be bought 12*s.* 4*d.*?

ind the value of 3 cwts. 1 qr. 12 lbs. of tea at 16 guineas t.

ake a bill of the following: 3¼ lbs. of almonds at 10*d.* .; 4¾ lbs. of candied peel at 1*s.* 4*d.* per lb.; 15½ lbs. of at 5½*d.* per lb.; and 3¾ lbs. of figs at 1*s.* 6*d.* per lb.

spend 15 guineas in 36 days; at the end of the year I find saved 100 guineas. What is my annual income?

II.

f 51 bushels of corn will feed 24 horses for 9 days, how bushels will be required to feed 24 horses for 15 days?

ind (by practice) the cost of excavating 379¼ cubic yards of £1 3*s.* 4*d.* per cubic yard?

ow many times is 13*s.* 4*d.* contained in £180 13*s.* 4*d.*?

silk-mercer bought 100 pieces of silk for 500 guineas, and em at a profit of 5 per cent; what was the selling price per

III.

farmer employed 4 men last year who mowed his fields in s; this year he wishes to have them mowed in 16 days; any men must he employ?

ind (by practice) the value of 768 rupees at 1*s.* 11½*d.* each.

n my butcher's book I read: "Joint of beef at 1*s.* per *s.* 3¾*d.*" Find the weight of the joint.

wine merchant bought 25 dozen of sherry at 3*s.* 4*d.* per eight bottles were broken; at what price per bottle must the remainder so as to gain 8 guineas on the whole?

IV.

5 men were engaged to build a wall in 9 days; 10 of the

men absented themselves; how long did it take the remaining men to complete the work?

2. Make a bill of the following: 5½ lbs. of coffee at 1*s.* 8*d.* per lb.; 2¾ lbs. of tea at 2*s.* 8*d.*; 17 lbs. of sugar at 3¾*d.* per lb.; and 4 ozs. of pepper at 6*d.* per lb.

3. What change shall I get out of a £5 note after paying for 1 cwt. 3 qrs. 16 lbs. of sugar at £2 2*s.* per cwt.?

4. What quantity of beef at 7*s.* 8*d.* per stone of 8 lbs. can be bought for £9 12*s.* 1¾*d.*?

V.

1. When eggs are sold at the rate of 23 for a shilling, what must I pay for 11½ dozen?

2. After paying for 14¼ lbs. of mutton at 11*d.* per lb., and 16 lbs. 6 ozs. of veal at 1*s.* per lb., I have £2 5*s.* 4½*d.* left; what money had I at first?

3. Find (by practice) the amount of the poor-rate at 3*s.* 7*d.* in the £, on a house rated at £84.

4. If I spend £54 in 135 days, and at the end of the year I owe £23, what is the amount of my yearly income?

VI.

1. How much duty will be charged upon 2 cwts. 3 qrs. 21 lbs. of tobacco at 6*s.* 4*d.* per packet of 3 lbs.?

2. A servant's wages are £18 per year; how much is that for 5 months?

3. When the interest on £100 is £3 15*s.*, what is the corresponding interest on £645 6*s.* 8*d.*?

4. Mrs. Jones bought of Draper & Co. 12¾ yards of cloth at 16*s.* 8*d.* per yard, 120¼ yards of calico at 5*d.* per yard, and 15 yards of linen at 9¾*d.* per yard. Make out the bill.

VII.

1. The shadow cast by a man 6 feet high is 8 feet 6 inches; at the same time of the day the shadow cast by a tree is 96 feet; what is the height of the tree?

2. Find the total cost of 163¾ yards of ribbon at 2*s.* 3*d.* per yard, and 3 gross of buttons at 1¼*d.* for 4.

3. If the fare by rail is 1¼*d.* per mile, how far can I travel for £3 8*s.* 4*d.*?

4. Find the rent of 25 acres 3 roods 20 poles of land at £2 6*s*. 8*d*. per acre.

VIII.

1. The interest obtained by putting £365 into the bank for 11 months is £9 7*s*. 6*d*.; what sum ought to gain £28 2*s*. 6*d*. in the same time?

2. Mrs. Thomas bought of Baker & Co. 7¾ lbs. of biscuits at 6*d*. per lb., 2 tins of biscuits, each weighing 8½ lbs., at 5*d*. per lb., 13 rolls at ¾*d*. each, and 5 loaves at 4¾*d*. Make out the bill, and say what change I ought to receive out of £5.

3. A mason is paid £9 11*s*. 3*d*. for building a wall. He received 5*s*. 9*d*. per day; how long was he in building the wall?

4. 1,862 yards of calico are bought for £30, and sold at the rate of 5½*d*. per yard; find the gain or loss.

IX.

1. If sugar is sold at £2 3*s*. 6*d*. per cwt., what quantity can be bought for £50 0*s*. 6*d*.?

2. If 25 men can do a piece of work in 75 days, how long will it take 15 men and 6 boys, supposing one man to do as much work as three boys?

3. Find by practice the interest on £196 10*s*. at 1*s*. 6*d*. per £.

4. A father divided £4,500 between his daughter and two sons. The sons had equal shares, but the daughter £500 more than a son's share; find the shares of each.

X.

1. Find (by practice) the value of 38,676 oranges at 1*s*. 6½*d*. per dozen.

2. How many horses at £53 each are equal in value to 1,060 sheep at £2 4*s*. each?

3. If an engine pump 1,000 tons of water from a mine in 14 hours, how long will it be in pumping 570 tons 5 cwts. 2 qrs. 24 lbs.?

4. Divide £111 among 15 persons, giving the first £3 3*s*. 9*d*. more than each of the others.

XI.

1. A bankrupt pays 14*s*. 6*d*. in the £; his liabilities are

£1,350 10*s.* 6*d.*; what are his assets? [Work this example by practice.]

2. If 6 farthings weigh ¾ oz., what will £14 6*s.* 3*d.* changed to farthings weigh?

3. 15 men can mow 150 acres in ten days; how many men can mow twice as much in 30 days at the same rate?

4. By selling one cwt. of sugar for two guineas I gain ½*d.* per lb.; what did the sugar cost me?

XII.

1. I tender 2 sovereigns to pay for 10¼ lbs. of cheese at 9½*d.* per lb., and 7 lbs. of tea at 2*s.* 11¼*d.* per lb., and I receive 10*s.* 7½*d.* change. Is this correct; and if not, what is the amount of the error?

2. What is the cost of 17 lbs. 5 ozs. of tea at 2½*d.* per oz.?

3. If 19 sheep cost £54 12*s.* 6*d.*, what will 11 cost?

4. A cistern which will hold 840 gallons is supplied by two pipes; the first pours in 56 gallons of water per minute, and the other 74 gallons per minute; there is also a discharge pipe which empties the cistern at the rate of 95 gallons per minute. Suppose, when the cistern is empty, the three pipes to be opened at once; how long will it take to fill the cistern?

XIII.

1. I bought from the butcher in one week 17 lbs. of beef at 11½*d.* per lb., 9 lbs. 6 ozs. of lamb at 1*s.* 2*d.*, and 8 lbs. 11 ozs. of mutton at 1*s.* Find the amount.

2. Divide £2 5*s.* 11*d.* among 7 men and 4 women, giving to each man half as much again as to each woman.

3. Find (by practice) the rent of 5 acres 3 roods 15 poles of land at £2 15*s.* 9*d.* per acre.

4. A garrison of 1,000 men have provisions for 26 days; after 11 days 250 of the men are sent away; how much longer will the provisions last the remainder?

XIV.

1. Find the value of 275¼ tons of iron at £6 19*s.* 6*d.* per ton.

2. I bought 27 horses for £2,079; at what price must I sell each horse so as to gain £85 1*s.* on the whole?

3. How many lbs. of butter at 1*s*. 10*d*. per lb. will cost as much 15 lbs. of cheese at £4 13*s*. 4*d*. per cwt. ?

4. One pipe pours water into a cistern at the rate of 50 gallons r minute, and another discharges it at the rate of 37 gallons per nute. Supposing there are 100 gallons of water in the cistern ien both pipes are opened, how many gallons will there be in the tern at the end of 12½ hours ?

XV.

1. If 5 girls can make 5 pinafores in 9 days, how many nafores can 14 girls make in the same time ?

2. A lad earns 10*s*. 8*d*. per week; he saves £7 10*s*. 7*d*. in a ar ; what does he spend per week ?

3. Which will cost the more, and by how much, 27 cwts. of tatoes at 1½*d*. per lb., or 448 lbs. of bacon at £4 13*s*. 4*d*. per t. ?

4. If a garrison of 1,500 men have provisions for 6 months, how any men must be sent away so that the provisions may last 9 onths ?

XVI.

1. Find (by practice) the value of 375 yards of silk at 15*s*. 9*d*. r yard.

2. My income is £550 per year; I pay income-tax on £350 at *l*. in the £, and save 100 guineas; how much money have I ft to spend ?

3. What is the annual payment on a life insurance for £500 at e rate of £3 19*s*. 6*d*. per cent. ?

4. Calculate the following :—
   13 dresses at 17*s*. 9½*d*. each.
   4½ yards of calico at 7½*d*. per yard.
   5 pieces of braid (12 yards each) at ¾*d*. per yard.
   6 reels of cotton at 1*s*. 3*d*. per dozen.

XVII.

1. Which will cost the more and by how much, 14 lbs. of beef 1*s*. 1*d*. per lb., or 14 lbs. 4 ozs. of mutton at 1*s*. per lb. ?

2. If one dress costs £1 16*s*. 7½*d*., how many can I buy for 318 12*s*. 9*d*. ?

3. Find the cost of 126 lbs. of soda at 4 lbs. for 5½*d.*

4. A tradesman in "selling off" wishes to mark his goc reduction of 1*s.* 9*d.* in the £; what price must be affixe article the former price of which was £7 15*s.*?

XVIII.

1. In a school the fees in one week amounted to £3 14 100 of the scholars paid 3*d.* and the remainder 2*d.* each; scholars who attended paid; find the number.

2. I bought a joint of beef for £1 8*s.* 9*d.* at 11½*d.* per lb. did it weigh?

3. A train which travels at the rate of 20 miles an hou from a certain station at 6 o'clock A.M.; a second train s 9 o'clock, travelling at the rate of 50 miles per hour distance will the trains have gone over when the second ov the first?

4. Which is the cheaper article: gloves at 3*s.* 6*d.* a pai last 8 weeks, or gloves at 2*s.* 3*d.* which last 5 weeks?

PART V.

FOR SENIOR SCHOOLS (*continued*).

STANDARD VI.

CHAPTER XXXV.

GREATEST COMMON MEASURE.

MEASURE.

oduce thus:—Place 12 pennies, or other objects, on ble, and ask the children to suggest into what groups divide or *measure out* 12 pennies, so as to have an umber in each group.

r one or two attempts we shall arrive at the following :—

ennies can be measured out into

6 equal groups of 2 each
4 " " 3 "
3 " " 4 "
2 " " 6 "

may also have 12 groups with 1 in a group, or one of 12.

, because we can *measure out* 12 pennies into 6 groups ach, 4 groups of 3 each, &c., we say that each of the rs, 6, 4, 3, 2, 1, 12, *measures* 12, or is a measure of 12. evident from this, that when one number is divided other and there is no remainder, the divisor must easure of the dividend.

Thus, 3 is a measure of 9
5 „ „ 15
7 „ „ 14
8 „ „ 32
11 „ „ 22

The quotient must also be a measure, because that also will divide the dividend without a remainder.

Hence, *a measure of a number is a factor of that number.*

Numbers like 5, 7, 11, 13, 17, &c., which have no measures except themselves and unity are called *prime* numbers. All other numbers are *composite.*

(B) Common Measure. (C. M.)

Next take 8 pennies and let the children divide or group these in a similar manner to the former grouping of 12 pennies.

We get 4 groups of 2 each
2 „ „ 4 „
or 8 „ „ 1 „
or 1 group of 8

Write on the black-board:—

The measures of 12 are 12, 6, 4, 3, 2, 1.
„ „ 8 „ 8, 4, 2, 1.

We see from this that 4, 2, and 1, measure both 12 and 8, that is, they are common to the two numbers; hence they are called *common measures.* We may also call them *common factors.*

Thus, 2 is a common measure of 4, 6, 8, &c.
3 „ „ „ 9, 12, 15, &c.
4 „ „ „ 8, 12, 16, &c.
5 „ „ „ 10, 15, 20, &c.

A common measure of two or more numbers is a factor common to each of the numbers.

(C) GREATEST COMMON MEASURE. (G. C. M.)

The measures of 12 are 12, 6, 4, 3, 2, 1.
„ „ 8 „ 8, 4, 2, 1.

That is, 4, 2, and 1 are the only measures of 12 and 8.

Of these common measures 4 is the greatest, and this is called the *Greatest Common Measure.*

The G. C. M. of two or more numbers is the greatest number which divides each of them exactly.

Example 1.—Required the G. C. M. of 18 and 12.

M. of 18 = 18, 9, 6, 2, 1.
„ 12 = 12, 6, 4, 3, 2, 1.
C. M. are 6 and 2.
∴ G. C. M. = 6.

Example 2.—What is the G. C. M. of 16, 48, and 54?

M. of 54 = 54, 27, 18, 9, 6, 3, 2, 1.
„ 48 = 48, 24, 16, 12, 8, 6, 4, 3, 2, 1.
„ 16 = 16, 8, 4, 2, 1.
G. C. M. = 2.

Example 3.—Is there a common measure to 15 and 32?

M. of 32 = 32, 16, 8, 4, 2, 1.
„ 15 = 15, 5, 3, 1.

There is no common measure except the unit. These numbers, 32 and 15, although not prime in themselves, are said to be prime to each other.

Example 4.—Find the G. C. M. of 1,379 and 2,401.

It is not easy to separate such high numbers into their component factors, and so to find the G. C. M. on inspection. We therefore adopt another method.

Divide the greater of the two numbers by the less; if there be a remainder divide it into the former divisor, if a remainder again, divide it into the former divisor, and so on, until there be no remainder; the last divisor is the G. C. M.*

---

* We think it better at this stage to omit any attempt to prove the truth of this rule.

```
1379)2401(1
     1379
     ----
     1022)1379(1
          1022
          ----
           357)1022(2
                714
               ----
                308)357(1
                    308
                    ---
                     49)308(6
                        294
                        ---
                         14)49(3
                            42
                            --
                             7)14(2
                               14
                               ==
```

G. C. M. = 7.

Example 5.—Find the G. C. M. of 133, 152, and 171.

In this case we find first the G. C. M. of any two of the numbers; and then the G. C. M. of the result and the other number.

Take the 1st and 2nd numbers.

```
133)152(1
    133
    ---
     19)133(7
        133
        ===
```

G. C. M. of 133 and 152 = 19.

To find the G. C. M. of 19 and 171.

```
19)171(9
   171
   ===
```

∴ G. C. M. of 133, 152, and 171 = 19.

Example 6.—A court-yard, 12 feet long and 9 feet oad, has to be paved with *square* flagstones. What is the rgest size that can be used?

M. of 12 = 12, 6, 4, 3, 2, 1.
„ 9 = 9, 3.

Therefore, the largest number which will divide each side ithout remainder is 3.

∴ size of square stones is 3 feet in the side.

Example 7.—What is the greatest number which will ivide 425 and 566, leaving a remainder of 2 in each case?

Here we must subtract 2 from each number, and so find he G. C. M. of 423 and 564. This is 141, and,

425 ÷ 141 = 3 and 2 remainder.
and 566 ÷ 141 = 4 „ 2 „

EXERCISES IN GREATEST COMMON MEASURE.

1. Find the G. C. M. of—

(1)	4	and	16	(2)	9	and	18
(3)	24	„	32	(4)	7	„	63
(5)	150	„	170	(6)	432	„	324
(7)	120	„	104	(8)	195	„	225
(9)	1302	„	1314	(10)	8008	„	9009
(11)	588	„	735	(12)	12838	„	18340

2. Find the greatest factor which will divide the following pairs numbers without remainder:—

(1)	60	and	68	(2)	72	and	64
(3)	52	„	72	(4)	156	„	216
(5)	95	„	228	(6)	1404	„	1944
(7)	5004	„	3792	(8)	6120	„	3768
(9)	2139	„	645	(10)	4808	„	6611
(11)	1176	„	2205	(12)	3304	„	3160

3. Find the G. C. M. of—

(1) 1854 and 1773
(2) 8687 „ 11169
(3) 2079 „ 1080
(4) 3816 „ 3492
(5) 12028 „ 42098
(6) 16, 36, and 48
(7) 2163 and 3605
(8) 3344 „ 4584
(9) 6170 „ 7404
(10) 3402 „ 2310
(11) 4242 „ 6565
(12) 24, 32, and 64

4. What are common measures of the following?

(1) 12, 18, and 32
(2) 64, 80, „ 100
(3) 27, 39, „ 21
(4) 72, 70, and 84
(5) 36, 80, „ 160
(6) 156 and 216

5. Find the greatest number which will divide 1,617 and 2,156 without a remainder.

6. Find the greatest number which will divide 339 and 791 giving a remainder of 3 in each case.

7. Find the greatest number which will divide 553 and 487 leaving remainders of 1 and 4 respectively.

8. I have two fields measuring respectively 24 and 32 acres; I wish to divide them into plots of equal size, and as large as possible. Find the number of plots and the size of each.

9. I have two cisterns holding 27 and 36 gallons respectively. Find the exact number of equal sized, but largest measures, I can fill from each.

10. I have 42 pennies in one bag, 56 in a second, 35 in a third; what is the smallest possible number of groups of equal size into which I can divide them, taking care to keep the pennies of one bag separate from those of the others?

11. What are the three smallest numbers which have 21 for their G. C. M.?

12. Find the G. C. M. of the even numbers up to 24 inclusive.

CHAPTER XXXVI.

LEAST COMMON MULTIPLE.

(A) MULTIPLE.

Write on the black-board—

$$12 = 3 \times 4$$

Here 3 and 4 are the factors of 12, and 12 is the product of these factors. We call 3 and 4 measures of 12 and we now call 12 a *multiple* of 3 and of 4, because it contains 3 or 4 an even number of times.

Any number which contains another number an exact number of times is a multiple of that number.

Thus 12 is a multiple of 3, 4, and 6
" 16 " " 8, 4, and 2
" 15 " " 5 and 3

And so on.

(B) COMMON MULTIPLE.

Write on the black-board—

The multiples of 2 are 2, 4, 6, 8, 10, 12, 14, &c.
" " 3 " 3, 6, 9, 12, 15, &c.

Here we see that 6 and 12 are multiples of both 2 and 3, and we say therefore that 6 and 12 are *common multiples* of these two numbers.

(C) LEAST COMMON MULTIPLE.

We see also from the same lines of figures that 6 is the lowest number which contains both 2 and 3. Hence 6 is the *Least Common Multiple* of 2 and 3 (L. C. M.).

The least number, which contains two or more numbers exactly, is said to be the Least Common Multiple of those numbers.

Example 1.—Required the L. C. M. of 12 and 16.

M. of 12 = 12, 24, 36, 48, &c.
„ 16 = 16, 32, 48, &c.

Hence, 48 is the L. C. M.

Example 2.—What is the L. C. M. of 3, 9, and 15 ?

M. of 3 = 3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, &c.
„ 9 = 9, 18, 27, 36, 45, &c.
„ 15 = 15, 30, 45, &c.

Hence, 45 is the L. C. M.

Example 3.—Required the L. C. M. of 3 and 5.

M. of 3 = 3, 6, 9, 12, 15, &c.
M. of 5 = 5, 10, 15, &c.
Here 15 is the L. C. M.
But 15 = 3 × 5.

Hence the L. C. M. of numbers prime to each other is their product.

Example 4.—Find the L. C. M. of 8, 9, 10, 12, 25, 32, 75, 80.

It would occupy too much time to find the measures of all these numbers until we come to the L. C. M., and we adopt the following method.*

Set down the numbers in a line, strike out any numbers contained exactly in any others, thus in 8, 9, 10, 12, 25, 32, 75, 80.

8 is contained in 32
10 „ „ „ 80
25 „ „ „ 75

Divide the other numbers by any number which will exactly divide any of them; place the quotient under the igure divided: any numbers which will not divide bring down.

---

* See note page 93.

Thus: divide by 3—

$$\begin{array}{r|cccccccc} 3 & \cancel{8}, & 9, & \cancel{10}, & 12, & \cancel{25}, & 32, & 75, & 80 \\ \hline & & 3 & & 4 & & 32, & 25, & 80 \end{array}$$

9, 12, and 75 divide by 3, and quotients are 3, 4, and 25. 32 and 80 do not divide.

Next divide by 4, next by 5, and then by 4.

$$\begin{array}{r|cccccccc} 3 & \cancel{8}, & 9, & \cancel{10}, & 12, & \cancel{25}, & 32, & 75, & 80 \\ \hline 4 & & 3 & & 4 & & 32 & 25 & 80 \\ \hline 5 & & 3 & & 1 & & 8 & 25 & 20 \\ \hline 4 & & 3 & & 1 & & 8 & 5 & 4 \\ \hline & & 3 & & 1 & & 2 & 5 & 1 \\ \hline \end{array}$$

The remaining quotients are all prime numbers. To find the L. C. M. multiply these and all the divisors together.

$$3 \times 4 \times 5 \times 4 \times 3 \times 2 \times 5 = 7{,}200$$

Example 5.—Three men start on a journey; the first walks 15, the second 20, and the third 25 miles per day; how far off is the first hotel where all three can put up?

The multiples of each number will give the distance got over at the end of each day.

Thus at 15 miles per day,—15, 30, 45, 60, 75, 90, 105, &c., up to the end of 20th day, when the distance walked will be 300 miles.

At 20 miles per day,—20, 40, 60, 80, 100, &c., up to the end of the 15th day, when the distance walked will be 300 miles.

At 25 miles per day,—25, 50, 75, 100, &c., up to the end of the 12th day, when the distance walked will be 300 miles.

In other words, the first resting-place for all three will be at a distance of 300 miles from the starting-point.

300 is the L. C. M. of 15, 20, and 25.

By shorter method—

$$5\underline{)15,\ 20,\ 25}$$
$$\underline{\ \ 3\quad 4\quad 5\ }$$
$$\text{and } 5 \times 3 \times 4 \times 5 = 300$$

Example 6.—A schoolmaster counts his boys 4 at a time, 5 at a time, and 6 at a time, and there is always one over. Find the least number of boys there can be in the school.

The L. C. M. of 4, 5, and 6 will be the smallest number which contains 4, 5, and 6 each, an even number of times, and one more than the number will always give one over.

L. C. M. of 4, 5, and 6 = 60.

∴ The required number is 61.

EXERCISES IN THE LEAST COMMON MULTIPLE.

1. Find the L. C. M. of—
   (1) 5, 15, 20
   (2) 3, 18, 36
   (3) 21, 16, 8, 7
   (4) 19, 16, 4, 38
   (5) 8, 16, 13, 15
   (6) 5, 7, 14, 15, 21
2. Find by inspection the L. C. M. of—
   (1) 6, 14
   (2) 8, 10, 23
   (3) 5, 2, 6, 12
   (4) 16, 14, 2, 7
   (5) 1, 2, 3, 4, 5
   (6) 7, 8, 9, 10
3. Find the L. C. M. of the following sets of numbers:—
   (1) 7, 19, 5, 6, 38
   (2) 42, 7, 3, 54, 84
   (3) 28, 17, 9, 34, 112
   (4) 48, 29, 58, 174, 56
   (5) 17, 16, 13, 32, 26
   (6) 13, 27, 39, 15, 9

4. Find the least number divisible by 7, 13, and 11, without a emainder.

5. Find the least number divisible by 3, 5, and 9, having a emainder of 1 in each case.

6. A, B, and C start together and walk round a field. A takes minutes, B 4 minutes, and C 6 minutes to go round; when will hey be together for the first time at the starting-point?

7. Find the least sum of money payable in pounds or guineas.

8. Find the least sum of money payable in half-crowns, crowns, nd florins.

9. A schoolmaster finds that he can arrange his boys in classes f 15, 25, or 30, and have none remaining; what is the smallest umber he can have in the school?

10. I have a certain number of shillings. I find on trial that I an arrange them in piles of 3, 4, 5, or 6, without having any over; ow many shillings have I?

11. What number is that which leaves a remainder of 2 when ivided either by 3, 5, or 7?

12. Find the least sum of money which contains an exact umber of shillings, florins, half-crowns, and half-guineas.

CHAPTER XXXVII.

VULGAR FRACTIONS.

SECTION I.

REDUCTION TO LOWEST TERMS.

Example 1. (*For the black-board.*)—Reduce $\frac{30}{105}$ to its west terms.

$$\frac{30}{105}=\frac{6}{21}=\frac{2}{7}$$

Ve divide numerator and denominator first by 5 and then y 3, *both common measures of* 30 *and* 105. We could have divided by 15, the G.C.M. of 30 and 105, with the same result.

Example 2.—Reduce $\frac{323}{493}$ to lowest terms. In this case it is not easy to find on inspection any common measure of 323 and 493, and here our rule for finding the G.C.M. becomes of service.

$$\begin{array}{l}
323)493(1\\
323\\
\overline{170})323(1\\
170\\
\overline{153})170(1\\
153\\
\overline{17})153(9\\
\phantom{323)170)153)17)}\underline{\underline{153}}
\end{array}$$

The G.C.M. is 17, and we can now reduce the fraction.

$$\frac{323 \div 17}{493 \div 17} = \frac{19}{29}$$

We can usually find common measures of the numerator and denominator by inspection, and so reduce the fraction. The following will assist us in finding common measures:—

1. The number is divisible by 2 if the figure in the *units* place is *even*.
2. The number is divisible by 4 if the *tens and units* be divisible by 4.
3. The number is divisible by 8 if the *hundreds, tens, and units* be divisible by 8.
4. The number is divisible by 5 if the figure in the *units* place is 5 or 0.
5. The number is divisible by 9 or 3 if the *sum* of the *digits* is divisible by 9 or 3.
6. A number which has three digits repeated in the same *order* is divisible by 7, 11, and 13.

EXERCISES ON REDUCTION OF FRACTIONS.

1. Reduce to improper fractions :—
   (1) $3\frac{1}{2}$, $2\frac{1}{9}$, $13\frac{2}{3}$, $14\frac{5}{8}$.
   (2) $17\frac{3}{4}$, $16\frac{1}{10}$, $19\frac{3}{5}$, $20\frac{7}{8}$.
   (3) $25\frac{5}{11}$, $18\frac{3}{7}$, $29\frac{8}{13}$, $37\frac{3}{5}$.
   (4) $16\frac{17}{21}$, $29\frac{8}{11}$, $183\frac{2}{5}$, $2\frac{115}{127}$.
   (5) $7\frac{131}{513}$, $9\frac{112}{113}$, $3\frac{11}{18}$, $12\frac{1}{5}$.
   (6) $4\frac{19}{21}$, $3\frac{17}{27}$, $14\frac{3}{8}$, $1\frac{316}{317}$.

2. Express as mixed numbers :—
   (1) $\frac{816}{27}$, $\frac{314}{29}$, $\frac{185}{21}$, $\frac{718}{33}$.
   (2) $\frac{229}{31}$, $\frac{364}{16}$, $\frac{280}{48}$, $\frac{197}{13}$.
   (3) $\frac{185}{14}$, $\frac{3002}{216}$, $\frac{187}{5}$, $\frac{1011}{301}$.
   (4) $\frac{314}{17}$, $\frac{8134}{216}$, $\frac{315}{31}$, $\frac{364}{21}$.
   (5) $\frac{81}{2}$, $\frac{3103}{5}$, $\frac{1984}{17}$, $\frac{816}{72}$.
   (6) $\frac{741}{15}$, $\frac{396}{11}$, $\frac{212}{8}$, $\frac{141}{5}$.

3. Reduce to their lowest terms :—
   (1) $\frac{48}{60}$, $\frac{16}{32}$, $\frac{19}{76}$, $\frac{84}{252}$.
   (2) $\frac{27}{63}$, $\frac{58}{174}$, $\frac{19}{171}$, $\frac{12}{84}$.
   (3) $\frac{35}{77}$, $\frac{21}{28}$, $\frac{14}{77}$, $\frac{28}{77}$.
   (4) $\frac{45}{99}$, $\frac{18}{54}$, $\frac{36}{72}$, $\frac{9}{117}$.
   (5) $\frac{18}{108}$, $\frac{150}{774}$, $\frac{192}{768}$.
   (6) $\frac{815}{1630}$, $\frac{339}{1695}$, $\frac{281}{5058}$.

4. Express with the least possible number of figures :—
   (1) $\frac{8}{72}$, $\frac{19}{38}$, $\frac{35}{77}$, $\frac{45}{120}$.
   (2) $\frac{55}{105}$, $\frac{74}{370}$, $\frac{152}{190}$.
   (3) $\frac{35}{100}$, $\frac{210}{1110}$, $\frac{15}{160}$.
   (4) $\frac{45}{100}$, $\frac{103}{721}$, $\frac{85}{170}$.

SECTION II.

ADDITION AND SUBTRACTION OF FRACTIONS.

Take an apple, or an orange, and divide into *eight* equal rts. Each part will be one-eighth, and we see that $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$, d $\frac{5}{8} + \frac{2}{8} = \frac{7}{8}$, and so on; that is, in adding these eighths we add only the numerators. Take another apple and cut into *four* equal parts. Each part will be one-fourth, and $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$, and so on. Now ask the children to add 2 fourths to 3 eighths. We get 5 pieces of apple, but they are not all fourths, or all eighths, and we cannot call them by either name any more than we can add shillings to pence, or dogs to cats, and call by either name.

If we wish to add shillings to pence we reduce the shillings to pence and then add, and so with our apple, we can cut the fourths to eighths, and then we can add.

To add fractions we must reduce them all to the same denominator.

Example 1.—Add $\frac{1}{3}$ to $\frac{1}{5}$.

$$\frac{1}{3} = \frac{5}{15}, \text{ and } \frac{1}{5} = \frac{3}{15}, \text{ and } \frac{5}{15} + \frac{3}{15} = \frac{8}{15}$$

Now 15 is the L. C. M. of 3 and 5, and here we find the use of our rule for finding the L. C. M.

If we wish to bring fractions to others with a common denominator, find the L. C. M. of the denominators, and this will be the common denominator.

Example 2.—Add $\frac{2}{3}$, $\frac{1}{9}$, and $\frac{3}{5}$.

L. C. M. of 3, 9, and 5 = 45.

Hence, 45 is the common denominator.

In $\frac{2}{3}$ how do I make the denominator (3) into 45?

By multiplying by 15. Hence, multiply numerator and denominator by 15 (this will not alter the value of the fraction), and we get

$$\frac{2 \times 15}{3 \times 15} = \frac{30}{45}$$

In $\frac{1}{9}$ multiply numerator and denominator by 5.

$$\frac{1 \times 5}{9 \times 5} = \frac{5}{45}$$

In $\frac{3}{5}$ multiply numerator and denominator by 9.

$$\frac{3 \times 9}{5 \times 9} = \frac{27}{45}$$

Hence—

$$\frac{2}{3} + \frac{1}{9} + \frac{3}{5} =$$

$$\frac{30}{45} + \frac{5}{45} + \frac{27}{45} =$$

$$\frac{30 + 5 + 27}{45} = \frac{62}{45} = 1\tfrac{17}{45}$$

Example 3.—Add $\frac{3}{5}$, $\frac{5}{11}$, $\frac{9}{22}$, $\frac{13}{40}$.

L. C. M. of 5, 11, 22, 40.

$$\begin{array}{r|rrrr} 5 & 5 & \cancel{11} & 22 & 40 \\ \hline 2 & 1 & & 22 & 8 \\ \hline 4 & & & 11 & 4 \\ \hline & & & 11 & 1 \end{array}$$

$5 \times 2 \times 4 \times 11 = 440.$

$$\frac{3 \times 88}{5 \times 88} = \frac{264}{440}$$

$$\frac{5 \times 40}{11 \times 40} = \frac{200}{440}$$

$$\frac{9 \times 20}{22 \times 20} = \frac{180}{440}$$

$$\frac{13 \times 11}{40 \times 11} = \frac{143}{440}$$

$$\frac{264 + 200 + 180 + 143}{440} = \frac{787}{440} = 1\tfrac{347}{440}.$$

Example 4.—From $\frac{7}{9}$ subtract $\frac{3}{5}$.

L. C. M. = 45.

$$\therefore \frac{7}{9} = \frac{35}{45} \text{ and } \frac{3}{5} = \frac{27}{45}$$

$$\frac{35 - 27}{45} = \frac{8}{45}$$

Example 5.—Find the value of $\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5}$.

G. C. M. = 60.

Hence—

$$\frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} =$$

$$\frac{30}{60} - \frac{20}{60} + \frac{15}{60} - \frac{12}{60} =$$

$$\frac{30 - 20 + 15 - 12}{60}$$

Add the numerators together which are to be added; add the numerators together which have to be subtracted; and subtract the second sum from the first.

$$\frac{(30 + 15) - (20 + 12)}{60} = \frac{45 - 32}{60} = \frac{13}{60}$$

By reducing to a common denominator we are able to compare fractions as to their relative size.

Example 6.—Which is the greatest of the following fractions, $\frac{5}{7}$, $\frac{24}{35}$, or $\frac{39}{56}$?

L. C. M. denominators = 280.

$$\frac{5}{7} = \frac{200}{280} : \frac{24}{35} = \frac{192}{280} : \frac{39}{56} = \frac{195}{280}$$

Therefore the $\frac{5}{7}$ is the greatest.

When mixed fractions have to be added, or subtracted, it is best to deal with the fractions first, and then with the whole numbers.

Example 7.—Add $3\frac{2}{9}$, $5\frac{1}{3}$, and $6\frac{8}{9}$—

$$\frac{2}{9} + \frac{1}{3} + \frac{8}{9} =$$

$$\frac{2}{9} + \frac{3}{9} + \frac{8}{9} = \frac{13}{9} = 1\frac{4}{9}$$

And $1\frac{4}{9} + 3 + 5 + 6 = 15\frac{4}{9}$.

In subtraction we may possibly have to borrow.

Example 8.—Take $7\frac{1}{5}$ from $12\frac{1}{6}$.

$\frac{1}{6} - \frac{1}{5} = \frac{5}{30} - \frac{6}{30}$ this is impossible.

$$\text{Take 1 from the } 12 = \frac{30}{30}$$

$$\text{which, with the } \frac{5}{30} = \frac{35}{30}$$

$$\text{and } \frac{35}{30} - \frac{6}{30} = \frac{29}{30}$$

$$11 - 7 = 4$$

$$\therefore\ 12\tfrac{1}{6} - 7\tfrac{1}{5} = 4\tfrac{29}{30}$$

EXERCISES IN ADDITION AND SUBTRACTION OF FRACTIONS.

1. Find the value of the following :—

(1) $\frac{3}{4} + \frac{1}{3} + \frac{2}{5}$; $\frac{1}{3} + \frac{5}{7} + \frac{2}{5}$.
(2) $\frac{3}{7} + \frac{6}{7} + \frac{3}{11}$; $\frac{4}{5} + \frac{3}{7} + \frac{1}{5}$.
(3) $\frac{2}{9} + \frac{1}{3} + \frac{7}{8}$; $\frac{4}{11} + \frac{2}{3} + 1\frac{1}{3}$.
(4) $\frac{1}{3} + \frac{5}{8} + 2\frac{1}{6}$; $14\frac{3}{5} + \frac{7}{8} + 3\frac{2}{15}$.
(5) $2\frac{11}{14} + 13\frac{3}{7} + 2\frac{1}{6} + 1\frac{3}{14}$.
(6) $13\frac{1}{3} + 1\frac{1}{4} + 2\frac{1}{8} + \frac{3}{4}$.

2. Take $\frac{1}{4}$ from $\frac{3}{4}$; and $\frac{3}{8}$ from $\frac{2}{3}$.

3. How much is $\frac{6}{7}$ greater than $\frac{4}{21}$?

4. A has to receive $\frac{2}{5}$ of his father's property, and B $\frac{1}{4}$ of it; what have A and B to receive together ?

5. How much is 1 greater than $\frac{17}{25}$?

6. Find the value of $\frac{1}{3} + \frac{2}{5} + \frac{1}{10}$ of a shilling.

7. Find the value of $3\frac{1}{2}$*d.* + $2\frac{6}{7}$*d.* + $3\frac{1}{4}$*d.*

8. Take $2\frac{1}{5}$*d.* from $13\frac{4}{11}$*d.*

9. What is the difference between $6\frac{1}{2}$ shillings and $3\frac{7}{13}$ shillings ?

10. How many yards of cloth are there in four pieces measuring $12\frac{1}{2}$ yds., $17\frac{3}{5}$ yds., $114\frac{3}{7}$ yds., $13\frac{1}{4}$ yds. ?

11. What weight in lbs. is there in four parcels containing respectively $5\frac{1}{2}$ lbs., $13\frac{3}{5}$ lbs., 17 lbs., $2\frac{13}{36}$ lbs. ?

12. After using $\frac{21}{19}$ of two parcels of tea, what part is left ?

13. What part of a cwt. must be added to $\frac{13}{120}$ to make $\frac{1}{3}$?

14. A woman sold $\frac{3}{8}$ of her eggs, and broke $\frac{2}{5}$ of the remainder; what part of the original number had she left?

15. What fraction must be added to $\frac{3}{15} + \frac{1}{30}$ to make 1?

16. What fraction added to $\frac{12}{125}$ will make $3\frac{1}{4}$?

17. Take the fifth of one from the third of one, and then add one-half to the remainder.

18. When I have spent $\frac{17}{250} + \frac{13}{140}$ of my money, what part is left?

19. What fraction added to $\frac{3}{7}$ will make $\frac{5}{8}$?

20. What part of my income added to $\frac{5}{8}$ of it will make $\frac{14}{15}$ of it?

21. How much is $\frac{2}{3}$ of my income greater than $\frac{1}{2}$ of it?

22. Find the sum and difference of $3\frac{1}{2}$ and $2\frac{7}{8}$.

23. I spent $\frac{2}{5}$ of my money on Monday, and $\frac{1}{7}$ of the remainder on Tuesday; what part was left?

24. A has $\frac{2}{7}$ of a farm, and B $\frac{3}{16}$ of it; what part has C who has the remainder?

25. Find the sum of $\frac{2}{3}$ of $\frac{1}{4}$, and $\frac{3}{4}$ of $\frac{1}{3}$, and take this sum from 1.

26. When $\frac{3}{5}$ of $\frac{2}{7}$ of the water in a cistern is consumed, what part is left?

27. One tap will fill a cistern in 20 minutes, and another in 36 minutes; what part of the cistern will be filled by both running together for 10 minutes?

SECTION III.

MULTIPLICATION OF FRACTIONS.

I.—BY A WHOLE NUMBER.

We have seen that multiplication is the addition of equal quantities, or numbers.

So that—

(1) 4 times $2 = 2 + 2 + 2 + 2 = 8$
(2) 4 „ $1 = 1 + 1 + 1 + 1 = 4$
(3) 4 „ $\frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = \frac{4}{2} = 2$

(4) 4 times $\frac{1}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4} = \frac{4}{4} = 1$
(5) 4 „ $\frac{2}{3} = \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$.

When we wish to *add* fractions we reduce to a common denominator and *add the numerators.* Now our numerators in the given examples are all *equal,* and instead of saying, for instance, in No. 5 above, $2 + 2 + 2 + 2 = 8$, we say at once $4 \times 2 = 8$. Hence, *to multiply a fraction by a whole number, multiply the numerator.*

Example 1.—Multiply $\frac{3}{4}$ by 4.

$$\frac{3 \times 4}{4} = \frac{12}{4} = 3$$

Example 2.—Multiply $\frac{5}{16}$ by 4.

$$\frac{5 \times 4}{16} = \frac{20}{16} = \frac{5}{4} = 1\frac{1}{4}$$

Instead of multiplying the numerator by 4, divide the denominator by 4, thus:—

$$\frac{5}{16 \div 4} = \frac{5}{4} = 1\frac{1}{4}$$

We see the result is the same, and the work is shorter. Of course we can only do this when the denominator is exactly divisible.

We may now write our rule thus: *To multiply a fraction by a whole number, multiply the numerator or divide the denominator.*

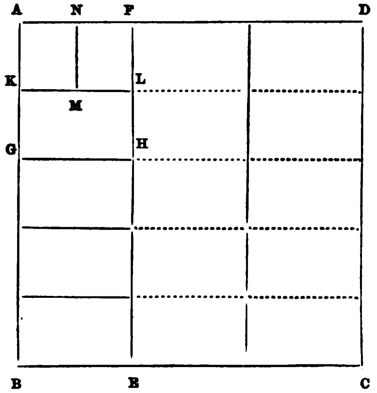
NOTE.—To show that $\frac{3}{4} \times 4 = \frac{3}{4}$ of 4.

We know that $\frac{1}{4}$ of $4 = 1$
also that $\frac{3}{4}$ of $4 = 3$
but we have seen that $\frac{3}{4} \times 4 = 3$.

Hence—$\frac{3}{4}$ of 4 and $\frac{3}{4}$ multiplied by 4 mean precisely the same thing, or "OF" = ×.

II.—By a Fraction.

Example 1.—Find the value of $\frac{2}{5} \times \frac{1}{3}$; $\frac{1}{2} \times \frac{2}{5} \times \frac{1}{3}$; $\frac{1}{2} \times \frac{1}{5} \times \frac{1}{3}$.



Draw a square, A B C D, and divide it into 3 equal parts. Then A B E F is $\frac{1}{3}$ of the whole. Divide this $\frac{1}{3}$ into 5 equal parts, and take two of these parts, viz. $\frac{2}{5}$: A G H F $= \frac{2}{5}$ of $\frac{1}{3}$.

What part is this $\frac{2}{5}$ of the whole figure? Clearly, $\frac{2}{15}$; for we have divided the whole into 15 parts, and taken 2 of these parts; hence, $\frac{2}{5}$ of $\frac{1}{3} = \frac{2}{15}$.

Again, the figure A K L F is $\frac{1}{2}$ of $\frac{2}{5}$ of $\frac{1}{3}$; what part is this of the whole?

$$\frac{1}{15}\text{; hence, } \frac{1}{2} \text{ of } \frac{2}{5} \text{ of } \frac{1}{3} = \frac{1}{15}.$$

Again, A K M N $= \frac{1}{2}$ of $\frac{1}{5}$ of $\frac{1}{3}$; what part is this of the whole figure? Clearly $\frac{1}{30}$.

$$\text{Hence, } \frac{1}{2} \text{ of } \frac{1}{5} \text{ of } \frac{1}{3} = \frac{1}{30}.$$

Now let us multiply the numerators together and the denominators together of these fractions, and compare the products with those we get from the figure.

$$\frac{2 \times 1}{5 \times 3} = \frac{2}{15}$$

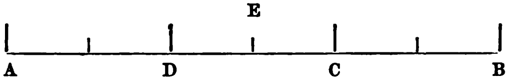
$$\frac{1 \times 2 \times 1}{2 \times 5 \times 3} = \frac{2}{30} = \frac{1}{15}$$

$$\frac{1 \times 1 \times 1}{2 \times 5 \times 3} = \frac{1}{30}$$

In every case the products are the same.

Example 2.—Show by the division of a line that $\frac{3}{4}$ of $= \frac{6}{12} = \frac{1}{2}$.

Draw the line A B and divide it into 3 equal parts—



Then A C $= \frac{2}{3}$. Divide A C into 4 equal parts. From A E $=$ 3 of these parts $= \frac{3}{4}$ of $\frac{2}{3}$.

$$\text{But A to E} = \frac{1}{2}\text{ A B}; \quad \therefore \frac{3}{4} \text{ of } \frac{2}{3} = \frac{1}{2} \text{ or } \frac{6}{12}.$$

Hence the rule: *To multiply a fraction by a fraction, mul- ly the numerators together for a new numerator, and the nominators for a new denominator.*

If the multipliers or multiplicands are mixed numbers, duce to improper fractions before multiplying. If the oduct is an improper fraction, reduce to a mixed number.

EXERCISES IN MULTIPLICATION OF FRACTIONS.

1. Find the value of the following:—

(1) $\frac{2}{5} \times \frac{3}{7}$; $\frac{4}{9} \times \frac{3}{5} \times 1$

(2) $\frac{5}{6} \times \frac{2}{7} \times \frac{1}{4}$; $\frac{2}{9} \times \frac{3}{7} \times \frac{1}{5}$.

(3) $\frac{4}{9} \times \frac{2}{11} \times \frac{1}{2}$; $\frac{3}{5} \times \frac{2}{7} \times \frac{1}{9}$.

(4) $\frac{8}{11} \times \frac{3}{4} \times \frac{22}{27}$; $\frac{16}{17} \times \frac{34}{48} \times \frac{2}{13}$.

(5) $\frac{1}{5} \times \frac{1}{6} \times \frac{1}{7}$; $\frac{2}{3} \times \frac{3}{4} \times \frac{4}{5} \times \frac{5}{6}$.

(6) $\frac{1}{5} \times \frac{3}{7} \times \frac{10}{21}$; $\frac{7}{8} \times \frac{8}{9} \times \frac{9}{10} \times \frac{10}{11}$.

2. Express as simple fractions :—

(1) $\frac{3}{4}$ of $\frac{5}{6}$; $\frac{2}{5}$ of $\frac{1}{3}$.
(2) $\frac{3}{8}$ of $\frac{7}{8}$; $\frac{1}{9}$ of $\frac{6}{7}$.
(3) $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{12}{13}$; $\frac{5}{9}$ of $\frac{3}{20}$ of $\frac{2}{7}$.
(4) $\frac{6}{7}$ of $\frac{12}{21}$; $\frac{29}{30}$ of 1.
(5) $\frac{1}{4}$ of $\frac{1}{5}$ of $\frac{1}{6}$; $\frac{3}{13}$ of $\frac{39}{33}$.
(6) $\frac{7}{5}$ of $\frac{5}{6}$ of $\frac{4}{3}$; $\frac{9}{4}$ of 1 of $\frac{1}{2}$.

3. Simplify the following :—

(1) $14\frac{1}{4} \times 12\frac{1}{12} \times \frac{1}{8}$.
(2) $\frac{2}{9}$ of $\frac{1}{2} \times 5\frac{3}{5}$ of $2\frac{1}{2}$.
(3) $7\frac{1}{9} \times \frac{2}{3}$ of $3\frac{1}{4}$ of 5.
(4) $2\frac{1}{2} \times 3\frac{1}{15}$ of $4 \times \frac{1}{4}$ of $\frac{1}{5}$.
(5) $\frac{2}{7}$ of $\frac{1}{9} \times \frac{3}{11}$ of $\frac{11}{14}$.
(6) $19\frac{1}{12} \times 13\frac{1}{16} \times \frac{1}{144}$ of $\frac{12}{104}$.

4. In a chest of tea there are $21\frac{2}{5}$ lbs.; how many pounds of tea in $5\frac{3}{4}$ such chests?

5. How many lbs. are there in $7\frac{2}{3}$ bars of soap, each weighing $4\frac{1}{5}$ lbs.?

6. Multiply £5 by $2\frac{2}{7}$.

7. Find the value of $\frac{3}{4}$ of £4; $\frac{6}{7}$ of £14; $1\frac{1}{5}$ of £1; $2\frac{2}{3}$ of 1*s.*

8. I want to give $\frac{1}{5}$ of a loaf of bread to each of 56 children; how many loaves shall I require?

9. Which is the greater, and by how much, $\frac{3}{5}$ of 10*s.*, or $\frac{2}{7}$ of a guinea?

10. Find the value of $\frac{5}{11}$ of $\frac{1}{10}$ of £2 2*s.*

11. If a boy can count 189 in a minute, how many can he count in $7\frac{6}{7}$ minutes?

12. A grocer sells $25\frac{3}{4}$ lbs. of soda per day; how many lbs. does he sell in $3\frac{3}{5}$ days at the same rate?

SECTION IV.

DIVISION OF FRACTIONS.

I.—BY A WHOLE NUMBER.

Elicit from the children and write on the black-board:—

4*s.* ÷ 4 means 4*s.* divided into 4 equal parts of 1*s.* each.

÷ 4 means 2*s.* divided into 4 equal parts, viz. 6*d.* or $\frac{1}{2}$*s.*

÷ 4 means 1*s.* divided into 4 equal parts of 3*d.* or ch.

÷ 4 means $\frac{1}{2}$*s.* or 6*d.* divided into 4 equal parts of 1$\frac{1}{2}$*d.* each.

÷ 4 means $\frac{1}{4}$*s.* or 3*d.* divided into 4 equal parts of $\frac{3}{4}$*d.* . each.

÷ 4 means $\frac{2}{3}$*s.* or 8*d.* divided into 4 equal parts of 2*d.* each.

÷ 4 means $\frac{5}{6}$*s.* or 10*d.* divided into 4 equal parts of r $\frac{5}{24}$*s.** each.

lecting all these results together we get—

Shillings. Shilling.

$$(1)\quad 4 \div 4 = 1$$

$$(2)\quad 2 \div 4 = \frac{1}{2}$$

$$(3)\quad 1 \div 4 = \frac{1}{4}$$

$$(4)\quad \frac{1}{2} \div 4 = \frac{1}{8}$$

$$(5)\quad \frac{1}{4} \div 4 = \frac{1}{16}$$

$$(6)\quad \frac{2}{3} \div 4 = \frac{1}{6}$$

$$(7)\quad \frac{5}{6} \div 4 = \frac{5}{24}$$

multiply the denominator of the fraction by the number in (4) (5) (6) (7), and compare the results with ust given.

$$(4)\quad \frac{1}{2 \times 4} = \frac{1}{8}$$

$$(5)\quad \frac{1}{4 \times 4} = \frac{1}{16}$$

---

* 5 half-pence is $\frac{5}{24}$ of 24 halfpence, or 1s.

$$(6)\quad \frac{2}{3 \times 4} = \frac{2}{12} = \frac{1}{6}$$

$$(7)\quad \frac{5}{6 \times 4} = \frac{5}{24}$$

The results are the same; hence, *To divide a fraction by a whole number multiply the denominator.*

Example 1.—Divide $\frac{4}{9}$ by 4.

$$\frac{4}{9 \times 4} = \frac{4}{36} = \frac{1}{9}$$

Now instead of multiplying the denominator divide the numerator.

$$\frac{4 \div 4}{9} = \frac{1}{9}$$

The result is the same, and we may enlarge our rule.

*To divide a fraction by a whole number, multiply the denominator or divide the numerator.**

If the numerator is not exactly divisible, multiply the denominator.

II.—By a Fraction.

The children are familiar with the fact that two factors multiplied together make a product, and that therefore this product divided by either factor will give the other factor for a quotient. Thus:—

$$\begin{array}{rl} \text{If } 6 \times 3 = 18 & \\ \text{then } 18 \div 6 = 3 & \quad (1) \\ \text{and } 18 \div 3 = 6 & \end{array}$$

Applying this principle to fractions there can be little difficulty in eliciting from the children that—

---

* Compare this with the rule given for multiplying a fraction by a whole number.

$$\begin{aligned}\text{If } \frac{1}{2} \times \frac{2}{3} &= \frac{2}{6} \\ \text{then } \frac{2}{6} \div \frac{2}{3} &= \frac{1}{2} \qquad (2) \\ \text{and } \frac{2}{6} \div \frac{1}{2} &= \frac{2}{3}\end{aligned}$$

gain, that—

$$\begin{aligned}\text{If } \frac{3}{5} \times \frac{2}{7} &= \frac{6}{35} \\ \text{then } \frac{6}{35} \div \frac{3}{5} &= \frac{2}{7} \qquad (3) \\ \text{and } \frac{6}{35} \div \frac{2}{7} &= \frac{3}{5}\end{aligned}$$

e any two of these divisions—

$$\begin{aligned}(1) \quad \frac{2}{6} \div \frac{2}{3} &= \frac{1}{2} \\ (2) \quad \frac{6}{35} \div \frac{3}{5} &= \frac{2}{7}\end{aligned}$$

may obtain the same result by dividing the *numerator*
idend by the numerator of the divisor, and the *denomi-*
of the dividend by the denominator of the divisor,
—

$$\begin{aligned}(1) \quad \frac{2 \div 2}{6 \div 3} &= \frac{1}{2} \\ (2) \quad \frac{6 \div 3}{35 \div 5} &= \frac{2}{7}\end{aligned}$$

ce, multiplication by a fraction stands thus:—

$$\frac{3}{5} \times \frac{2}{7} = \frac{3 \times 2}{5 \times 7} = \frac{6}{35}$$

division by a fraction stands thus:—

$$\frac{6}{35} \div \frac{3}{5} = \frac{6 \div 3}{35 \div 5} = \frac{2}{7}$$

That is, the division is the exact converse of the multiplication.

Example 2. (*For black-board.*)—

$$(1)\quad \frac{3}{35} \div \frac{3}{7} = \frac{3 \div 3}{35 \div 7} = \frac{1}{5}$$

$$(2)\quad \frac{45}{77} \div \frac{9}{11} = \frac{45 \div 9}{77 \div 11} = \frac{5}{7}$$

$$(3)\quad \frac{17}{57} \div \frac{17}{19} = \frac{17 \div 17}{57 \div 19} = \frac{1}{3}$$

It usually happens that the numerator or denominator (or both) is not exactly divisible—and we must adopt another method. Take the three examples just given, exchange the numerators and denominators of the divisors: for $\frac{3}{7}$ take $\frac{7}{3}$, for $\frac{9}{11}$ take $\frac{11}{9}$, and for $\frac{17}{19}$ take $\frac{19}{17}$; *multiply*, and compare the results.

$$(1)\quad \frac{3}{35} \times \frac{7}{3} = \frac{3 \times 7}{35 \times 3} = \frac{21}{105} = \frac{3}{15} = \frac{1}{5}$$

$$(2)\quad \frac{45}{77} \times \frac{11}{9} = \frac{45 \times 11}{77 \times 9} = \frac{495}{693} = \frac{45}{63} = \frac{5}{7}$$

$$(3)\quad \frac{17}{57} \times \frac{19}{17} = \frac{17 \times 19}{57 \times 17} = \frac{323}{969} = \frac{10}{57} = \frac{1}{3}$$

The results are the same.

This appears a very long method compared with the former, but we can often shorten our work by cancelling. Thus—

$$(1)\quad \frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{35}}} \times \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{3}}} = \frac{1}{5}$$

$$(2)\quad \frac{\overset{5}{\cancel{45}}}{\underset{7}{\cancel{77}}} \times \frac{\overset{1}{\cancel{11}}}{\underset{1}{\cancel{9}}} = \frac{5}{7}$$

$$(3)\quad \frac{\overset{1}{\cancel{17}}}{\underset{3}{\cancel{57}}} = \frac{\overset{1}{\cancel{19}}}{\underset{1}{\cancel{17}}} = \frac{1}{3}$$

for division by fractions: *Invert the divisor and mul-*

This is not strictly accurate, for if we invert the
isor as a whole, our figures will be upside down. It
l be better to teach the children the meaning of the
n Reciprocal,* and state the rule thus: *To divide by*
action multiply by its reciprocal.

XERCISES IN DIVISIONS OF FRACTIONS.

d the value of the following :—

$\frac{3}{4} \div 2$; $\frac{4}{5} \div 2$.
$\frac{9}{10} \div \frac{1}{5}$; $\frac{6}{7} \div 3$.
$\frac{13}{15} \div 26$; $\frac{17}{19} \div 1$.
$\frac{3}{5} \div \frac{2}{3}$; $\frac{6}{11} \div \frac{3}{10}$.
$\frac{8}{9} \div \frac{8}{9}$; $\frac{11}{5} \div \frac{5}{11}$.
$\frac{1}{2}$ of $\frac{1}{4} \div \frac{1}{4}$; $\frac{3}{7} \div \frac{1}{3}$ of $\frac{1}{5}$

press as simple fractions :—

$3\frac{1}{15} \div \frac{1}{10}$; $\frac{1}{10} \div 3\frac{1}{15}$.
$\frac{3}{16} \div 3$; $3 \div \frac{6}{16}$.
$33 \div 4\frac{1}{3}$; $4\frac{1}{3} \div 33$.
$1 \div \frac{1}{6}$ of $\frac{1}{7}$; $\frac{1}{6}$ of $\frac{1}{7} \div 1$.
$3\frac{2}{13} \div 1\frac{1}{13}$; $8\frac{1}{2} \div 2\frac{3}{4} \times \frac{1}{5}$.
$\frac{2}{3}$ of $9 \div \frac{3}{5}$; $84\frac{5}{8} \div 19\frac{3}{12}$.

plify the following :—

$\frac{2}{7}$ of $\frac{6}{7}$ of $\frac{1}{7} \div 1\frac{1}{6}$ of $\frac{2}{7}$.
$\frac{31}{42} \times \frac{3}{62} \div \frac{21}{31} \times \frac{119}{17}$.

---

product of two numbers is unity, each is called the *reciprocal* of

$\frac{2}{3} \times \frac{3}{2} = 1 \;\therefore\; \frac{3}{2}$ is the reciprocal of $\frac{2}{3}$ and $\frac{2}{3}$ of $\frac{3}{2}$.
$2 \times \frac{1}{2} = 1 \;\therefore\; \frac{1}{2}$ is the reciprocal of 2 and 2 of $\frac{1}{2}$.

(3) $\frac{5}{11}$ of $\frac{3}{5}$ of $\frac{11}{14} \div 2\frac{1}{5}$ of $3\frac{1}{14}$ of 6.
(4) $\frac{2}{11}$ of $3\frac{1}{4}$ of $\frac{2}{13} \div \frac{1}{11}$ of $2\frac{1}{4}$ of $\frac{2}{9}$.
(5) $7\frac{1}{9}$ of $2\frac{1}{6} \times 3 \div \frac{184}{185}$ of $\frac{5}{92} \times \frac{37}{50}$.
(6) $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$ of $\frac{1}{5} \div \frac{1}{6}$ of $\frac{1}{7}$ of $\frac{1}{8}$ of $\frac{1}{9}$.

4. How often is $\frac{7}{9}$ contained in $\frac{21}{45}$? and $\frac{1}{7}$ in $\frac{1}{21}$?

5. If $10\frac{1}{3}$ is the product, and $2\frac{1}{5}$ one of the factors, find the other factor.

6. Divide $\frac{3}{4}$ by 9, and multiply the quotient by itself.

7. Divide £5 by $2\frac{2}{7}$.

8. Find the value of £6 $\div 1\frac{1}{3}$; 5*s.* $\div 2\frac{3}{4}$, and $\frac{2}{3}$ of 2*s.* $\div 1\frac{1}{5}$.

9. I give $\frac{1}{5}$ of a loaf of bread to each boy, and thus give away $27\frac{2}{5}$ loaves, how many boys were there?

10. Which is the greater, and by how much, 1*s.* $\div \frac{1}{5}$, or £1 $\times \frac{1}{4}$?

11. A horse trots $55\frac{2}{10}$ miles in $5\frac{1}{3}$ hours; find the rate per hour.

12. A landlord distributed 14 tons $12\frac{1}{2}$ cwts. of coal among his tenants, giving $6\frac{1}{2}$ cwts. to each; how many tenants were there?

SECTION V.

SIMPLIFICATION OF FRACTIONS.*

When the scholars thoroughly understand all the principles involved in addition, subtraction, multiplication, and division of fractions, they should be exercised in examples which involve two or more of these processes.

Exercises to be worked by the teacher and scholars on the black-board:—

Example 1.—Simplify $$\frac{\frac{1}{2} \text{ of } \frac{1}{4} \text{ of } \frac{1}{5}}{\frac{3}{8} + \frac{1}{5} - \frac{17}{40}}$$

---

* That is, reduce to simple expressions. The answers should be whole numbers, or mixed numbers with fractions in lowest terms, or proper fractions in the lowest terms.

$$\frac{\frac{1}{2}\times\frac{1}{4}\times\frac{1}{5}}{\frac{15}{40}+\frac{8}{40}-\frac{17}{40}} = \frac{\frac{1}{40}}{\frac{23}{40}-\frac{17}{40}} =$$

$$\frac{\frac{1}{40}}{\frac{6}{40}} = \frac{1}{40}\div\frac{6}{40} = \frac{1}{40}\times\frac{40}{6} = \frac{1}{6}$$

Example 2.—Simplify $\dfrac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\text{ of }\frac{3}{11}}$

$$\frac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\times\frac{3}{11}} = \frac{\frac{44+21}{77}}{1-\frac{12}{77}} =$$

$$\frac{\frac{65}{77}}{\frac{77}{77}-\frac{12}{77}} = \frac{\frac{65}{77}}{\frac{65}{77}} =$$

$$\frac{65}{77}\times\frac{77}{65} = 1$$

The attention of the scholars must be directed to the fact that $\frac{4}{7}$ of $\frac{3}{11}$ is one fraction, and must be simplified before subtracting from 1. We cannot subtract $\frac{4}{7}$ from 1 and then multiply by $\frac{3}{11}$, an error into which children often fall.

All possible multiplications and divisions must be got rid of before adding or subtracting.

Example 3.—What quantity must be subtracted from $5\frac{4}{9}$ to leave $\frac{1}{2}$ of $3\frac{1}{5}$?

$$5\frac{4}{9} - \frac{1}{2}\text{ of }3\frac{1}{5} =$$

$$\frac{49}{9} - \frac{1}{2}\times\frac{16}{5} =$$

$$\frac{49}{9} - \frac{8}{5} = \frac{245 - 72}{45} =$$

$$\frac{173}{45} = 3\tfrac{38}{45}$$

Example 4.—Arrange in descending order of magnitud $\frac{5}{6}, \frac{7}{9}, \frac{13}{15}$.

L. C. M. of 6, 9, 15 = 90.

$$\frac{5}{6} = \frac{75}{90} \qquad \frac{7}{9} = \frac{70}{90} \qquad \frac{13}{15} = \frac{78}{90}$$

$$\therefore \text{ the order is } \frac{13}{15}, \frac{5}{6}, \frac{7}{9}.$$

Example 5.—From 9 take its fourth, its fifth, and it sixth, and divide the remainder by $1\frac{7}{40}$.

$$\frac{1}{4} \text{ of } 9 = \frac{9}{4}$$

$$\frac{1}{5} \text{ of } 9 = \frac{9}{5}$$

$$\frac{1}{6} \text{ of } 9 = \frac{9}{6}$$

We have first to subtract $\frac{9}{4} + \frac{9}{5} + \frac{9}{6}$ from 9.

$$\frac{9}{4} + \frac{9}{5} + \frac{9}{6} = \frac{135 + 108 + 90}{60} = \frac{333}{60}$$

$$9 - \frac{333}{60} = \frac{540 - 333}{60} = \frac{207}{60}$$

$$\frac{207}{60} \div 1\frac{7}{40} = \frac{207}{60} \div \frac{47}{40}$$

$$= \frac{\overset{69}{\cancel{207}}}{\underset{3}{\cancel{60}}} \times \frac{\overset{2}{\cancel{40}}}{47} = \frac{138}{47} = 2\frac{44}{47}$$

xample 6.—Add the sum and difference of $\frac{3}{5}$ and $\frac{2}{3}$.

$$\text{Sum} = \frac{3}{5} + \frac{2}{3} = \frac{9}{15} + \frac{10}{15} = \frac{19}{15}$$

$$\text{Diff.} = \frac{10}{15} - \frac{9}{15} = \frac{1}{15}$$

$$\text{And } \frac{19}{15} + \frac{1}{15} = \frac{20}{15} = \frac{4}{3} = 1\tfrac{1}{3}$$

xample 7.—Find the value of $\dfrac{\frac{1}{3} + \frac{1}{4}}{\frac{1}{3} - \frac{1}{4}} \times \dfrac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3} + \frac{1}{4}}$.

$$\frac{+\frac{1}{4}}{-\frac{1}{4}} \times \frac{\frac{1}{3} - \frac{1}{4}}{\frac{1}{3} + \frac{1}{4}} = \frac{\frac{4}{12} + \frac{3}{12}}{\frac{4}{12} - \frac{3}{12}} \times \frac{\frac{4}{12} - \frac{3}{12}}{\frac{4}{12} + \frac{3}{12}}$$

$$= \frac{\frac{7}{12}}{\frac{1}{12}} \times \frac{\frac{1}{12}}{\frac{7}{12}} = \frac{7}{12} \div \frac{1}{12} \times \frac{1}{12} \div \frac{7}{12}$$

$$= \frac{\overset{1}{\cancel{7}}}{\underset{1}{\cancel{12}}} \times \frac{\overset{1}{\cancel{12}}}{1} \times \frac{1}{\underset{1}{\cancel{12}}} \times \frac{\overset{1}{\cancel{12}}}{\underset{1}{\cancel{7}}} = 1$$

xample 8.—Which is the greater, and by how much,
$\frac{-1\frac{2}{3}}{-2\frac{2}{3}}$, or $\frac{3\frac{1}{3} - 2\frac{2}{5}}{4\frac{5}{6} - 3\frac{2}{3}}$?

$$\frac{2\frac{1}{4} - 1\frac{2}{3}}{3\frac{2}{5} - 2\frac{2}{3}} = \frac{\frac{9}{4} - \frac{5}{3}}{\frac{17}{5} - \frac{8}{3}} = \frac{\frac{27 - 20}{12}}{\frac{51 - 40}{15}} =$$

$$\frac{7}{12} \div \frac{11}{15} = \frac{7}{\underset{4}{\cancel{12}}} \times \frac{\overset{5}{\cancel{15}}}{11} = \frac{35}{44}$$

(2) $$\frac{3\frac{1}{3} - 2\frac{2}{5}}{4\frac{5}{6} - 3\frac{2}{3}} = \frac{\frac{10}{3} - \frac{12}{5}}{\frac{29}{6} - \frac{11}{3}} = \frac{\frac{50 - 36}{15}}{\frac{29 - 22}{6}} =$$

$$\frac{14}{15} \div \frac{7}{6} = \frac{\overset{2}{\cancel{14}}}{\underset{5}{\cancel{15}}} \times \frac{\overset{2}{\cancel{6}}}{\underset{1}{\cancel{7}}} = \frac{4}{5}$$

We have now to find which is the greater $\frac{35}{44}$ or $\frac{4}{5}$.

$$\frac{35}{44} = \frac{175}{220}\text{; and } \frac{4}{5} = \frac{176}{220}.$$

Therefore the second fraction is the greater, and by $\frac{1}{220}$.

Example 9.—Reduce the following fraction to its simples form :— $2\frac{1}{2} \times \dfrac{1}{3\frac{1}{4} + \dfrac{1}{4\frac{1}{4}}}$

$$2\frac{1}{2} \times \frac{1}{3\frac{1}{4} + \frac{1}{4\frac{1}{4}}} = 2\frac{1}{2} \times \frac{1}{3\frac{1}{4} + \frac{1}{\frac{17}{4}}}$$

$$= 2\frac{1}{2} \times \frac{1}{\frac{13}{4} + \frac{4}{17}}$$

$$\left(\text{and } \frac{13}{4} + \frac{4}{17} = \frac{221}{68} + \frac{16}{68} = \frac{237}{68}\right)$$

$$= 2\frac{1}{2} \times \frac{1}{\frac{237}{68}} = \frac{5}{\cancel{2}} \times \frac{\overset{34}{\cancel{68}}}{237} = \frac{170}{237}$$

xample 10.—A owns $\frac{3}{5}$ of a field and B the remainder. difference between their shares is 4 acres. Find the area he field?

owns $\frac{3}{5}$; and $\frac{3}{5}$ from $\frac{5}{5}$, or the whole, is B's share, $\frac{2}{5}$.

ifference $= \frac{3}{5} - \frac{2}{5} = \frac{1}{5}$.

$$\frac{1}{5} = 4 \text{ acs.} \therefore \frac{5}{5} \text{ or the whole} = 20 \text{ acs.}$$

XERCISES IN SIMPLIFYING COMPLEX FRACTIONS.

. Simplify the following fractions :—

) $\dfrac{3+\frac{1}{2}}{\frac{2}{5}\text{ of }5}$; $\dfrac{\frac{1}{2}\text{ of }\frac{3}{4}}{\frac{1}{2}+\frac{3}{4}}$ (2) $\dfrac{3\frac{3}{4}\text{ of }\frac{2}{5}}{3\frac{3}{4}+\frac{2}{5}}$; $\dfrac{\frac{3}{7}\text{ of }\frac{1}{8}}{\frac{1}{3}\text{ of }\frac{9}{10}}$

) $\dfrac{\frac{5}{11}\text{ of }3\frac{1}{4}}{\frac{2}{3}+\frac{1}{6}}$; $\dfrac{\frac{1}{2}-\frac{3}{7}}{\frac{11}{14}}$ (4) $\dfrac{7\frac{1}{2}-\frac{3}{4}}{\frac{1}{4}}$; $\dfrac{\frac{2}{7}-\frac{1}{8}}{\frac{1}{9}\text{ of }\frac{2}{3}}$

(5) $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{2+3+4}$; $\dfrac{\frac{3}{10}\text{ of }\frac{1}{3}}{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}$

(6) $\dfrac{\frac{7}{11}\text{ of }\frac{2}{3}\text{ of }16}{\frac{1}{4}\text{ of }2\frac{1}{5}}$; $\dfrac{\frac{7}{15}\text{ of }\frac{1}{6}\text{ of }\frac{3}{7}}{4+\frac{1}{2}+5}$

(7) $\dfrac{\frac{1}{3}+\frac{3}{4}-\frac{1}{2}}{\frac{1}{6}+\frac{3}{4}+\frac{1}{2}}$; $\dfrac{\frac{2}{7}\text{ of }\frac{4}{15}\text{ of }\frac{7}{18}}{1+\frac{3}{16}\times 2\frac{1}{2}}$

) $\dfrac{3\frac{1}{4}+\frac{1}{3}}{2-\frac{1}{4}}$; $\dfrac{3+\frac{2}{5}}{3-\frac{2}{5}}$ (9) $\dfrac{3\frac{1}{15}}{1\frac{1}{4}} \div \dfrac{3\frac{1}{15}\times 1\frac{1}{4}}{3\frac{1}{15}-1\frac{1}{4}}$

) $\dfrac{3\frac{1}{4}\text{ of }\frac{1}{13}-\frac{5}{8}\text{ of }\frac{1}{5}}{\frac{3}{4}\text{ of }1\frac{1}{2}}$ (11) $\dfrac{3\frac{1}{4}\text{ of }\frac{4}{5}\times 1\frac{3}{7}\text{ of }\frac{21}{40}}{4\frac{1}{2}+2\frac{1}{16}}$

2) $\dfrac{\frac{2}{7}\text{ of }\frac{7}{8}-\frac{5}{16}\text{ of }\frac{4}{5}}{\frac{1}{2}\times\frac{1}{3}+\frac{1}{4}}$ (13) $\dfrac{3\frac{7}{10}+1}{3\frac{7}{10}-1} - \dfrac{4\frac{2}{5}+1}{4\frac{2}{5}-1}$

(14) $\dfrac{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}}{1\frac{1}{2}+3\frac{1}{3}+4\frac{1}{4}} \div \dfrac{\frac{2}{5} \text{ of } \frac{5}{6}}{\frac{2}{5}+\frac{5}{6}}$

(15) $\dfrac{3+(\frac{2}{3} \text{ of } \frac{1}{6})}{4+(\frac{4}{7} \text{ of } \frac{7}{8})} \div \dfrac{3\frac{1}{2}}{4\frac{1}{4}} + 1$ (16) $\dfrac{\frac{7}{16}-\frac{3}{32}}{1+\frac{3}{16}} - (\frac{1}{15} \text{ of } \frac{1}{4}$

(17) $\dfrac{2\frac{1}{6} \text{ of } \frac{3}{8}}{\frac{5}{9} \text{ of } \frac{1}{22}} - \dfrac{\frac{3}{5} \text{ of } \frac{2}{3}}{\frac{1}{8} \text{ of } \frac{2}{7}}$ (18) $\dfrac{3\frac{1}{7} \text{ of } \frac{1}{5}}{\frac{2}{5} \text{ of } \frac{7}{11}} - \dfrac{\frac{1}{15} \text{ of } \frac{1}{3}}{4}$

(19) $\dfrac{\dfrac{1\frac{3}{5} \text{ of } \frac{3}{5}}{1\frac{3}{5}+\frac{3}{5}}}{\dfrac{2\frac{1}{7} \text{ of } \frac{1}{4}}{1\frac{1}{4} \text{ of } \frac{2}{7}}}$ (20) $\dfrac{3\frac{1}{7} \text{ of } 2\frac{1}{5} \text{ of } 1\frac{3}{4}}{1\frac{1}{2} \text{ of } \frac{1}{5} \text{ of } \frac{3}{9}} \times 1\frac{3}{5}$

(21) $\dfrac{\dfrac{1\frac{1}{4}+2\frac{1}{2}}{3\frac{1}{2} \text{ of } 1\frac{1}{4}}}{\dfrac{2\frac{1}{3} \text{ of } 1\frac{1}{5}}{\frac{1}{3}+\frac{1}{5}}} + 2\frac{1}{4}$ (22) $\dfrac{1+1\frac{1}{3}}{1\frac{1}{3} \text{ of } \frac{1}{3}} \div \dfrac{2\frac{1}{3}}{\frac{3}{4}}$

(23) $\dfrac{1+\dfrac{2\frac{1}{2}}{3\frac{1}{4}}}{2-\dfrac{1\frac{1}{4}}{1\frac{2}{5}}}$ (24) $\dfrac{3+1\frac{1}{5} \div \frac{4}{15}}{2-\frac{1}{15} \div \frac{3}{15}}$

(25) $\dfrac{\frac{2}{7}+\frac{3}{4} \text{ of } (\frac{1}{2}-\frac{1}{3})}{1+\dfrac{3\frac{2}{5}}{\frac{1}{3}}}$ (26) $\dfrac{2\frac{1}{2}-\dfrac{1\frac{1}{4}+2}{4\frac{1}{4}-2}}{\frac{6}{5}+(\frac{1}{3} \text{ of } \frac{1}{5})} \div \frac{3}{8}$

(27) $\dfrac{3\frac{1}{7} \text{ of } 2\frac{1}{4} \text{ of } 16\frac{1}{8}}{\frac{1}{5}+\frac{1}{3}} \times \dfrac{1\frac{1}{4}}{3\frac{1}{7} \text{ of } 16\frac{1}{8} \text{ of } 2\frac{1}{4}}$

(28) $\dfrac{15-\dfrac{2\frac{1}{5} \text{ of } \frac{1}{11}}{4\frac{1}{7} \text{ of } \frac{1}{29}}}{1+\dfrac{1\frac{1}{8} \text{ of } 4}{4+1\frac{1}{8}}}$

2. Add the sum and difference of $\frac{5}{7}$ and $\frac{7}{9}$, and divide this su by $\frac{35}{36}$.

3. Find the sum, difference, and product of $\frac{3}{13}$ and $\frac{5}{11}$.

4. What fraction must be added to $1\frac{3}{7}$ to make $2\frac{4}{11}$?

5. Compare the following fractions as to their magnitude, $\frac{6}{7}$, $\frac{16}{1}$ and $\frac{54}{57}$.

6. $\frac{1}{5}$ of $\frac{3}{4}$ of a ship and cargo are worth 1,000 guineas, what is the value of the whole?

7. Four artisans earn together a certain sum of money; A has to receive $\frac{1}{4}$ of the whole, B $\frac{1}{3}$ of the remainder, C $\frac{1}{2}$ of what still remains, and D what is left, viz. 5 guineas; what was the total sum?

8. Simplify $\dfrac{\frac{1}{3} \text{ of } 3\frac{3}{5} \text{ of } 4\frac{1}{3} - \dfrac{2\frac{1}{4}}{3\frac{1}{5}}}{4\frac{23}{40}}$.

9. A labourer spends $\frac{1}{2}$ of his weekly earnings in food, $\frac{1}{6}$ in rent and taxes, $\frac{1}{15}$ in coals, $\frac{1}{15}$ in clothes, and $\frac{1}{8}$ in sundries; he is thus enabled to save 2*s*. 3*d*. per week; what are his wages?

10. When $\frac{3}{5}$ of a sum of money is spent, and also $\frac{1}{7}$ of the remainder, what fraction of the original sum is left?

11. I spend on Monday $\frac{2}{7}$ of my money, and on Tuesday $\frac{3}{7}$ of the original sum; I have £25 11*s*. left; what was the original sum?

12. A business is bought by three partners for £42,000; A provides $\frac{4}{7}$ of $\frac{3}{5}$ of the money, B $2\frac{1}{3}$ of $\frac{2}{7}$ of what A provides, and C the remainder; find the sum paid by each.

13. If I spend $\frac{3}{5}$ of my money on Wednesday, $\frac{5}{8}$ of the remainder on Thursday, and $\frac{4}{5}$ of what still remains on Friday, what fraction of the whole shall I have left?

14. William had twice as many marbles as Henry. They played a game and William won $\frac{1}{3}$ of Henry's. Express the ratio of the number of marbles they had after the game.

15. $\frac{3}{5}$ of $\frac{5}{9}$ of a number is 525; find the number.

16. A gentleman, who owned $\frac{3}{7}$ of a mine, sold $\frac{1}{6}$ of his share for 500 guineas; what was the value of the whole mine?

17. Show that $\frac{3}{5}$ of $\frac{5}{11}$ of $6\frac{10}{13}$ is equal to $\dfrac{\frac{1}{2} + \frac{1}{3} - \frac{1}{6}}{\frac{13}{36}}$.

18. When 50 gallons of water are poured into a cistern $\frac{3}{5}$ full, it is found to be $\frac{5}{6}$ full; how much does the cistern hold?

SECTION VI.

REDUCTION OF FRACTIONS.

Example for the black-board:—

What fraction of 10*s*. is 2*s*. 6*d*.? and what fraction is 6*s*. 8*d*. of a £?

We know that 2*s*. 6*d*. is ¼ of 10*s*. But we may find the fraction thus—

$$\frac{2s.\ 6d.}{10s.\ 0d.} = \frac{5 \text{ sixpences}}{20 \text{ sixpences}} = \frac{1}{4}$$

Again we know that 6*s*. 8*d*. is ⅓ of a £.

We find this fraction thus—

$$\frac{6s.\ 8d.}{£1\ 0s.\ 0d.} = \frac{80 \text{ pence}}{240 \text{ pence}} = \frac{1}{3}$$

From these examples we deduce the rule for converting quantities, or fractions of quantities, to fractions of other quantities.

Draw a line; place ABOVE *the line the quantity to be reduced, and* BELOW *the line the quantity we require the fraction of. Reduce to the same name and then the fraction to its lowest terms.*

Example 1.—Reduce 3*s*. 4*d*. to the fraction of 6*s*. 8*d*.

$$\frac{3s.\ 4d.}{6s.\ 8d.} = \frac{10 \text{ fourpences}}{20 \text{ fourpences}} = \frac{1}{2}$$

Example 2.—What fraction is 8*s*. 4*d*. of 19*s*. 4*d*.?

$$\frac{8s.\ 4d.}{19s.\ 4d.} = \frac{25 \text{ fourpences}}{58 \text{ fourpences}} = \frac{25}{58}$$

xample 3.—Reduce £1 to the fraction of 1*d.*, and 1*d.* ie fraction of £1.

$$\frac{£1}{1d.} = \frac{240}{1} = 240$$

$$\frac{1d.}{£1} = \frac{1}{240}$$

xample 4.—What fraction is 3 dwts. 12 grs. of 1 lb. 5 dwts.?

$$\frac{3\text{ dwts. }12\text{ grs.}}{1\text{ lb. }5\text{ dwts.}} = \frac{3\frac{1}{2}\text{ dwts.}}{12 \times 20 + 5} = \frac{\frac{7}{2}}{\frac{245}{1}} =$$

$$\frac{7}{2} \div \frac{245}{1} = \frac{\cancel{7}}{2} \times \frac{1}{\underset{35}{\cancel{245}}} = \frac{1}{70} \quad \textit{Ans.}$$

xample 5.—Express $\frac{7}{8}$ of £1 6*s.* 8*d.* as the fraction of £5.

$$\frac{\frac{7}{8}\text{ of }£1\ 6s.\ 8d.}{\frac{3}{5}\text{ of }£5} = \frac{\frac{7}{8} \times 1\frac{1}{3}}{\frac{3}{5} \times \frac{5}{1}} = \frac{\frac{7}{\underset{2}{\cancel{8}}} \times \frac{\overset{1}{\cancel{4}}}{3}}{\frac{3}{\cancel{5}} \times \frac{\cancel{5}}{1}}$$

$$= \frac{7}{6} \div \frac{3}{1} = \frac{7}{6} \times \frac{1}{3} = \frac{7}{18}$$

xample 6.—What fraction of £5 10*s.* will produce $3\frac{1}{5}$ $1\frac{1}{10}$?

$$3\tfrac{1}{5}\text{ of }£1\tfrac{1}{10} = \frac{16}{5} \times \frac{11}{10} = \frac{176}{50}$$

nd $\frac{176}{50} \div 5\frac{1}{2} = \frac{176}{50} \div \frac{11}{2} = \frac{\overset{16}{\cancel{176}}}{\underset{25}{\cancel{50}}} \times \frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{11}}} = \frac{16}{25}$

EXERCISES IN REDUCTION OF FRACTIONS.

1. Reduce 2*s*. 6*d*. to the fraction of £1; and 2*s*. to the fraction of 15*s*.

2. Express 3*s*. 6*d*. as a fraction of a guinea; and 6*s*. 8*d*. as a fraction of £2.

3. Express 7$\frac{1}{2}$*d*. as the fraction of 5*s*., and 4*s*. 3*d*. as the fraction of 2*s*. 8*d*.

4. What fraction is £1 6*s*. 8*d*. of £1 16*s*. 9*d*.; and £1 16*s*. 9*d*. of £1 6*s*. 8*d*.?

5. What fraction is £7 1*s*. 8*d*. of £7 18*s*. 4*d*.?

6. Reduce 2 qrs. 14 lbs. to the fraction of a cwt.

7. What fraction is 1 lb. troy of 1 lb. avoirdupois?

8. Express $\frac{1}{4}$*d*., $\frac{3}{4}$*d*., $\frac{1}{8}$*d*., $\frac{3}{7}$*d*. in the denomination of shillings.

9. Express £$\frac{1}{5}$, £$\frac{3}{17}$, £$\frac{2}{11}$, £$\frac{5}{13}$ in the denomination of shillings.

10. Reduce £$\frac{2}{3}$, £$\frac{3}{16}$, $\frac{3}{4}$ of 5*s*. to the denomination of pence.

11. Express $\frac{1}{3}$*s*., $\frac{1}{2}\frac{8}{3}$*s*., £$\frac{3}{5}$, 25 farthings in the denomination of pence.

12. Reduce $\frac{3}{4}$*d*., $\frac{7}{18}$*d*., £$\frac{1}{5}$, $\frac{2}{11}$*d*. to the denomination of shillings.

13. Express 3$\frac{1}{2}$*d*., $\frac{2}{5}$*s*., and $\frac{5}{6}$ of a crown in the denomination of £'s.

14. Write $\frac{3}{7}$ lb., 3$\frac{1}{2}$ cwts., 3 qrs. respectively, as fractions of a ton.

15. Reduce 13 weeks 3 days to the fraction of a year, and then to the fraction of 2 years.

16. What fraction of a cwt. is $\frac{3}{5}$ oz.?

17. Express $\frac{3}{5}$ guinea, $\frac{7}{15}$ crown, and $\frac{2}{3}$ of half a guinea as fractions of a crown.

18. What fractions of an English ell are 1 yd., $\frac{1}{3}$ yd., $\frac{2}{5}$ yd. respectively?

19. Write $\frac{2}{3}$ nail, $\frac{1}{6}$ qr., $\frac{1}{5}$ yd., and $\frac{2}{7}$ in. respectively, in the fraction of an English ell.

20. Express $\frac{1}{1876}$ mile in the denomination of inches.

21. Express $\frac{1}{315}$ perch as the fraction of an acre, and $\frac{1}{315}$ of an acre as the fraction of a perch.

22. What fraction of £2$\frac{1}{2}$ is 3$\frac{1}{2}$ shillings?

23. What fraction of £1 is 15*s*. 1*d*.; and what part of £2 is 11*s*. 4$\frac{1}{2}$*d*.?

24. What fraction of a furlong is 3 ft.?
25. Express 73 days as a fraction of 1 year and 76 days.
26. Reduce $1\frac{1}{2}d.$ to the fraction of one guinea.
27. What part of 24 hours is 5 minutes?
28. What fraction of a guinea is equivalent to $\frac{1}{3}$ of $\frac{3}{4}$ of a sovereign?
29. What fraction of 5 m. 4 fur. and 10 yds. is 2 m. 3 fur. 5 yds.?
30. Express $\frac{9}{10}$ of a cubic foot as a fraction of $\frac{1}{2}$ a cubic yard.
31. What fraction is $\frac{3}{5}$ of a shilling of $\frac{9}{10}$ of a florin?
32. Reduce $\frac{5}{9}$ of 1*s.* to the fraction of $\frac{2}{3}$ of 3*s.* 9*d.*
33. Reduce $\frac{3}{5}$ of a dwt. to the fraction of 4 ozs.
34. Reduce $5\frac{1}{2}$ sq. yds. to the fraction of a sq. foot.
35. Find the difference between $\frac{2}{7}$ of $\frac{3}{5}$ of $2\frac{1}{2}d.$, and $\frac{1}{7}$ of $\frac{2}{3}$ of $3\frac{3}{4}d.$, and express as a fraction of 1*s.*
36. Compare the values of $\frac{7}{8}$ of a guinea, $\frac{5}{8}$ of a £, and $3\frac{3}{4}$ of a crown.

SECTION VII.

MISCELLANEOUS EXERCISES IN VULGAR FRACTIONS.

Specimen examples to be worked on the black-board by the teacher and scholars.

Example 1.—A can do a piece of work in 6 days, and B can do the same work in 4 days. In what time can they do it working together?

A can do $\frac{1}{6}$ of the work in 1 day

B ,, $\frac{1}{4}$,, ,, 1 ,,

Hence A and B ,, $\frac{1}{6}+\frac{1}{4}$,, 1 ,,

,, $=\frac{5}{12}$,, ,, 1 ,,

,, can do 5 times the work in 12 days

$$\text{or whole work in } \frac{12}{5} \text{ days}$$
$$= 2\tfrac{2}{5} \quad „$$

Example 2.—A can do a piece of work in 5 days, B in 6 days, and C in 7 days. In what time, working together, will they do $2\frac{2}{3}$ of such pieces of work?

$$\text{A can do } \frac{1}{5} \text{ of the work in a day}$$
$$\text{B} \quad „ \quad \frac{1}{6} \quad „ \quad „$$
$$\text{C} \quad „ \quad \frac{1}{7} \quad „ \quad „$$

∴ A, B, and C can do $\frac{1}{5} + \frac{1}{6} + \frac{1}{7}$ of the whole work in one day $= \dfrac{42 + 35 + 30}{210} = \dfrac{107}{210}$

Hence A, B, and C will do the whole piece of work in $\dfrac{210}{107}$ days, and $2\frac{2}{3}$ such pieces in $\dfrac{\overset{70}{\cancel{210}}}{107} \times \dfrac{8}{\underset{1}{\cancel{3}}} = \dfrac{560}{107}$

$= 5\frac{25}{107}$ days.

Example 3.—A and B can do a piece of work in 12 days; B working alone can do it in 18 days; in what time can A do it working alone?

$$\text{A and B do } \frac{1}{12} \text{ of the work in one day}$$
$$\text{B does} \quad \frac{1}{18} \quad „ \quad „$$
$$\therefore \quad \text{A does } \frac{1}{12} - \frac{1}{18} \quad „ \quad „$$

$$= \frac{3-2}{36} = \frac{1}{36} \text{ in one day.}$$

$\therefore$ A can do the whole in 36 days.

Example 4.—A can do a piece of work in 16 days, B can do it in 20 days; A works 4 days and then B joins him; in what more time will they complete the work.

A and B can do $\frac{1}{16} + \frac{1}{20}$ of the work in one day $=$

$$\frac{5+4}{80} = \frac{9}{80}$$

Hence they could do the whole work in $\frac{80}{9}$ days.

But $\frac{1}{4}$ of the work has already been accomplished by A, and $\frac{3}{4}$ of the work remaining can be done by both in

$$\frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \text{ of } \frac{\overset{20}{\cancel{80}}}{\underset{3}{\cancel{9}}} \text{ days, or } \frac{20}{3}, \text{ or } 6\tfrac{2}{3} \text{ days.}$$

Example 5.—A can reap 3 acres of wheat in 4 days; B can reap 2 acres in 3 days; how long will it take them to reap one acre working together?

A can reap 3 acs. in 4 days
or „ $\frac{3}{4}$ „ 1 day
B can reap 2 acs. in 3 days
or „ $\frac{2}{3}$ „ 1 day

$\therefore$ A and B can reap $\frac{3}{4} + \frac{2}{3}$ acs. in 1 day, viz. $\frac{9+8}{12}$ or $\frac{17}{12}$ acs.

If they can reap $\frac{17}{12}$ acs. in one day, they can reap 17 acs. in 12 days, or they can reap 1 ac. in $\frac{12}{17}$ days.

Example 6.—A cistern holding 500 gallons is emptied by one pipe in 6 minutes and by another in 7 minutes. In what time will both pipes empty the cistern?

The first pipe empties $\frac{1}{6}$ in a minute

the second „ $\frac{1}{7}$ „

$\therefore$ the two empty $\frac{1}{6} + \frac{1}{7}$ in a minute, viz. $\frac{13}{42}$

and therefore will empty the whole in $\frac{42}{13}$ or $3\frac{3}{13}$ min.

Example 7.—From a cistern $\frac{3}{5}$ full of water 30 gallons are drawn; when 90 gallons are added it is found to be $\frac{5}{6}$ full; what does the cistern hold?

30 gallons are added and 90 withdrawn; this is equivalent to adding 60 gallons, and this 60 gallons makes the *difference* between $\frac{3}{5}$ full and $\frac{5}{6}$ full.

$$\text{Hence } \frac{5}{6} - \frac{3}{5} \text{ or } \frac{25-18}{30} \text{ or } \frac{7}{30} = 60 \text{ gals.}$$

$$\text{If } \frac{7}{30} = 60 \text{ gals.}$$

$$\frac{1}{30} = \frac{60}{7} \text{ gals.}$$

$$\text{and the whole} = \frac{60 \times 30}{7} =$$

$$\frac{1800}{7} = 257\tfrac{1}{7} \text{ gals.}$$

Example 8.—A gentleman left $\frac{5}{12}$ of his property to his son, and $\frac{2}{9}$ each to his two daughters, and the remainder, viz. *£500, to* charities; what was the value of his estate?

$$+ \frac{4}{9} = \frac{15 + 16}{36} = \frac{31}{36} \text{ was left to the children}$$

$$\text{And } \frac{36}{36} - \frac{31}{36} = \frac{5}{36}$$

$$\text{And } \frac{5}{36} = £500$$

$$\therefore \frac{1}{36} = \frac{500}{5} = 100 \text{; and the whole} =$$

$$100 \times 36 = £3600.$$

xample 9.—The hour and minute hand of a clock are ether at 12 o'clock; when will they be together again? The face of the clock is divided into 60 minute spaces. 1 o'clock the hour hand has travelled over 5 and the nute hand over 60 of these spaces. That is, the minute hand ns 55 spaces in travelling over 60, but at 1 o'clock the nute hand is 5 spaces behind; that is, it has to *gain* 5 ces before it overtakes the hour hand.

55 spaces are gained in travelling over 60 spaces

∴ 5 are gained in travelling over $\frac{60}{11} = 5\frac{5}{11}$.

In other words, the minute hand will overtake the hour nd at $5\frac{5}{11}$ minutes past 1 o'clock.

MISCELLANEOUS EXERCISES IN FRACTIONS.

. Add together £$\frac{3}{7}$, $\frac{3}{4}$ shilling, $\frac{4}{9}$ crown, $\frac{3}{7}$ of half a crown, and f half a guinea, and give the answer in the denomination of a ling.

. Take $\frac{1}{5}$ of a shilling from $\frac{1}{7}$ of £1, and find what fraction of hilling must be added to make a crown.

. What fraction of a shilling must be added to $\frac{3}{7}$ of a penny to ke a shilling?

. By what fraction of a shilling does $1\frac{3}{4}d.$ exceed $\frac{3}{7}d.$?

5. Having $\frac{3}{5}$ yd. of cloth, what fraction of a yard do I want to make an English ell?

6. By what fraction of a £ must you reduce $2\frac{1}{15}$ shillings to make it $\frac{2}{15}$*s.*?

7. How much greater is £$\frac{1}{5}$ + $\frac{2}{5}$*s.* than $2\frac{4}{5}$*s.*?

8. Find the difference between $\frac{3}{4}$ of $\frac{4}{21}$ and $\frac{3}{4} + \frac{4}{21}$.

9. What is the difference between $\frac{3}{4}$ of 1 and $1 - \frac{3}{4}$?

10. A boy was told to find $\frac{3}{5}$ of 2*s.* $7\frac{1}{4}$*d.* and he *divided* 2*s.* $7\frac{1}{4}$*d.* by $\frac{3}{5}$. What was the difference between his answer and the correct one?

11. What fraction of a £ must you add to 13*s.* $6\frac{1}{2}$*d.* to make it one guinea?

12. Divide $\frac{3}{4}$ of 16 ft. 4 in. by $\frac{1}{12}$.

13. Find $\frac{3}{8}$ of $\frac{1}{3}$ of $\frac{4}{5}$ of 15 years 3 months 3 weeks, and then the difference between this result and 3 years, giving your answer in weeks.

14. Find the value of $3\frac{1}{4}$ times 5 acs. 3 roods divided by $1\frac{3}{8}$.

15. What must be taken from $3\frac{1}{3}$ tons to leave as a remainder $5\frac{1}{10}$ cwts.?

16. By what must $\frac{3}{45}$ be divided to produce 1 as a quotient?

17. What must be added to $\frac{3}{5}$ of an hour to make 1 day?

18. By what fraction of a £ does $\frac{3}{8}$ of £5 differ from $\frac{3}{8}$ of 5 guineas?

19. What is the least number into which you can divide a dozen, a score, a gross, and a hundred an exact number of times?

20. What is the greatest number that will divide 400 and 450 without a remainder?

21. Find the value of $\frac{3}{5}$ of 21*s.* + $\frac{7}{8}$ of 5*s.* + $\frac{3}{4}$ of three half-crowns — $\frac{2}{7}$ of 1*s.* 2*d.*

22. If $\frac{3}{4}$ of $\frac{2}{15}$ of a house cost 315 guineas, what is the value of $\frac{1}{3}$ of it?

23. Multiply the sum of $\frac{1}{3}$ and $1\frac{1}{15}$ by their difference and then divide this product by $\frac{1}{3}$.

24. Four casks contain $17\frac{1}{2}$ gals., $20\frac{1}{4}$ gals., $18\frac{3}{7}$ gals., and $6\frac{7}{8}$ gals. respectively; what is the average number of quarts in each cask?

25. If $2\frac{3}{4}$ of an acre be worth £5,001 6*s.* 8*d.*, what is the value of $\frac{3}{4}$ *of* an acre?

26. Simplify—

$$\frac{1\frac{2}{5} + 3\frac{1}{4}}{3\frac{1}{4} - 1\frac{2}{5}} + \frac{2}{7} \text{ of } (3\tfrac{1}{2} - \tfrac{1}{4})$$

27. What number multiplied by $3\frac{1}{2}$ will produce $17\frac{3}{4}$?

28. What must be added to $3\frac{1}{2}$ to make a number 5 times as great as itself?

29. What fraction of a £ must be added to $\frac{3}{4}$*s.* to produce the $\frac{1}{8}$ of a £?

30. Find the value of $13\frac{1}{4}$ yards of stuff at 1*s.* $3\frac{1}{3}$*d.* + $17\frac{1}{10}$ yds. of silk at 6*s.* $4\frac{1}{6}$*d.*

31. A owned $\frac{3}{5}$ of a certain property; he sold $\frac{4}{7}$ of his share to B, and B sold $\frac{1}{6}$ of his share to C. If the whole property was worth £10,000, what was the value of C's share?

32. After walking $\frac{2}{7}$ of my journey, I found I had 15 miles farther to walk; what was the length of the journey?

33. A can do a certain piece of work in 4 days, B in 5 days, C in 6 days, and D in 7 days; in what time will they do it working together?

34. A house and furniture are worth £1,047 13*s.* 4*d.*; the house is worth 3 times as much as the furniture; what is the value of the furniture?

35. If a coach wheel is $15\frac{1}{7}$ ft. in circumference, how many turns does it make in $1\frac{3}{4}$ mile?

36. An estate is worth 5,000 guineas; what fractional part of the estate will 350 guineas purchase?

37. A person owned $\frac{3}{16}$ of an estate; he sold $\frac{1}{7}$ of $\frac{3}{16}$ of his share for £1,500; what part of the estate has he left and what is its value at the same rate?

38. A can mow a certain field in 9 days, B in 8 days, and C in 10 days; in how much less time will A and B do it than B and C, working together?

39. After sending away $\frac{2}{3}$ of $\frac{1}{2}$ + $\frac{3}{4}$ of $\frac{1}{7}$ of my money, what part is left?

40. Two men can reap a field in 9 days; at the end of five days one man is taken ill; how long will it take the other to complete the work?

41. By what number must you multiply $\frac{1}{15}$ of $\frac{3}{4}$ of $\frac{20}{21}$ to produce 2 as a product?

42. Divide $172\frac{1}{5}$ lbs. of tea into 2 parcels, putting $38\frac{2}{5}$ lbs. more in one parcel than in the other.

43. By working $13\frac{2}{5}$ hours per day a man can complete a work in 5 days; in what time will he complete it working $5\frac{3}{5}$ hours per day?

44. A and B working together can build a wall in 20 days; A working alone can do the work in 30 days; in what time could B do it?

45. A man does $\frac{1}{5}$ of a piece of work in 2 days; in how many more days will he complete the work?

46. A can do a piece of work in 10 days, B in 15 days; B works 6 days and then A joins him; in how many days will the work be completed?

47. A can dig $\frac{3}{7}$ of a garden in $2\frac{1}{3}$ days, and B $\frac{2}{7}$ in $1\frac{3}{4}$ day; in what time can they dig the whole?

48. At what time will the hands of a clock be together for the first time after 4 o'clock?

49. At what time will the hands of a clock be at right angles to each other for the first time after 6 o'clock?

50. At what time will the hands of a clock be opposite to each other for the first time after 5 o'clock?

CHAPTER XXXVIII.

DECIMAL FRACTIONS.

SECTION I.

WE have hitherto been dealing with fractions with varying denominators, we now propose to deal with fractions which always have 10 or some *power* of 10 for denominators; such fractions are called *decimals* (*deci*, "ten").

The children are perfectly familiar with the fact that each figure beyond the units is 10 times the same figure to the right, and a tenth part of the same figure to the left. Thus the 1 in the tens place is 10 times the 1 in the units place, and a tenth part of 1 in the hundreds place.

We will now continue a similar notation downwards, separating our new figures by a dot.

Thus :—

Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.
1	1	1	1	·	1	1	1

The first figure 1 after the units will stand for $\frac{1}{10}$ of a unit, the second figure 1 will be $\frac{1}{10}$ of $\frac{1}{10}$, or $\frac{1}{100}$, the third $\frac{1}{10}$ of $\frac{1}{100}$, or $\frac{1}{1000}$, and so on.

We may write the given number thus :—

$$1000 + 100 + 10 + 1 + \tfrac{1}{10} + \tfrac{1}{100} + \tfrac{1}{1000}, \text{ \&c.}$$

In writing numbers in figures we omit the words hundreds, tens, &c., because these are shown by the *position* of the figures; and so in writing decimal fractions we omit the denominator because that is always indicated by the position of the figures, that is, by the number of places distant from the units figure.

Exercises for the black-board :—

Example 1.—Explain the meaning of 2·04.

2 means 2 units
0 „ that we have no tenths
4 „ four-hundredths

Written as a vulgar fraction $= 2\frac{4}{100}$ or $2\frac{1}{25}$.

Example 2.—What is the value of the cipher in the following:—

·04, 40, ·40, 04.

(1) ·04. In this number the cipher makes the $4 = \frac{4}{100}$ instead of $\frac{4}{10}$. It will be seen from this that a 0 placed to left of a decimal *divides* it by 10.

(2) $40 = 4$ tens and 0 units; the cipher makes the 4 into 40. A cipher placed to the right of an integer *multiplies* it by 10.

(3) $\cdot 40 = \frac{4}{10} + \frac{0}{100}$, therefore the 0 is of no value. A cipher placed to the right of a decimal does not alter its value.

(4) $04 = 4$ units and 0 tens. A cipher to the left of an integer does not alter its value.

Example 3.—Express as vulgar fractions ·01, ·42, 2·1, 1·001.

$$\cdot 01 = \frac{0}{10} + \frac{1}{100} = \frac{1}{100}$$

$$\cdot 42 = \frac{4}{10} + \frac{2}{100} = \frac{42}{100} = \frac{21}{50}$$

$$2\cdot 1 = 2 + \frac{1}{10} = 2\frac{1}{10}$$

$$1\cdot 001 = 1 + \frac{1}{1000} = 1\frac{1}{1000}$$

Example 4.—Express as decimal fractions $\frac{4}{10}$, $\frac{5}{100}$, $\frac{6}{1000}$, $\frac{2}{5}$, $\frac{6}{25}$.

(1) $\frac{4}{10} = \cdot 4.$ (2) $\frac{5}{100} = \cdot 05.$

(3) $\frac{6}{1000} = \cdot 006.$ (4) $\frac{2}{5} = \frac{4}{10} = \cdot 4.$

(5) $\frac{6}{25} = \frac{24}{100} = \cdot 24.$

EXERCISES ON DECIMAL NOTATION.

1. From the following numbers take the figure 3, and write it to show its value, in the form of a vulgar fraction: ·3, ·03, ·003, ·0003, ·00003.

2. Write each of the figures in the following number in the form of a vulgar fraction: ·32756.

3. Change to vulgar fractions: ·05, ·006, ·0007.

4. Write in the decimal form one-hundredth, five-thousandth, three-tenths, and two ten-thousandths.

5. Write as vulgar fractions ·17, 1·7, ·017.

6. What is the value of the figure 5 in each of the following decimal fractions: ·05, 3·5, 21·60504, ·752, 1·052?

7. Place a cipher on the left hand of each of the following numbers, and say how the value is changed, ·4, ·05, 4·2, ·056, 5.

8. What is the effect of the cipher in each of the following numbers: ·03, 30, 03·5, 2·150?

9. Multiply each of the following numbers by 10: ·05, ·006, 1·062, 66·021, 207·1.

10. Divide each of the following numbers by 10: ·05, ·201, ·2401, ·1, ·01, ·001, ·101.

11. Multiply ·007, 2·105, 30·01, ·82, ·0004, ·00001, each by 100.

12. Divide ·7, 210·5, 3001, 82, ·04, ·001, each by 100.

SECTION II.

ADDITION AND SUBTRACTION OF DECIMAL FRACTIONS.

Addition and subtraction of decimal fractions present no more difficulty than the addition and subtraction of integers.

We have to bear in mind that as hundreds and units have their proper columns, so have the tenths and the hundredths, &c.

Example 1.—Add 35·42, ·647, ·004, 100·37.

	Tenths.	Hundredths.	Thousandths.
35 ·	4	2	
·	6	4	7
·	0	0	4
100 ·	3	7	
136 ·	4	4	1

7 + 4 = 11, viz. 11 thousandths = 1 hundredth and 1 thousandth; put down 1 thousandth and carry 1 hundredth to the next or hundredth column.

1 + 7 + 4 + 2 = 14, viz. 14 hundredths = 1 tenth and 4 hundredths; put down 4 hundredths and carry 1 tenth.

1 + 3 + 6 + 4 = 14, viz. 14 tenths = 1 unit and 4 tenths; put down 4 tenths and carry 1 unit to the units column. Place the dot to separate tenths from units, and complete as in ordinary addition.

Example 2.—From 104·011 subtract 15·6042.

104·0110
 15·6042
 88·4068

We cannot subtract 2 ten-thousandths from 0, we change the one thousandth to tens of thousandths.

2 from 10 leaves 8; 4 from 0 impossible, change the 1 hundredth to 10 thousandths; 4 from 10 leaves 6; 0 from 0 leaves 0; 6 from 0 impossible, change one of the 4 units to tenths; 6 from 10 leaves 4; complete as in ordinary subtraction.

EXERCISES IN ADDITION AND SUBTRACTION OF DECIMAL FRACTIONS.

1. Find the sum of 14·3, 3·03, 25·005, 1·025, 250·5.

2. Add together 1, ·1, 9, ·9, 1001, ·1001, 180·3, and ·1803.

3. 8·3 + 83·3 + 183·03 + 3·183 + 5.

4. 1·59 + 59·1 + 3·159 + 159·3 + ·001.

5. Find the sum of 108, ·108, 1·08, and 10·8.

6. Add $\frac{2}{10000}$, $\frac{19}{100}$, $\frac{3}{100000}$, $\frac{75}{10000}$.

7. Reduce to decimals, and add $\frac{2}{100}$, $\frac{3}{10000}$, $\frac{154}{100}$, $\frac{85}{100000}$, $\frac{17}{100}$, $\frac{1}{10}$.

8. Find the value of 3·56 + $\frac{2}{100}$ + 4·013 + $\frac{1}{1000}$ + 15·1.

9. What is the sum of £5·3, £·18, £·53, and £42·05?

10. Add £$\frac{5}{1000}$, £·25, £$\frac{106}{10}$, £3·15.

11. Take ·5 from 5; ·15 from 15; ·051 from 15; 31·65 from 316; ·805 from 8.

12. Find the difference between 183 and ·183; 105 and ·105; 1 and ·1; 13 and ·0013.

13. Take ·365 from 365, and add ·005 to the remainder.

14. Add decimally $\frac{3}{1000}$ and $\frac{3}{10}$, and find the difference between this sum and 1.

15. What is the difference between £3 and £·3; between 3*s*. and ·03*s*.; between 15 yds. and ·15 yds.?

16. How much greater is ·5 than ·05? Give your answer in a vulgar as well as in a decimal fraction.

17. From the hundredth part of £15 take the thousandth part of £5. (Work by decimals.)

18. Find the difference between £8 and £·808, and between 5*s*. and ·05*s*.

SECTION III.

MULTIPLICATION OF DECIMAL FRACTIONS.

I. TO MULTIPLY BY 10 OR ANY POWER OF 10.

We have seen that to move the decimal point one *place* to the *right* is equivalent to multiplying by 10.

$$\text{Thus}\quad 3{\cdot}75 \times 10 = 37{\cdot}5$$
$$\text{and}\quad {\cdot}005 \times 10 = {\cdot}05$$

Now if we multiply these products again by 10 we shall change the decimal point one more place to the right.

$$\text{Thus}\quad 37{\cdot}5 \times 10 = 375$$
$$\text{and}\quad {\cdot}05 \times 10 = {\cdot}5$$

But to multiply by 10, and the product again by 10, is to multiply by 100. Hence to multiply by 100 move the decimal point *two* places to the right; and to multiply by 1000 *three* places to the right; and so on.

Now if we compare the number of ciphers in the multipliers with the number of places we move the decimal point, we are able to deduce the following rule:—

To multiply a decimal by 10 *or any power of* 10, ***shift*** *the decimal point as many places to the right as there are ciphers in the multiplier.*

II. TO MULTIPLY DECIMAL FRACTIONS BY ANY OTHER DECIMAL FRACTIONS.

Example 1.—Multiply 2·3 by 3·2, and 4·01 by 6·5.

$$(1)\quad 2{\cdot}3 = 2\frac{3}{10} = \frac{23}{10}$$

$$3{\cdot}2 = 3\frac{2}{10} = \frac{32}{10}.$$

$$\frac{23}{10} \times \frac{32}{10} = \frac{736}{100} = 7{\cdot}36.$$

$$(2)\quad 4{\cdot}01 = 4\frac{1}{100} = \frac{401}{100}.$$

$$6{\cdot}5 = 6\frac{5}{10} = \frac{65}{10}.$$

$$\frac{401}{100} \times \frac{65}{10} = \frac{26065}{1000} = 26{\cdot}065.$$

Now compare the number of decimal places in the multiplier and multiplicand *together*, with the number of decimal places in the product, in each case they are the same.

Hence deduce the rule :—

Multiply as in ordinary multiplication, and from the product mark off—from the right—as many decimal places as there are decimal places in the multiplier and multiplicand together.

Example 2.—Multiply 3·047 by 35·04.

$$\begin{array}{r} 3{\cdot}045 \\ \underline{35{\cdot}04} \\ 12180 \\ 15225 \\ \underline{9135} \\ \underline{\underline{106{\cdot}69680}} \end{array}$$

After having found the position of the decimal point in he product, we strike out the cipher on the right as not ffecting the value of the decimal.

EXERCISES ON MULTIPLICATION OF DECIMAL FRACTIONS.

1. Multiply ·016 by 10; ·354 by 100; ·781 by 1000; ·2 by 0,000.
2. Multiply ·35 by 10,000; ·35 by 100; ·351 by 10; 2·4 by 00.
3. Multiply 2·34 by 100; 1·75 by 10,000; 3·51 by 100; 2·7 y 100.
4. Multiply ·01 by 1000; ·1 by 100; ·02 by 1000.
5. Multiply ·4 by 5; ·18 by 3; 3·9 by 7; 5 by ·4; 35 by ·18; 2 by 3·9.
6. Multiply 71·3 by 16·2; 18·4 by ·003; 29·1 by ·008; 2·17 y ·0016.
7. Multiply 81·3 by ·37; and take the thousandth part of 15 om the product.
8. Find the product of ·309 and ·0197.

9. Find the value of 8·17 × 3·502; 38·6 × 2·94; 49·2 × 35.
10. Find the value of 78·5 × 3·96; 85·2 × 3·004; 4·216 × 209.
11. Find the value of 8·016 × 6·018; 8·134 × ·4831; 23·4 × ·0072.
12. Find the value of 3·806 × 8·036; 4·201 × ·00008; ·5 × 5000.
13. Multiply 82·9 by 72·6, and take 500 from the product.
14. Multiply 3·10002 by itself, and find the difference between the product and 10.
15. Multiply the sum of ·53 and 5·31 by the difference.
16. Find the difference between ·0038 and ·38 × 1.
17. Multiply the sum of ·4 and ·004 by their difference.
18. Multiply £81·013 by 2·03.
19. Find the value of 235 yds. of cloth at 2·53 shillings per yd.
20. What is the value of 350 times £2·54 in pounds?

SECTION IV.

DIVISION OF DECIMAL FRACTIONS.

TO DIVIDE BY 10, OR ANY POWER OF 10.

We have seen that to move the decimal point one place to the *left* is equivalent to division by 10.

Thus $3{\cdot}2 \div 10 = {\cdot}32$
and $\;{\cdot}6 \div 10 = {\cdot}06$

Now divide these quotients again by 10.

Thus $\;{\cdot}32 \div 10 = {\cdot}032$
$\;{\cdot}06 \div 10 = {\cdot}006$

But to divide by 10, and the quotient again by 10, is to divide by 100. Hence to divide by 100 move the decimal point *two* places to the left, and to divide by 1000 *three* places to the left, and so on (add ciphers to the left, if necessary, to make enough places).

From such examples deduce the rule :—

To divide a decimal fraction by 10, *or any power of* 10, *shift the decimal point as many places to the left as there are ciphers in the divisor.*

TO DIVIDE A DECIMAL FRACTION BY ANY OTHER DECIMAL FRACTION.

FIRST METHOD.

Example 1.—Divide ·068921 by ·041.

This may be written as a vulgar fraction, thus :—

$$\frac{\cdot068921}{\cdot041};$$

and since we may multiply both numerator and denominator of a fraction without altering its value, in this case multiply by 1000, and so get rid of all the decimal places in the denominator or divisor, thus :—

$$\frac{\cdot068921 \times 1000}{\cdot041 \times 1000} = \frac{68\cdot921}{41}$$

We may now divide as in ordinary division ; all the figures brought down from the dividend before the decimal point will give integers in the quotient, all after will be decimals.

$$\begin{array}{l}
41)68\cdot921(1\cdot681 \\
\quad\;\; \underline{41}\ \cdots \\
\quad\;\; 279 \\
\quad\;\; \underline{246} \\
\quad\;\;\; 332 \\
\quad\;\;\; \underline{328} \\
\quad\;\;\;\; \cdot41 \\
\quad\;\;\;\;\; \underline{\underline{41}}
\end{array}$$

Example 2.—Divide ·001 by ·0002.

$$\frac{\cdot001\times10000}{\cdot0002\times10000}=\frac{10}{2}=5.$$

From these, and similar examples, we may deduce the following rule: *Remove the decimal point of the divisor to the right of the right hand figure, making the divisor to consist of integers. Remove the decimal point of the dividend as many places to the right as it has been moved in the divisor. Divide as in ordinary division.*

SECOND METHOD.

Example 1.—Divide 7·36 by 3·2, and 31·968 by 7·4.

$$(1)\ 7\cdot36=7\frac{36}{100}=\frac{736}{100}.\qquad 3\cdot2=3\frac{2}{10}=\frac{32}{10}.$$

$$\text{And }\frac{736}{100}\div\frac{32}{10}=\frac{\overset{\overset{23}{\cancel{92}}}{\cancel{736}}}{\underset{10}{\cancel{100}}}\times\frac{\overset{1}{\cancel{10}}}{\underset{\underset{1}{\cancel{4}}}{\cancel{32}}}=\frac{23}{10}=2\cdot3.$$

$$(2)\ 31\cdot968=31\frac{968}{1000}=\frac{31968}{1000}.\quad 7\cdot4=7\frac{4}{10}=\frac{74}{10}.$$

$$\text{And }\frac{31968}{1000}\div\frac{74}{10}=\frac{\overset{\overset{432}{\cancel{15984}}}{\cancel{31968}}}{100\cancel{0}}\times\frac{\overset{1}{\cancel{10}}}{\underset{\underset{1}{\cancel{37}}}{\cancel{74}}}=\frac{432}{100}=4\cdot32.$$

Compare the number of decimal places in the divisor, dividend, and quotient. It will be seen that in each case the number in the quotient is the number in the dividend *less* the number in the divisor.

Hence, from these and other examples, deduce the rule:—*Divide as in ordinary division, and mark off as many decimal* *places in the quotient as there are in the dividend, less the number in the divisor.*

If there are not a sufficient number of decimal places in the dividend indicate more by ciphers, thus:—

Example 2.—Divide 6·15625 by ·000625.

In this example we have *one* more decimal place in the divisor than in the dividend, we add one cipher to the dividend, and then the number of decimal places in the dividend less the number in the divisor will be nothing, that is, we have no decimals in our quotient.

```
·000625)6·156250(9850·
        5625 · · ·
        ————
         5312
         5000
         ————
          3125
          3125
          ————
          · · · · 0
          ————
```

EXERCISES IN DIVISION OF DECIMAL FRACTIONS.

1. Divide 5·324 by 10; 2·006 by 100; 3·017 by 10; ·01 by 1000.
2. Divide ·82 by 100; 82 by 100; ·082 by 1000; ·3 by 10000.
3. Divide ·54 by 100; 73·2 by 10; ·732 by 10.
4. Divide ·82 by 1000; 84 by 1000; 32 by 10.
5. Divide ·31 by 100; 3·7 by 1000; ·9 by 1000; 1·43 by 10.
6. Divide ·1 by 100; ·01 by 100.
7. Divide 35 by 3; 74 by ·32; 21 by ·36.
8. Divide 71 by ·04; 310 by ·52; 815 by 2·4.
9. Divide 316·4 by ·81; 725·3 by ·29; 27·4 by ·71; 29·84 by ·386.
10. Divide 29·8 by 496; ·304 by 304; 1·01 by 101; 7 by ·865.
11. Divide ·19 by 199; ·26 by 264; ·73 by 734; 8 by 9·862.
12. Divide 9·84 by 1001; 2·36 by 7·864; 1·02 by 908; 8·34 by 5000·7.
13. Find the value of (7·95 + 1·047 − ·467) ÷ ·81.

14. Find the value of (3·46 + ·0015 — ·471) ÷ 79.
15. Find the value of (3·164 — 2·987 + 4·019) ÷ 2·07.
16. Divide £·078 into 4 equal parts.
17. If 350 sheep cost £753·294, what is the price of one ?
18. Simplify 3·86 of 14·4 ÷ $2\frac{6}{100}$.
19. Simplify (18·9 + 3·016 — 2·84) ÷ ·5.
20. What is the fourth part of 37 ? 1·001 ? 7·105 ? 8·116 ?

SECTION V.

TO CONVERT VULGAR TO DECIMAL FRACTIONS. RECURRING DECIMALS.

Examples for the black-board :—

(1) $\frac{2}{5} = \frac{4}{10} = \cdot 4.$

(2) $\frac{3}{4} = \frac{15}{20} = \frac{75}{100} = \cdot 75.$

(3) $\frac{6}{25} = \frac{24}{100} = \cdot 24.$

We may obtain these same results by dividing the numerator by the denominator, adding decimal places if required. Thus—

(1) 5)2·0
·4

(2) 4)3·00
·75

(3) 25)6·00
·24

[illegible]ce the rule : *Divide the numerator by the denominator.*

mple 1.—Reduce $\frac{137}{256}$ to a decimal fraction.

```
256)137·00000000(·53515625
    1280
    ----
      900
      768
      ----
      1320
      1280
      ----
        400
        256
        ----
        1440
        1280
        ----
         1600
         1536
         ----
          ·640
           512
           ----
           1280
           1280
           ----
           ....
           ====
```

mple 2.—Reduce the following vulgar to decimal ns : $\frac{1}{3}$, $\frac{6}{11}$, $\frac{5}{12}$, $\frac{25}{54}$.

```
(1)      3)1·0000000
            ·3333333, &c.

(2)     11)6·00000000
            ·54545454, &c.

(3)     12)5·00000000
            ·41666666, &c.

(4)    54)25·0000000
            ·4629629, &c.
```

hese examples we see that the quotient never ends. nnot therefore accurately express the given vulgar as l fractions. Because the figures of the quotient repeat themselves we call such decimals *recurring*, *repeating*, or *circulating* decimals.

When *all* the figures recur, as in (1) and (2) above, the decimal fractions are called *pure circulators*; when a part only recur, as in (3) and (4), we call the fractions *mixed circulators*.

To save writing the figures over and over again we place a dot over the recurring figure, or if there is more than one recurring figure a dot over the first and last figure of the recurring period.

Thus (1) $\frac{1}{3} = \cdot\dot{3}.$

(2) $\frac{6}{11} = \cdot\dot{5}\dot{4}.$

(3) $\frac{5}{12} = \cdot41\dot{6}.$

(4) $\frac{25}{54} = \cdot4\dot{6}2\dot{9}.$

NOTE 1.—We have seen that a decimal fraction has some power of 10 for its denominator. But 2 and 5 are the only factors of 10. Hence any vulgar fraction whose denominator consists of the factors 2 and 5, or of the powers of 2 and 5, is reducible to an exact decimal fraction. Such decimal fractions are *finite*. If any other factor such as 3, 7, 9, 11, 13, &c., be in the denominator the fraction is not exactly reducible, and the decimal is *indeterminate*.

NOTE 2.—The work may sometimes be shortened by so multiplying or dividing the numerator and denominator of the fraction as to get the denominator in the form of a factor multiplied into 10 or some power of 10.

Thus:—Reduce $\frac{47}{160}$ and $\frac{47}{480}$ to decimal fractions.

$\frac{47}{160}$. Multiply numerator and denominator by 5, and by 100.

$$\frac{47 \times 5}{160 \times 5} = \frac{235 \div 100}{800 \div 100} = \frac{2{\cdot}35}{8} = {\cdot}29375$$

(2) $\frac{47}{480}$. Divide numerator and denominator by 8, multiply y 5, and divide by 100.

$$\frac{47 \div 8}{480 \div 8} = \frac{5{\cdot}875 \times 5}{60 \times 5} = \frac{29{\cdot}375 \div 100}{300 \div 100} = \frac{{\cdot}29375}{3}$$
$$= {\cdot}09791\dot{6}.$$

EXERCISES IN CONVERTING VULGAR TO DECIMAL FRACTIONS.

1. In the following fractions, which will produce finite decimals? $\frac{1}{2}$, $\frac{2}{5}$, $\frac{2}{3}$, $\frac{5}{9}$, $3\frac{1}{7}$, $16\frac{1}{4}$, $\frac{17}{32}$.

2. Reduce to decimals $5\frac{1}{4}$, $6\frac{2}{5}$, $\frac{13}{20}$, $\frac{19}{64}$, $\frac{3}{125}$.

3. Express as decimals $\frac{13}{25}$, $\frac{13}{16}$, $1\frac{5}{6}$, $\frac{2}{3}$ of $\frac{5}{8}$, $\frac{3}{2\frac{1}{2}}$.

4. Write in the decimal form $\frac{1}{100}$, $\frac{3}{10000}$, $\frac{6}{100000}$, $\frac{23}{3000}$.

5. Reduce to decimals $\frac{11}{111}$, $5\frac{29}{37}$.

6. Express as decimals $\frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4}}{\frac{1}{2} \text{ of } \frac{1}{3}}$, and $\frac{\frac{2}{3} \text{ of } 3\frac{3}{4}}{\frac{1}{2} + 2\frac{1}{6}}$.

SECTION VI.

O CONVERT DECIMAL TO VULGAR FRACTIONS. RECURRING DECIMALS.

Examples for the black-board :—

Reduce the following decimals to vulgar fractions ·4, ·14, 05, 4·004.

(1) $\cdot 4 = \frac{4}{10} = \frac{2}{5}$.

(2) $\cdot 14 = \frac{1}{10} + \frac{4}{100} = \frac{10}{100} + \frac{4}{100} = \frac{14}{100} = \frac{7}{50}$.

(3) $\cdot 205 = \frac{2}{10} + \frac{0}{100} + \frac{5}{1000} = \frac{205}{1000} = \frac{41}{200}$.

(4) $4{\cdot}004 = 4 + \frac{4}{1000} = \frac{4000}{1000} + \frac{4}{1000} = \frac{4004}{1000} = \frac{1001}{250}$.

From examples such as these the following rule may be elicited from the children:—

To reduce a decimal to a vulgar fraction, write the decimal (or whole number and decimal) for the numerator, and for the denominator 1 *followed by as many ciphers as there are decimal places in the fraction to be reduced. Reduce the fraction to its lowest terms.*

Note.—This rule only holds good for *finite* decimals.

$$\text{Thus—}\quad 3\cdot47 = \tfrac{347}{100}$$
$$62\cdot001 = \tfrac{62001}{1000}$$
$$\cdot001 = \tfrac{1}{1000}$$
$$\cdot0001 = \tfrac{1}{10000}.$$

To reduce pure recurring decimals to fractions the teacher should take such examples as the following:—

$$\frac{5}{9} = \cdot\dot{5} \qquad \frac{54}{99} = \cdot\dot{5}\dot{4} \qquad \frac{627}{999} = \cdot\dot{6}2\dot{7}$$

and show that:—

1. A pure circulator of one figure has one nine for its denominator in its equivalent vulgar fraction.

2. A pure circulator of two figures has two nines for its denominator.

3. A pure circulator of three figures has three nines for its denominator, and so on.

And hence deduce the rule:—

To reduce a pure circulating decimal to a vulgar fraction, make the figure or figures which recur the numerator, and write for the denominator as many nines as there are digits in the period or "repetend."

$$\text{Thus—}\quad \cdot\dot{3}\dot{7} = \frac{37}{99} \qquad \cdot\dot{4}01\dot{4} = \frac{4014}{9999}$$

$$\cdot\dot{0}05\dot{4} = \frac{0054}{9999} = \frac{54}{9999}$$

The rule for reducing mixed circulators may be deduced by a similar method.

We give a second method which is applicable also to the reduction of pure circulators.

Let it be required to reduce $\cdot 4\dot{6}\dot{2}$ to its equivalent vulgar fraction.

The fraction $= \cdot 4626262$, &c.

(1) Hence 1000 times the fraction $= 462\cdot 6262$, &c.

(2) And 10 times the fraction $= 4\cdot 6262$, &c.

Subtracting (2) from (1) we get 990 times the fraction $= 458$; that is, the fraction $= \frac{458}{990}$.

In a similar way it may be shown that in all cases of recurring mixed circulators to vulgar fractions we find:—

1. The numerator, by taking the fraction as far as the end of the first period ($\cdot 4\dot{6}\dot{2}$) and subtracting the figures that do not recur ($462 - 4 = 458$).

2. The denominator, by taking as many nines as there are digits recurring, and adding as many ciphers as there are figures that do not recur.

Thus—Reduce $\cdot 085\dot{4}0\dot{3}$, $\cdot 00\dot{5}$, $\cdot 00\dot{1}\dot{7}$ to fractions.

(1) $\cdot 085\dot{4}0\dot{3} - 85 = 85318 =$ numerator; $999000 =$ denominator; $\therefore\ \cdot 085\dot{4}0\dot{3} = \frac{85318}{999000}$.

(2) $\cdot 00\dot{5} = \frac{5}{900}$.
(3) $\cdot 00\dot{1}\dot{7} = \frac{17}{9900}$.

NOTE.—For the addition, subtraction, multiplication, and division of recurring decimals, it is best for young pupils to reduce the equivalent vulgar fractions.

Addition and subtraction may be effected to any assigned degree of accuracy by extending the recurring figures and adding and subtracting in the usual way.

Thus—Add $4\cdot\dot{5}\dot{9}$, $37\cdot\dot{0}2\dot{1}$, $\cdot\dot{5}60\dot{7}$, and subtract $40\cdot\dot{5}40\dot{7}$. Give the answer correct to five places of decimals.

4·59595 959
37·02102 102
56075 607

42·17773 668
40·54075 407

1·63698 261 *Ans.* 1·63698

EXERCISES IN RECURRING DECIMALS.

1. Write the following recurring decimals in the form of vul
fractions :—

(1) $\cdot\dot{3}$; $\dot{3}\dot{5}$; $\cdot\dot{4}0\dot{5}$; $\cdot\dot{1}\dot{8}$; $\cdot 02\dot{5}$.
(2) $3\cdot\dot{7}$; $\cdot\dot{2}\dot{6}$; $12\cdot 2\dot{9}\dot{1}$; $\cdot 1\dot{4}$.
(3) $14\cdot\dot{3}0\dot{7}$; $21\cdot 1\dot{5}$; $74\cdot\dot{1}0\dot{4}$; $3\cdot 1\dot{6}$.
(4) $\cdot 0\dot{2}$; $17\cdot\dot{0}0\dot{5}$; $\cdot\dot{0}00\dot{5}$; $18\cdot\dot{5}$.
(5) $13\cdot\dot{1}\dot{3}$; $13\cdot\dot{0}1\dot{3}$; $13\cdot\dot{0}01\dot{3}$; $13\cdot 00\dot{1}\dot{3}$.
(6) $14\cdot 1\dot{2}$; $\cdot 8\dot{4}$; $192\cdot 1\dot{3}\dot{8}$; $\cdot 7\dot{6}$.
(7) $31\cdot 2\dot{9}8\dot{6}$; $25\cdot 2\dot{4}7\dot{9}$; $4\cdot\dot{8}36\dot{2}$.
(8) $4\cdot 7\dot{1}3\dot{9}$; $2\cdot 103\dot{5}$; $7\cdot 8\dot{1}4\dot{6}$.
(9) $2\cdot\dot{0}0\dot{5}$; $17\cdot 3\dot{1}\dot{2}$; $2\cdot 9\dot{8}6\dot{4}$.
(10) $\cdot 000\dot{5}\dot{3}$; $\cdot 00\dot{2}\dot{1}$; $2\cdot 00\dot{8}\dot{7}$.
(11) $5\cdot 8\dot{2}\dot{7}$; $\cdot 1\dot{0}\dot{9}$; $3\cdot 1\dot{4}\dot{6}$.
(12) $7\cdot 1\dot{3}\dot{5}$; $\cdot 0\dot{4}\dot{6}$; $\cdot 7\dot{1}\dot{3}$; $\cdot 2\dot{6}$.
(13) $315\cdot 1\dot{0}\dot{6}$; $\cdot 40\dot{1}\dot{7}$; $3\cdot 0\dot{1}\dot{6}$; $\cdot 2\dot{5}\dot{1}$.
(14) $374\cdot 1\dot{1}\dot{6}$; $\cdot 0\dot{1}3\dot{4}$; $181\cdot\dot{0}01\dot{6}$.
(15) $53\cdot\dot{0}01\dot{5}$; $4\cdot 4\dot{1}\dot{6}$; $\cdot 0\dot{1}\dot{3}$.

2. Add together $4\cdot\dot{2}$, $3\cdot\dot{1}\dot{6}$, and $7\cdot\dot{4}\cdot$

3. Find the sum of $\cdot\dot{4}\dot{6}$, $\cdot 327$, $1\cdot\dot{0}\dot{8}$.

4. Take $\cdot\dot{8}1\dot{3}$ from $9\cdot 17$.

5. From $18\cdot\dot{5}$ take $4\cdot 1\dot{6}$.

6. Find the value of $(3{\cdot}\dot{7} + 2{\cdot}\dot{3}) \times {\cdot}00\dot{5}$.

7. Multiply $3{\cdot}56$ by $3{\cdot}0\dot{5}$; $4{\cdot}\dot{2}\dot{1}$ by $2{\cdot}\dot{8}$.

8. Multiply $4{\cdot}3\dot{2}$ by $1{\cdot}\dot{4}$; ${\cdot}3\dot{1}\dot{9}$ by $2{\cdot}\dot{6}$.

9. Divide ${\cdot}2\dot{6}\dot{2}$ by ${\cdot}3\dot{4}$; $4{\cdot}1\dot{3}$ by $2{\cdot}\dot{7}$.

10. Divide ${\cdot}4\dot{8}\dot{1}$ by $3{\cdot}\dot{5}$; $2{\cdot}\dot{1}\dot{7}$ by ${\cdot}\dot{9}\dot{4}$.

11. Divide $3{\cdot}\dot{1}\dot{6}$ by $2{\cdot}\dot{4}$; $7{\cdot}\dot{1}\dot{9}$ by $3{\cdot}\dot{4}$.

12. Multiply $8{\cdot}1\dot{4}$ by ${\cdot}2\dot{1}\dot{3}$; $7{\cdot}\dot{1}\dot{3}$ by $2{\cdot}\dot{4}$.

13. Divide $4{\cdot}\dot{3}$ by $2{\cdot}\dot{5}\dot{4}$; $2{\cdot}1\dot{7}$ by ${\cdot}\dot{3}1\dot{6}$.

14. Take $8{\cdot}1\dot{4}$ from $12{\cdot}\dot{1}\dot{5}$, and divide the remainder by ${\cdot}\dot{2}$.

15. Add $3{\cdot}1\dot{4}$ to ${\cdot}3\dot{1}\dot{4}$ and multiply the sum by ${\cdot}\dot{7}$.

16. Take $8{\cdot}\dot{0}3\dot{7}$ from $12{\cdot}\dot{1}4\dot{5}$.

SECTION VII.

REDUCTION OF DECIMAL FRACTIONS.

To reduce a decimal fraction of one denomination to its quivalent decimal fraction of another denomination, proceed in ordinary reduction.

Example 1.—Reduce £·376 to pence.

$$\begin{array}{rl} {\cdot}376 & \\ 20 & \\ \hline 7{\cdot}520 & \text{shillings} \\ 12 & \\ \hline 90{\cdot}240 & \text{pence} = 7s.\ 6{\cdot}24d. \\ \hline\hline \end{array}$$

If we remove the integers at each step, we get the value of e decimal in terms of the different units of like kind.

Example 2.—Find the value of £1·226.

$$\begin{array}{r} \pounds 1{\cdot}226 \\ 12 \\ \hline 2{\cdot}712s. \\ 4 \\ \hline 2{\cdot}848d. \\ \hline\hline \end{array}$$

Hence £1·226 = £1 2*s.* 2·848*d.*

Example 3.—Reduce ·492 inches to yards.

$$\begin{array}{r l} 12) & {\cdot}492 \text{ inches} \\ \hline 3) & {\cdot}041 \text{ feet} \\ \hline & {\cdot}013\dot{6} \text{ yards.} \\ \hline\hline \end{array}$$

To reduce quantities of one denomination to the decimal of another, first reduce to vulgar fractions as already explained, and then reduce to decimal fractions.

Example 1.—Reduce £39 18*s.* 1½*d.* to the decimal of £47 17*s.* 9*d.*

$$\frac{\pounds 39\ 18s.\ 1\tfrac{1}{2}d.}{\pounds 47\ 17s.\ 9d.} = \tfrac{19155}{22986} \text{ half-pence} = {\cdot}8\dot{3}.$$

$$\begin{array}{r} 22986)19155{\cdot}00({\cdot}8\dot{3} \\ 18388\ 8\phantom{0(\cdot 8\dot{3}} \\ \hline 766\ 20\phantom{(\cdot 8\dot{3}} \\ 689\ 58\phantom{(\cdot 8\dot{3}} \\ \hline 76\ 62\phantom{(\cdot 8\dot{3}} \\ \hline\hline \end{array}$$

Example 2.—Express 1 cwt. 3 qrs. 7 lbs. as the decimal of 15 cwts.

$$\frac{1 \text{ cwt. } 3 \text{ qrs. } 7 \text{ lbs.}}{15 \text{ cwts.}} = \frac{203}{1680} = \frac{20{\cdot}3}{168} = {\cdot}1208\dot{3}$$

$$\begin{array}{l}
168)20{\cdot}300000({\cdot}1208\dot{3}\\
\underline{16\,8}\\
\underline{3\,50}\\
\underline{3\,36}\\
\phantom{168)20{\cdot}}\underline{1400}\\
\phantom{168)20{\cdot}}\underline{1344}\\
\phantom{168)20{\cdot}3}\underline{560}\\
\phantom{168)20{\cdot}3}\underline{504}\\
\phantom{168)20{\cdot}30}\underline{\underline{56}}
\end{array}$$

EXERCISES IN REDUCTION OF DECIMALS.

1. Find the value of :—

(1) £·186 ; £·37 ; £4·37.
(2) £·324 ; £·345 ; ·48 shilling ; £·15.
(3) £·487 ; ·296 crown ; ·78 guinea.
(4) £·834 ; ·95 fourpence ; £·817.
(5) £·371 ; ·58 cwt. ; ·345 qr.
(6) 2·91 ozs. ; 7·54 cwts. ; 9·83 lbs.
(7) 4·67 yds. ; ·953 acre.

2. Find the value of 3·72 qrs. + 4·57 cwts. + 3·7 qrs.

3. Find the value of £3·79 + £2·83 + 4·5*s.*

4. Find the value of £12·398, and subtract the result from 13 guineas.

5. Express £·5832 in terms of shillings and pence.

6. How many cwts., qrs., and lbs. are there in ·983 ton ?

7. Express ·983 yd. in the denomination of feet and inches.

8. What is the value, in pence, of £123 + ·24 shilling + 14·8 pence ?

9. Express ·526 acre in terms of roods and perches.

10. What is the difference in value between £13·875 and £5 3*s.* 4½*d.* ?

11. How many shillings and pence are equal to ·89 crown ?

12. Find the value of 5·4 of £·87, and give your answer in £ *s.* *d.*

13. Find the value of ·46 shilling + £·872 + ·46 pence.

14. Take £·934 from £2·946, and add £1 3*s*. 9½*d*. to the remainder.

15. Reduce 3·84 shillings to £'s; 8·75 pence to £'s; and 3*s*. 9¼*d*. to £'s.

16. Reduce 3 days to weeks; ·73 week to months; 3·65 days to years; and 8·34 months to years.

17. Express 4*s*. 7½*d*. in £'s; 2*s*. 9¼*d*. in £'s; and ·836 farthing in £'s.

18. Reduce to the decimal fraction of a £: £1 3*s*. 6¾*d*.; £2 18*s*. 9½*d*.; and £1 13*s*. 5¾*d*.

19. Reduce to the decimal of an acre: 133 sq. feet; ·139 sq. perch; and ·87 rood.

20. What is the difference between £1 4*s*. 2½*d*. and £·837?

21. Give the value in £'s of 5*s*. 3*d*. + £2 3*s*. 4½*d*. + 1*s*. 6¾*d*. + 2¾*d*.

22. Express the difference between half a guinea and 4*s*. 3½*d*. in the denomination of £'s.

23. Reduce ⅓ of £1 10*s*. 4½*d*. to the decimal of a guinea.

24. What decimal fraction is 12*s*. 6½*d*. of £5 12*s*. 10½*d*.?

25. Express £3,167 9*s*. as the decimal of £3,749 16*s*. 11¾*d*.

26. What decimal of a guinea is ·3 of 10*s*.?

27. Reduce 8*s*. 9*d*. to the decimal of £1 6*s*. 3*d*. and £1 0*s*. 3*d*. to the decimal of 8*s*. 9*d*.

SECTION VIII.

MISCELLANEOUS EXAMPLES IN VULGAR AND DECIMAL FRACTIONS.

Examples to be worked on the black-board by the teacher assisted by the scholars.

Example 1—Find the least vulgar fraction which may be added to 3·75, 2·405, 3·03 to produce a whole number.

$$\begin{array}{r} 3{\cdot}75 \\ 2{\cdot}405 \\ \underline{3{\cdot}03} \\ \underline{\underline{9{\cdot}185}} \end{array} = 9\tfrac{185}{1000}$$

To make the fraction $\frac{185}{1000}$ a whole number we must add its difference from one.

$$\frac{1000}{1000} - \frac{185}{1000} = \frac{815}{1000}$$

$$\text{And } 9\frac{185}{1000} + \frac{815}{1000} = 10$$

Hence the required fraction is $\frac{815}{1000}$ or $\frac{163}{200}$.

Example 2.—What must be added to 1·35 of £3 7*s.* 6*d.* to make up $\frac{3}{5}$ of $2\frac{1}{3}$ of 5 guineas?

$$1{\cdot}35 = 1\frac{35}{100} = 1\frac{7}{20}$$

$$\text{And } 1\frac{7}{20} \text{ of £3 } 7s.\ 6d. = \text{£4 } 11s.\ 1\tfrac{1}{2}d.$$

$$\text{Again } \frac{3}{5} \text{ of } 2\frac{1}{3} = \frac{3}{5} \times \frac{7}{3} = \frac{7}{5}$$

$$\frac{7}{5} \text{ of £5 } 5s. = \text{£7 } 7s.\ 0d.$$

$$\text{and £7 } 7s.\ 0d. - \text{£4 } 11s.\ 1\tfrac{1}{2}d. = \text{£2 } 15s.\ 10\tfrac{1}{2}d.$$

Hence difference = £2 15*s.* 10½*d.*, and this is the amount to be added to 1·35 of £3 7*s.* 6*d.*

Example 3.—In a speculation three men, A, B, and C, arn £375·45; A receives ·3054 of the whole, B ·4446, and C he remainder; what was C's share?

A and B together received ·3054 + ·4446 = ·75 or ¾ of he whole. Hence C receives ¼ or £93·8625.

Example 4.—When the price of a lb. of sugar is 2·75*d.*, what is the value of 950·047 lbs.?

$$\begin{array}{r@{}l}
950{\cdot}047d. & \\
2{\cdot}75 & \\
\hline
4750235 & \\
6650329 & \\
1900094 & \\
\hline
2612{\cdot}62925d. & \text{ or } £10\ 17s.\ 8{\cdot}62925d.
\end{array}$$

Example 5.—A man bought a house for 1000 guineas, and sold at a profit of $\cdot\dot{2}8571\dot{4}$ of the cost; required the selling price.

$\cdot\dot{2}8571\dot{4} = \frac{285714}{999999} = \frac{2}{7}$. And $\frac{2}{7}$ of 1000 guineas $= \frac{2}{7}$ of £1050 = £300. Hence the selling price = £1050 + £300 = £1350.

Example 6.—Simplify $\dfrac{\cdot75 \times \cdot125 + \cdot90625}{2\cdot375 - \cdot05}$

$$\frac{\cdot75 \times \cdot125 + \cdot90625}{2\cdot375 - \cdot05} = \frac{\cdot09375 + \cdot90625}{2\cdot375 - \cdot05}$$

$$= \frac{1}{2\cdot325} = \frac{1}{\frac{2325}{1000}} = \frac{1000}{2325} = \frac{40}{93}$$

Example 7.—Subtract $\cdot02\dot{5}\dot{4}$ of £134 1*s.* 3*d.* from $1\frac{3}{5}$ of £5 3*s.* $1\frac{1}{2}$*d.*, and express the result as a decimal of £3.

$$\cdot02\dot{5}\dot{4} = \frac{252}{9900} = \frac{7}{275}$$

$$\frac{13}{15} \text{ of } £5\ 3s.\ 1\tfrac{1}{2}d. \quad = \quad £4\ 9s.\ 4\tfrac{1}{2}d.$$

$$\text{And } \frac{7}{275} \text{ of } £134\ 1s.\ 3d. = \quad \underline{£3\ 8s.\ 3d.}$$

$$\text{Difference} = \quad \underline{\underline{£1\ 1s.\ 1\tfrac{1}{2}d.}}$$

$$\frac{£1\ 1s.\ 1\tfrac{1}{2}d.}{£3} = \frac{507}{1440} = \cdot 35208\dot{3}$$

ample 8.—Find the value of $\frac{5}{6}$ of ·075 of £1 + $\frac{4}{7}$

$\dot{3}\dot{9}$ of £1 0s. 7½d. + $\frac{1}{\cdot 2\dot{3}}$ of $10\frac{1}{2}$

$$\frac{5}{6} \text{ of } \cdot 075 \text{ of } £1 = \frac{\overset{1}{\cancel{5}}}{\underset{2}{\cancel{6}}} \text{ of } \frac{\cancel{3}}{\underset{8}{\cancel{40}}} \text{ of } £1 = \frac{1}{16} \text{ of}$$

$= 1s.\ 3d.$

$$\frac{4}{7} \text{ of } \cdot 5\dot{3}\dot{9} \text{ of } £1\ 0s.\ 7\tfrac{1}{2}d. = \frac{4}{7} \text{ of } \frac{534}{990} \text{ of}$$

$$.\ 7\tfrac{1}{2}d. = \frac{4}{7} \times \frac{534}{\cancel{990}} \times \frac{\cancel{990}}{1} \text{ farthings} = \frac{2136}{7}$$

$.\ 4\tfrac{2}{7}d.$

$$\frac{1}{\cdot 2\dot{3}} \text{ of } 10\frac{1}{2} = \frac{1}{\frac{21}{90}} \text{ of } 10\frac{1}{2} = \frac{\overset{45}{\cancel{90}}}{\cancel{21}} \times \frac{\cancel{21}}{2} =$$

$= 3s.\ 9d.$

Hence the total $= 11s.\ 4\tfrac{2}{7}d.$

MISCELLANEOUS EXERCISES.

Find the sum of $\frac{2}{7}$, $\frac{3}{4}$, and $\frac{8}{43}$, and from this sum take the
ict of $\frac{2}{7}$ and $\frac{3}{4}$.

What is the least number which can be divided by 7, 9, 25,
7, and 21 without a remainder ?

3. What must be taken from £$\frac{4}{7}$ to leave $\frac{3}{7}$*s.* ?

4. Divide ·125 by ·004, ·016, and 2·5, and add the quotients together.

5. What is the value of $\frac{3}{7}$ guinea + £$\frac{2}{9}$ + 5$\frac{1}{14}$ shillings + 8$\frac{1}{4}$ pence ?

6. Simplify $\dfrac{\frac{1}{2} + \frac{1}{4} + \frac{1}{6}}{\cdot\dot{6} + \frac{1}{3}}$

7. Find the difference between £3 14*s.* 7$\frac{3}{4}$*d.* and £1 2*s.* 3·27*d.*

8. What is the difference between £1 13*s.* 5$\frac{1}{4}$*d.* and £·794 ?

9. Having spent $\cdot\dot{4} + \cdot\dot{2}8571\dot{4}$ of my money, what part of the original sum remains ?

10. Reduce 2 pks. 1 gal. 2 qts. to the decimal of a bushel.

11. How many 50ths are there in $\frac{3}{7}$?

12. Add five-sixths of 27$\frac{1}{4}$ to fifteen-sevenths of 13$\frac{1}{4}$.

13. What fraction of 108 is 13$\frac{4}{5}$?

14. Reduce 2*s.* 7*d.* to the decimal of £1 3*s.* 3*d.*

15. How many fifths of a £ are there in 19*s.* 6*d.* ?

16. What fraction of £5 subtracted from $\frac{4}{7}$ of a shilling will leave $\frac{3}{28}$ of a penny ?

17. Divide 64·075 by $\cdot 2\dot{3}\dot{5}$.

18. Express as vulgar fractions 37·04, 21·007, $\cdot 037\dot{1}$, and $\cdot 00\dot{5}$.

19. What is the difference, in inches, between $\frac{3}{5}$ of a yard and $\frac{1}{4}$ of a foot ?

20. $\frac{1}{3}$ of $\frac{1}{4}$ of my money is the same as the eighth of 2*s.* 8*d.*; how much money have I ?

21. I spent $\frac{1}{3}$ of my money yesterday, and $\frac{3}{7}$ of the remainder to-day, and I have 15*s.* 9*d.* left; how much money had I at first ?

22. Take ·27*s.* from a crown, and find what decimal of a £ is left.

23. If ·35 of an acre be subtracted from 1 acre, how many roods and perches are left ?

24. What is the value of ·436 cwt. of soap at 4$\frac{3}{4}$*d.* per lb. ?

25. If $\frac{3}{5}$ of an oz. of tea be worth $\frac{1}{2}$ lb. of sugar, how much sugar will a chest of tea containing 45$\frac{3}{7}$ lbs. be worth ?

26. Calculate the value of 15·75 lbs. of tea at 2·45 shillings *per lb.*

27. If 14·7 lbs. of silver be worth £35·7, what will a spoon eighing 3·42 ozs. be worth?

28. The $\frac{1}{5}$ of a number is £·465, what decimal fraction of a uinea is $\frac{3}{8}$ of the number?

29. Find the value of ·36 of an oz. Troy if 1·5 lbs. cost 4*s.* $3\frac{1}{2}$*d.*

30. What decimal fraction of 5 guineas is £$\frac{1}{3}$ + 2*s.* 6*d.* + £·54 – 1*s.* $6\frac{1}{2}$*d.*?

31. If ·85 of my income be £315 16*s.* 8*d.*, what is my whole ncome?

32. If $\frac{3}{8}$ of a lb. of sugar be worth $\frac{2}{27}$ of a lb. of tea, find the uantity of tea equal in value to 1 cwt. of sugar.

33. If a parcel of tea weighing 5 lbs. 4 ozs. be worth half a uinea, what will a parcel 5·12 lbs. greater than this be worth?

34. What sum of money is ·432 of £1 10*s.* $4\frac{1}{2}$*d.*?

35. If the interest on £100 be £3·47, what is the interest on £$450\frac{1}{2}$?

36. $\frac{1}{3}$ of my money is greater than $\frac{1}{5}$ of $\frac{2}{3}$ by 3*s.* 9*d.*; what is my whole money?

37. If my age were $\frac{2}{3}$ of itself less I should be 21; how old am I?

38. If I spend in 3 years $2\frac{3}{5}$ of my annual income, what part do I save in 1 year?

39. Simplify $\dfrac{3\frac{1}{2} \text{ of } 2\frac{2}{7}}{14} + \dfrac{2\frac{1}{4} \text{ of } 3\frac{1}{5}}{1\frac{1}{3} + \frac{1}{6}}$.

40. Simplify $\dfrac{3\frac{1}{8} \text{ of } \frac{7}{15}}{1\frac{1}{4} \text{ of } 1} + 3\frac{1}{2} \text{ of } \frac{2}{3}$.

41. Simplify $\dfrac{\cdot 53 + \frac{1}{4} - \cdot 13}{\frac{1}{2} \text{ of } \frac{3}{5}}$.

42. Simplify $\dfrac{5\cdot 6\ (2\cdot 3 \text{ of } 1\cdot 6)}{4\frac{1}{2} + \cdot 72}$.

43. To how many persons can I give 3·7 shillings out of 15·355 sovereigns?

44. From £7 I paid for 15 yds. of cloth at 7·35 shillings per yd.; what decimal fraction of the original sum remained?

45. If I have spent $\frac{3}{8}$ of ·52 of ·4 of my money and have 15·674 shillings left, what sum had I at first?

46. If a yd. of cloth cost 3·7 shillings, find the value, in guineas, of 17 pieces each measuring on an average 56·4 yds.

47. If $\frac{3}{5}$ of ·56 of a cistern contain 79 gallons, what will the whole contain ?

48. Divide ·30476 into two parts such that one is ·0025 greater than the other.

49. What number multiplied by ·2 of ·375 will produce ·3 ?

50. What vulgar fraction divided by $\cdot\dot{4}2857\dot{1}$ of 6 will give $\frac{16}{21}$ as quotient ?

CHAPTER XXXIX.

COMPOUND PROPORTION.

We have seen that in simple proportion there is always *one pair* of terms and an odd term. When *two or more pairs* of terms are given and an odd one, the question is said to be in compound proportion.

When there are two or more pairs of terms we may obtain the answer by two or more distinct and separate applications of the "*rule of three.*" Hence compound proportion is sometimes called the "*double rule of three.*"

Example 1.—If 6 horses consume 18 bushels of oats in 4 days, how many bushels will 20 horses consume in 3 days?

Here we have two pairs of terms, horses and days.

We will solve the question first without reference to the number of days.

The first question then will read thus: If 6 horses consume 18 bushels in a certain time, how many bushels will 20 horses consume in the same time?

20 horses will consume more than 6 horses.

Hence— $6 : 20 : : 18 : x$

$$x = \frac{20 \times \overset{3}{\cancel{18}}}{\cancel{6}} = 60 \text{ bushels.}$$

Our second question then will be (omitting the number of horses): If a certain number of horses consume 60 bushels of oats in 4 days, how many bushels will the same number of horses consume in 3 days?

Less will be consumed in 3 days than in 4 days.

Hence— $4 : 3 :: 60 : x$

$$x = \frac{3 \times 60}{4} = 45 \text{ bushels.}$$

Compound proportion helps us to solve the two simple proportions in a shorter way.

1. The odd term must occupy the *third* place and x the fourth.
2. Take either pair of terms and state as in simple proportion without reference to the other pair or pairs of terms.
3. Treat the next pair in the same way.
4. Multiply the first terms together and the second terms together to form new first and second terms (this is called *compounding* the ratios). Applying these rules to the given example—

$$\left.\begin{matrix} 6 : 20 \\ 4 : 3 \end{matrix}\right\} :: 18 : x$$

$$24 : 60 :: 18 : x$$

$$x = \frac{\overset{15}{\cancel{60}} \times \overset{3}{\cancel{18}}}{\underset{4}{\cancel{24}}} = 45$$

We obtain the same result as before.

It is best for convenience in cancelling not to actually multiply the first terms together, and the second together, but merely to indicate multiplication by its sign, thus—

$$x = \frac{3 \times \overset{5}{\cancel{20}} \times \overset{3}{\cancel{18}}}{\cancel{4} \times \cancel{6}} = 45 \text{ bushels. } \textit{Ans.}$$

Example 2.—If 25 men can do a piece of work in 24 days working 8 hours a day, how many hours a day must 30 men work in order to complete the same piece of work in 16 days?

In this example—

(1) 30 men will require to work a less number of hours per day than 25 to do a certain piece of work in the same number of days;

(2) they have to work more hours per day to do a piece of work in 16 days than in 24 days.

Hence—

$$\left.\begin{matrix}30 : 25\\16 : 24\end{matrix}\right\} :: 8 : x$$

$$\text{And } x = \frac{\overset{\overset{2}{\cancel{4}}}{\cancel{24}} \times \overset{5}{\cancel{25}} \times \cancel{8}}{\underset{\cancel{2}}{\cancel{16}} \times \underset{\cancel{6}}{\cancel{30}}} = 10 \text{ hours. } \textit{Ans.}$$

Example 3.—If 14 horses and 125 sheep can be kept for 8 days for £50, what will it cost to keep 10 horses and 184 sheep for 2 days, supposing 3 horses eat as much as 15 sheep?

In this example we must first find the value of the horses' keep in terms of the sheep or *vice versâ*.

3 horses eat as much as 15 sheep;
1 horse will eat as much as 5 sheep;
14 horses will eat as much as 70 sheep;
and 10 horses will eat as much as 50 sheep.

∴ 14 horses and 125 sheep = 195 sheep,
and 10 horses and 184 sheep = 234 sheep.

We have now only two pairs of terms and the odd term.

(1) 234 sheep will cost more to keep than 195.

(2) It will cost less to keep them 2 days than 8 days.

$$\text{Hence}\quad \left.\begin{array}{r} 195 : 234 \\ 8 : 2 \end{array}\right\} :: £50 : x$$

$$x = \frac{\overset{\overset{3}{\cancel{6}}}{\cancel{234}} \times \overset{1}{\cancel{2}} \times \overset{\overset{5}{\cancel{10}}}{\cancel{50}}}{\underset{\cancel{39}}{\cancel{195}} \times \underset{\underset{\underset{1}{\cancel{2}}}{\cancel{4}}}{\cancel{8}}} = £15.\ \textit{Ans.}$$

Example 4.—If $\frac{5}{12}$ of 1 cwt. of sugar cost $\frac{3}{4}$ of $\frac{7}{8}$ of £1 when the duty is $\frac{5}{13}$ of £1 per cwt., what must be paid for $\frac{8}{9}$ of a ton when the duty is $\frac{7}{12}$ of £1 per cwt.?

Take (1) cost. $\frac{8}{9}$ of a ton will cost more.
(2) duty. duty greater ∴ greater cost.

$$\left.\begin{array}{ccc} \text{cwt.} & & \text{cwt.} \\ \frac{5}{12} & : & \frac{8 \times 20}{9} \\ \frac{5}{13} & : & \frac{7}{12} \end{array}\right\} :: \frac{3}{4} \text{ of } \frac{7}{8} : x$$

$$x = \frac{\frac{8 \times 20}{9} \times \frac{7}{12} \times \frac{3}{4} \times \frac{7}{8}}{\frac{5}{12} \times \frac{5}{13}}$$

$$= \frac{\overset{1}{\cancel{8}} \times \overset{\overset{1}{\cancel{4}}}{20}}{\underset{3}{\cancel{9}}} \times \frac{7}{\underset{1}{\cancel{12}}} \times \frac{\overset{1}{\cancel{3}}}{\underset{1}{\cancel{4}}} \times \frac{7}{\underset{1}{\cancel{8}}} \times \frac{\overset{1}{\cancel{12}}}{\underset{1}{\cancel{5}}} \times \frac{13}{5}$$

$$= \frac{637}{15} = £42\ 9s.\ 4d.\ \textit{Ans.}$$

Example 5.—If 3 men can earn as much in a day as 5 women, and 1 woman as much as 2 boys, and if 4 men, 7 women, and 6 boys working together can earn £25 in 10 days, what will be the earnings of 5 men, 3 women, and 4 boys working together for 12 days?

In this example it will be found most convenient to get the work of the women and boys in terms of the men.

5 women do the work of 3 men.

$\therefore$ 1 woman does ,, $\frac{3}{5}$ * man.

and 1 boy ,, ,, $\frac{3}{10}$ * man.

Hence, 4 men, 7 women, and 6 boys will do the same amount of work as—

$$4 + \frac{21}{5} + \frac{18}{10} = \frac{20}{5} + \frac{21}{5} + \frac{9}{5} = \frac{50}{5} = 10 \text{ men.}$$

And 5 men, 3 women, and 4 boys will do the same amount of work as—

$$5 + \frac{9}{5} + \frac{12}{10} = \frac{25}{5} + \frac{9}{5} + \frac{6}{5} = \frac{40}{5} = 8 \text{ men.}$$

We have now only two *pairs* of terms, viz. men and days.

(1) 8 men will earn less than 10 men in a given time.

(2) A given number of men will earn more in 12 days than in 10.

$$\left.\begin{matrix} 10 : 8 \\ 10 : 12 \end{matrix}\right\} :: £25 : x$$

$$x = \frac{8 \times \overset{3}{\cancel{12}} \times \overset{\cancel{5}}{\cancel{25}}}{\underset{\cancel{2}}{\cancel{10}} \times \underset{\cancel{2}}{\cancel{10}}} = £24. \quad \textit{Ans.}$$

Example 6.—A railway company charges $\cdot\dot{6}$ of £2·375 for the carriage of $\cdot 41\dot{6}$ of 3·75 cwts. of goods for 100 miles; what weight ought to be carried 35 miles for £1·05 at the same rate?

(1) A greater weight ought to be carried for a shorter distance.

---

* This must be interpreted as the work of 1 man in $\frac{3}{5}$ and $\frac{3}{10}$ of a day.

(2) Less weight for less money.

$$\left.\begin{array}{r} 35 : 100 \\ \cdot\dot{6} \times 2{\cdot}375 : 1{\cdot}0\dot{5} \end{array}\right\} :: \cdot 41\dot{6} \times 3{\cdot}75 : x$$

$$x = \frac{100 \times 1{\cdot}0\dot{5} \times \cdot 41\dot{6} \times 3{\cdot}75}{\cdot\dot{6} \times 2{\cdot}375 \times 35}$$

$$= \frac{100 \times 1\frac{5}{90} \times \frac{5}{12} \times 3\frac{3}{4}}{\frac{2}{3} \times 2\frac{3}{8} \times 35}$$

$$= \frac{\frac{100}{1} \times \frac{19}{18} \times \frac{5}{12} \times \frac{15}{4}}{\frac{2}{3} \times \frac{19}{8} \times 35}$$

$$= \frac{\overset{25}{\cancel{100}}}{1} \times \frac{\cancel{19}}{\underset{6}{\cancel{18}}} \times \frac{5}{\underset{\underset{1}{\cancel{4}}}{\cancel{12}}} \times \frac{\overset{\cancel{3}}{\cancel{15}}}{\cancel{4}} \times \frac{\cancel{3}}{\cancel{2}} \times \frac{\cancel{8}}{\cancel{19}} \times \frac{1}{\underset{7}{\cancel{35}}}$$

$$= \frac{25 \times 5}{6 \times 7} = \frac{125}{42} = 2\frac{41}{42} \text{ cwts. } \textit{Ans.}$$

EXERCISES IN COMPOUND PROPORTION.

1. If 15 horses can be kept 20 days for £10 15*s.* 6*d.*, how many can be kept 60 days for £32 6*s.* 6*d.* ?

2. If 7 men reap 20 acres of wheat in 5 days, how many acres can 18 men reap in 8 days ?

3. If a man travel 150 miles in 6 days, walking 9 hours a day, how far will he travel in 4 days, walking 12 hours a day ?

4. If 17 people spend £35 14*s.* in 9 months, how many will spend £321 6*s.* in 27 months at the same rate ?

5. If 17 pecks of wheat serve a family consisting of 5 persons 20 days, how many pecks will serve a family of 15 persons for 12 days ?

6. If 16 horses plough 19 acres in 10 days, how many will plough 76 acres in 5 days ?

7. If when wheat is 50*s*. a quarter the 7*d*. loaf weighs 30 ozs., what will the 1*s*. loaf weigh when wheat is 55*s*. per quarter?

8. If a railway company undertakes to carry 120 cwts. a distance of 365 miles for £2 10*s*. 6*d*., for how much will it carry 365 cwts. a distance of 50 miles?

9. If 5 men mow 80 acres of grass in 18 days, how long will 38 men be mowing 76 acres?

10. If the 4*d*. loaf weighs 4 lbs. 1 oz., when wheat is 56*s*. a quarter, what will the 9*d*. loaf weigh when wheat is 65*s*. a quarter?

11. If 17 horses eat 23 bushels of oats in 10 days, how many horses will eat 207 bushels in 30 days at the same rate?

12. If it costs £38 to keep 15 cows for 2 months, how much will it cost to keep 38 cows for 17 days (28 days = 1 month)?

13. If 56 men can erect 5 houses in 3 months working 6 days in a week, how many men would be required to put them up in 7 months, working only 4 days a week?

14. What sum will 27 men earn in 10 days, working 8 hours a day, when 12 men can earn £250 in 9 days, working 8 hours a day?

15. If 17 packages, each weighing 12 lbs., cost £20 15*s*., what will 84 packages, weighing 18 lbs. each, cost?

16. Goods weighing 17 cwts. 2 qrs. are carried a distance of 70 miles for £1 15*s*.; what weight will be carried 280 miles for 37*s*.?

17. If 100 men can build a wall 150 yards long and 7 feet high in 14 days, in how many days will 210 men build a wall 75 yards long and 9 feet high?

18. A family of 6 persons can be supplied for 3 weeks 4 days for £12 15*s*. when wheat is 60*s*. a quarter; for what time will a family of 8 persons be supplied for £20 8*s*. when wheat is 7*s*. per bushel?

19. If when bricks are 35*s*. a thousand, it costs £59 10*s*. to build a wall 60 yards long and 5 feet high, how much will it cost to build a wall 30 yards long and 7 feet high when bricks cost ½ as much again per thousand?

20. If 19 men reap 50 acres in 55 hours, how many men will reap 180 acres in 114 hours?

21. A tradesman with a capital of £5,060 gains £298 in 8 months; what capital will he require to gain £745 in 4 months?

22. If 15 men manufacture 33 articles in 4 days, working 11 hours per day, how many days will 8 men take to manufacture 39 such articles, working 9 hours per day?

23. If 12 men dig a trench 84 yards long in 7 days of 9 hours each, how many yards of this trench may be dug by 3 men in 8 days of 10 hours each?

24. If the interest on £100 for 1 year be £6, what will be the interest on £35 for 8 months?

25. A man, walking 12 hours a day, can reach a town 250 miles off in 12 days; in what number of days will he walk $\frac{1}{3}$ of the distance travelling 8 hours a day?

26. If 12 fires consume $3\frac{1}{4}$ tons of coals in $19\frac{1}{2}$ days, burning $10\frac{1}{2}$ hours a day, how many fires will consume 13 tons of coals in $4\frac{1}{3}$ days, burning 12 hours a day?

27. If 11 fires consume $2\frac{3}{4}$ tons of coals in $16\frac{1}{2}$ days, burning $10\frac{1}{2}$ hours a day, how many tons of coals will 4 fires consume in $15\frac{2}{5}$ days, burning $12\frac{3}{5}$ hours a day?

28. If 20 men can reap a field of $35\frac{3}{4}$ acres in $7\frac{3}{5}$ days, working $13\frac{1}{2}$ hours a day, how many men can reap a field of $\frac{2}{5}$ the number of acres in $14\frac{2}{5}$ days, working only $9\frac{1}{2}$ hours a day?

29. A field can be ploughed by 14 horses in $2\frac{6}{7}$ days, working 9 hours a day; in how many days can it be ploughed by 18 horses working $11\frac{2}{5}$ hours a day?

30. If the interest on £150 for $\frac{3}{4}$ of a year amount to £8 14*s*., what will the interest on £260 for $2\frac{1}{2}$ years amount to?

31. A man can walk $150\frac{1}{2}$ miles in $6\frac{1}{5}$ days, when he walks $9\frac{1}{4}$ hours each day; in how many days can he walk $251\frac{1}{4}$ miles, when he walks 6 hours 10 minutes each day?

32. 7 men, working 8 hours a day, earn £13 13*s*. 4*d*. in $5\frac{1}{4}$ days; how many hours a day must 9 men work to earn £61 10*s*. in 14 days?

33. If 7 men earn £$12\frac{2}{5}$ in $7\frac{3}{4}$ days, working $9\frac{1}{6}$ hours a day, how much will 15 men earn in $8\frac{2}{5}$ days, working $6\frac{1}{4}$ hours a day?

34. If $\frac{5}{8}$ cwt. sugar cost $\frac{1}{3}$ of $\frac{3}{5}$ of £2 when the duty is $1\frac{1}{2}$ shillings per cwt., what will $\frac{5}{8}$ ton cost when the duty is $1\frac{3}{5}$ shillings per cwt.?

35. If in 16·4 days 3 men can earn £7·35, working 13·5 hours a day, how much will 8 men earn in 12·5 days if they work 9·8 hours a day?

36. If 800 men can throw up an embankment in 3·15 days, working $13\frac{2}{7}$ hours a day, how many men would be required to do half the work in 4·5 days, working $6\frac{9}{14}$ hours a day?

37. If £3·5 be the wages of 5 men for 5·95 days, what will be the wages of 18 men for $3\frac{1}{2}$ weeks of $4\frac{1}{4}$ days in each week?

38. If I pay £6·34 for the loan of £75·8 for 9·2 months, what shall I have to pay for £36·5 for 7·45 months at the same rate of interest?

39. If 18 men can do a piece of work in 18·35 days, working 7·35 hours a day, how many men will be required to do a third of the work in ·25 of the time?

40. If 18 cows consume £52·8 worth of hay in 4·8 months, when hay is 8*s*. per cwt., what will be the cost of keeping 8 cows and 6 horses for 8·4 months when hay is 6·7 shillings per cwt., supposing 2 cows eat as much as 3 horses?

CHAPTER XL.

EXAMINATION PAPERS.

STANDARD VI.

I.

1. What is the greatest number which will divide 96, 124, and 60 without a remainder?

2. How much is $\frac{1}{3}$ of $\frac{2}{5}$ greater than $1-\frac{8}{9}$?

3. Reduce 1*s*. $11\frac{1}{2}$*d*. to the fraction of 8*s*. $9\frac{3}{4}$*d*.

4. When I have paid income-tax on my income at 2*d*. in the £ I have £247 18*s*. 4*d*. left; what is my income?

II.

1. A greengrocer has a basket of onions which he can tie in bundles of 6, 7, or 8 and have none over; find the smallest number of onions which the basket may contain.

2. Express as a simple fraction $\frac{3\frac{2}{3}\text{ of }6\frac{3}{7}}{4\frac{4}{5}}$.

3. Find the value of $1\frac{2}{5}$ of 1*s*. 3*d*. + $3\frac{2}{7}$ of 3 guineas + $9\frac{5}{11}$ of $5\frac{1}{2}$*d*.

4. In what time will £340 10*s*. amount to £442 13*s*. at 5 per cent. per annum?

III.

1. Find the least sum of money payable in half-sovereigns, half-crowns, or florins.

2. Simplify $\frac{2\frac{1}{4} - 1\frac{2}{3}}{3\frac{1}{3} + 1\frac{3}{4}} \div \frac{5\frac{2}{5} + 2\frac{1}{10}}{6\frac{2}{3} - 4\frac{5}{6}}$.

3. What fraction of $1\frac{1}{3}$ cwt. is $1\frac{1}{2}$ lb.?

4. At what rate per cent. per annum will £340 10*s*. amount to £442 13*s*. in 6 years?

IV.

1. What number is that which leaves 3 as a remainder when divided by either 7, 6, or 9?

2. From 5 subtract its fourth and its sixth, add the remainders, and divide by $4\frac{7}{12}$.

3. How much money must be added to £2 6*s*. $5\frac{1}{2}$*d*. to make £4 2*s*. $1\frac{2}{3}$*d*.?

4. If a garrison of 1,400 men have provisions for 12 weeks, how many men must be sent away so that the provisions may last 21 weeks?

V.

1. Find the value of $\frac{4}{5} - \frac{7}{10} + \frac{2}{9} - \frac{1}{5}$.

2. Which is the greater, and by how much, 5*s*. ÷ $\frac{2}{3}$ of $\frac{1}{4}$, or £1 × $1\frac{2}{3}$ of $1\frac{1}{4}$?

3. Express $\frac{5}{6}$ of half-a-crown as the fraction of 7*s*. 1*d*.

4. If $1\frac{5}{7}$ of a yard of cloth cost $3\frac{3}{7}$*s*., what is the value of $15\frac{3}{8}$ yards?

VI.

1. Arrange the following fractions in the order of their magnitude: $\frac{10}{3}$, $\frac{17}{5}$, $\frac{24}{7}$.

2. Find the value of $\frac{2}{3}$ of £5 + $\frac{5}{7}$ of 14*s*. − $1\frac{1}{3}$ of 10*s*. − $3\frac{1}{3}$ of 1*s*.

3. Divide £$2\frac{1}{5}$ by $\dfrac{\frac{1}{2}-\frac{1}{3}}{\frac{1}{3}-\frac{1}{4}}$.

4. If 15 men can do a piece of work in 9 days, working 8 hours per day, how many hours per day must 18 men work in order to complete the same piece of work in 6 days ?

VII.

1. If $\frac{1}{2}$ of $\frac{2}{3}$ of a ship's cargo be worth £5,000, what is the value of the whole ?

2. Add together £4 7*s.* $2\frac{1}{3}$*d.*, £5 19*s.* $6\frac{2}{3}$*d.*, 16*s.* $7\frac{4}{9}$*d.*, 3*s.* $11\frac{5}{7}$*d.*

3. A man walks $21\frac{1}{4}$ miles in $6\frac{1}{4}$ hours ; find the rate per hour.

4. A can complete a piece of work in 9 days, and B can do the same in 8 days ; in what time, working together, will they complete the work ?

VIII.

1. Add the sum and difference of $\frac{5}{6}$ and $\frac{6}{7}$.

2. What fraction of 13 acres 3 roods is 2 acres 3 roods ?

3. Compare the values of $\frac{5}{8}$ of 3 guineas and 30*s.* $\div$ $\frac{1}{2}$.

4. If the interest on £1 for 1 year be 1*s.*, what is the interest on £525 for 3 years ?

IX.

1. What part of 21 hours is 2 hours 6 minutes ?

2. Simplify $\dfrac{5\frac{5}{21}+7\frac{19}{21}}{9\frac{2}{7}-8\frac{5}{42}} \div 2\frac{18}{37}$.

3. Divide $26\frac{2}{7}$ yards into equal lengths of $3\frac{2}{7}$ yards.

4. Find the value of $19\frac{6}{11}$ chests of tea, each containing $17\frac{3}{5}$ lbs., at 3 lbs. 2 ozs. for 9*s.* $4\frac{1}{2}$*d.*

X.

1. Subtract—using decimal fractions—the thousandth part of £15 from the hundredth part of £150.

2. Find the value of $\frac{1}{3}$ of 12 + $\frac{1}{3}$ of $12\frac{1}{2}$ + $\frac{1}{3}$ of $13\frac{1}{2}$.

3. Add together 2·6 of a day and ·85 of an hour, and give the answer in minutes.

4. If either 50 oxen *or* 40 horses will eat up the grass of a field in 25 days, in what time will 12 oxen *and* 8 horses eat up the *same ?*

XI.

1. Divide 6 furlongs 108 yards 1 foot by ·305.

2. Add $33\frac{1}{3}$, $37\frac{1}{2}$, $55\frac{3}{4}$, $66\frac{2}{3}$.

3. Reduce 2*s*. $8\frac{4}{10}$*d*. to the decimal of £1.

4. If the wages of 20 men for 6 days amount to £27, how many men should work 19 days for £47 0*s*. 6*d*.?

XII.

1. Divide $\frac{3}{5}$ of $6\frac{1}{4}$ by $\frac{2}{3}$ of $7\frac{1}{2}$.

2. Reduce $10\frac{3}{4}$*d*. to the decimal of 1*s*. 6*d*.

3. Which is the greater, and by how much, $1\frac{2}{3} \times 2\frac{5}{8}$ or $·018 \times 21·6$?

4. A and B together own 396 acres of land, and $\frac{3}{5}$ of A's farm equals $\frac{3}{4}$ of B's; how many acres are owned by each?

XIII.

1. From a cask containing $45\frac{1}{2}$ gallons of oil, an oilman sold $16\frac{3}{4}$ gallons to one customer and $21\frac{3}{8}$ gallons to another; how many gallons remained unsold?

2. Find the difference between ·31595 of a guinea and 5·12295 of a shilling.

3. Find the value of $\dfrac{2\frac{1}{2} \times 2\frac{1}{2} - 1}{3\frac{1}{5} - 2\frac{2}{3} + \frac{13}{15}}$.

4. A house is rated at $\frac{9}{10}$ of its rent, and the rate, at 5*d*. in the £, is 18*s*. 9*d*.; what is the rent?

XIV.

1. Divide 47·014 by ·001, by ·01, and by ·1.

2. By what fraction must $\frac{3}{5}$ be multiplied so that the product may be $26\frac{2}{3}$?

3. What part of a pound of gold is ·25 of an ounce?

4. If 5 men build a wall 5 feet high and 100 yards long in 8 days, how long will it take 7 men to build a wall 200 yards long and $3\frac{1}{2}$ feet high?

XV.

1. Multiply 3 acres 2 roods 19 poles by ·325.

2. Simplify $\dfrac{6\frac{1}{2} \times 3\frac{1}{3}}{4\frac{1}{3} \div 5\frac{1}{3}}$.

3. From 25·6 ÷ ·064 subtract 32·4 × ·015.

4. If the wages of 8 men for 3 days amount to £4 2*s.*, what will be the wages of 29 men for 5·25 days ?

XVI.

1. Reduce 1·408 feet to the decimal of a mile.

2. Multiply $\frac{5}{8} + \frac{3}{8}$ of $\frac{3}{4}$ by $\frac{5}{8} - \frac{3}{8}$ of $\frac{3}{4}$.

3. Divide $\cdot78\dot{9}6\dot{2}$ by $\cdot\dot{3}$.

4. If 9 men do a piece of work in 8 days, how many will do $\frac{1}{3}$ of the work in $2\frac{2}{3}$ days ?

XVII.

1. Divide 67·814 by ·000213.

2. A man receives a salary of £30 14*s.* 7*d.* per quarter, and he has received £14 15*s.* in advance ; what fraction of his quarter's salary has he still to receive ?

3. A boy has 4 dozen marbles ; in one game he loses a third of them, in another a fourth of the remainder ; how many has he then left ?

4. A hare is $157\frac{1}{2}$ yards before a greyhound, but the greyhound runs 7 yards whilst the hare runs 5 ; how far must the greyhound run to catch the hare ?

XVIII.

1. Find the G.C.M. of 6·21 and 1·173.

2. Reduce 5*s.* 4*d.* to the decimal of 6*s.* 8*d.*

3. What decimal multiplied by 175 will give the sum of $\frac{1}{4} + 1\frac{1}{3} + 3\frac{1}{3}$?

4. A and B perform together $\frac{9}{10}$ of a piece of work in 2 days; B then leaves off, and A completes the work in $\frac{1}{2}$ a day ; in what time can each do it alone ?

PART VI.*

CHAPTER XLI.

are certain questions relating to interest, insurance, age, averages, exchange, profit and loss, shares, stocks, ll of which may be solved by the aid of the rules y given, but more especially by simple and compound tion.

have already given incidentally some solutions of les in interest and commission. In this part we shall nore fully with per-centage computations and com- l transactions generally.

MMISSION, BROKERAGE, INSURANCE, PROFIT AND LOSS.

SSION is the charge made of so much per cent. for or selling goods. Manufacturers, for instance, em- en to travel through the country to sell their wares. men usually receive so much *commission* for every worth of goods they sell.

KERAGE is commission paid to a *broker* for buying and *shares* or merchandise.

URANCE is a contract to pay so much per annum—the *m*—in return for which the insurers (generally a com-

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is part will be found suitable for scholars of Public Elementary who have passed through the six Standards.

pany) agree to bear all losses caused by fire, death, accident, or shipwreck, &c. Insurers of ships and their cargoes are called *underwriters*.

PER-CENT., so much for every hundred (*centum*).

Examples for the black-board:—

Example 1.—I insure my life for £500 at £2 3*s*. 4*d*. per cent. premium; what is my annual payment?

£100 gives a premium of £2 3*s*. 4*d*.
∴ £500 gives a premium of £2 3*s*. 4*d*. × 5
and £2 3*s*. 4*d*. × 5 = £10 16*s*. 8*d*. *Ans.*

Example 2.—Find the insurance on a ship's cargo, worth £1,040 10*s*., at 3¼ per cent.

$$\text{£}100 \text{ gives } \text{£}3\tfrac{1}{4}$$

$$\text{£}1 \text{ gives } \frac{3\frac{1}{4}}{100}$$

$$\text{£}1040\ 10s. \text{ gives } \frac{3\frac{1}{4} \times 1040\frac{1}{2}}{100}$$

$$= \frac{\frac{13}{4} \times \frac{2081}{2}}{100} = \frac{13 \times 2081}{2 \times 4 \times 100} = \text{£}33\ 16s.\ 3\tfrac{9}{10}d. \quad \textit{Ans.}$$

Example 3.—My house and furniture are insured for £660, for this I pay annually 16*s*. 6*d*.; what is my premium per cent.?

£660 gives a premium of 16*s*. 6*d*.

$$\therefore \text{£}1 \text{ gives a premium of } \frac{16s.\ 6d.}{660}$$

$$\text{and } \text{£}100 \text{ gives a premium of } \frac{16s.\ 6d. \times 100}{660}$$

$$= \frac{16\tfrac{1}{2}s. \times \overset{5}{\cancel{100}}}{\underset{33}{\cancel{660}}} = \frac{\cancel{33} \times 5}{2 \times \cancel{33}} = \frac{5}{2} = 2s.\ 6d. \quad \textit{Ans.}$$

Example 4.—Find the gain on goods bought for £640 at a profit of $15\frac{7}{8}$ per cent.

$$£100 \text{ gives a profit of } 15\tfrac{7}{8}$$

$$£1 \text{ gives a profit of } \frac{15\frac{7}{8}}{100}$$

$$£640 \text{ gives a profit of } \frac{15\frac{7}{8} \times 640}{100}$$

$$= \frac{127 \times \overset{4}{\overset{\cancel{8}}{\cancel{640}}}}{\underset{1}{\cancel{8}} \times \underset{5}{\cancel{100}}} = \frac{508}{5} = £101\ 12s. \quad \textit{Ans.}$$

Example 5.—Goods bought for £640 are sold at a loss of 7 per cent.; for how much were they sold?

$$£100 \text{ worth of goods was sold for } £93$$

$$\therefore\ £1 \text{ worth of goods was sold for } \frac{93}{100}$$

$$£640 \text{ worth of goods was sold for } \frac{93 \times \overset{32}{\cancel{640}}}{\underset{5}{\cancel{100}}}$$

$$= \frac{93 \times 32}{5} = £595\ 4s. \quad \textit{Ans.}$$

Example 6.—The population of a certain town was 350,000 at the commencement of one year; the increase being $2\frac{1}{2}$ per cent., what was the population at the end of the year?

$$100 \text{ gives an increase of } 2\tfrac{1}{2}$$

$$1 \text{ gives an increase of } \frac{2\frac{1}{2}}{100}$$

$$350{,}000 \text{ gives an increase of } \frac{2\frac{1}{2} \times 350{,}000}{100}$$

$$= \frac{5}{2} \times 3{,}500 = 8{,}750$$

The total increase is 8,750
and ∴ the population = 358,750. *Ans.*

Example 7.—By selling a book for 11*s.* 4½*d.*, a bookseller made a gain of 5 per cent.; find the prime cost of the book.

At 5 per cent. £105 gives £100 as the cost price,

$$\therefore \text{£}1 \quad ,, \quad \text{£}\frac{100}{105} = \frac{20}{21} \quad ,, \quad ,,$$

$$\text{and } 1s. \quad ,, \quad \text{£}\frac{20}{21 \times 20} \quad ,, \quad ,,$$

$$\tfrac{1}{2}d. \quad ,, \quad \text{£}\frac{20}{21 \times 20 \times 24} \quad ,, \quad ,,$$

(11*s.* 4½*d.* = 273 halfpence)

$$\therefore\ 273 \text{ halfpence gives } \text{£}\frac{20 \times 273}{21 \times 20 \times 24} \text{ as the cost price}$$

$$= \frac{\cancel{20} \times \overset{\overset{13}{\cancel{39}}}{\cancel{273}}}{\underset{3}{\cancel{21}} \times \cancel{20} \times 24} = \text{£}\frac{13}{24} = 10s.\ 10d.$$

Prime cost = 10*s.* 10*d.* *Ans.*

Or by proportion—

$$105 : 11s.\ 4\tfrac{1}{2}d. :: 100 : x$$

$$x = \frac{11s.\ 4\frac{1}{2}d. \times 100}{105} = 10s.\ 10d.\ \textit{Ans.}$$

Example 8.—A horse is sold for 80 guineas at a loss of 10 per cent.; required the selling price to gain 10 per cent.

(*a*) With a loss of 10 per cent.

£90 selling price gives £100 as cost price.

Or £1 ,, ,, $£\frac{100}{90}$,,

£84 ,, ,, $£\frac{10\cancel{0} \times 84}{9\cancel{0}}$,,

$$= £\frac{840}{9} = \text{cost price.}$$

(*b*) To gain 10 per cent.

£100 cost price gives £110 as selling price.

And £1 ,, ,, $£\frac{110}{100}$,,

$\therefore£\frac{840}{9}$,, ,, $£\frac{11\cancel{0} \times 84\cancel{0}}{1\cancel{00} \times 9}$,,

$$= £\frac{924}{9} = £102\ 13s.\ 4d. \quad \textit{Ans.}$$

as the selling price to gain 10 per cent.

Or by proportion—

$$\begin{array}{lcccccccc} & \text{Selling price.} & & \text{Selling price.} & & \text{Cost price.} & & \text{Cost price.} \\ (a) & 90 & : & 84 & :: & 100 & : & x \end{array}$$

$$x = \frac{84 \times 100}{90} = \frac{840}{9}$$

$$\begin{array}{lcccccccc} & \text{Cost price.} & & \text{Cost price.} & & \text{Selling price.} & & \text{Selling price.} \\ (b) & 100 & : & \frac{840}{9} & :: & 110 & : & x \end{array}$$

$$x = \frac{840 \times 110}{100 \times 9} = \frac{924}{9} = £102\ 13s.\ 4d.$$

as the selling price to gain 10 per cent.

EXERCISES.

1. What shall I have to pay for insuring a house worth £850, at 2*s*. 6*d*. per cent. ?

2. What will a person, 25 years of age, pay to insure his life for £350, when the premium charged by the Company for this age is £2 9*s*. 3*d*. per cent. ?

3. What will be the cost of insuring a house worth £3,050, at 1*s*. 6*d*. per cent. ?

4. For what sum can a ship's cargo worth £11,800 be insured at £2 10*s*. 6*d*. per cent. ?

5. What will it cost me to insure my life for £3,000 when the premium at my age is £2 3*s*. 4*d*. per cent. ?

6. The population of a town in 1875 was 16,000 ; in 1877 it is 24,000 ; how much per cent. has it increased ?

7. How much per cent. does a tradesman gain by selling tea at 3*s*. 6*d*. per lb. which cost him 2*s*. 8*d*. per lb. ?

8. For insuring property to the amount of £3,050 I paid £7 12*s*. 6*d*. ; what is the rate per cent. of the premium ?

9. A farmer insured a hay-rick for £560 ; what premium has he to pay at 2*s*. 6*d*. per cent ?

10. By selling cloth at 5*s*. per yard a tradesman gains 10 per cent. ; what was the cost price per yard ?

11. A bookseller allows 3*d*. in the shilling on books bought; what is that per cent. ?

12. What commission ought an agent to receive for the sale of goods to the amount of £3,890 at 1¼ per cent. ?

13. If an agent receives a commission of £27 for selling goods to the amount of £600, what is that per cent. ?

14. A grocer purchases 84 lbs. of tea for £12 12*s*. ; at what price per lb. must he sell it to gain 12½ per cent. ?

15. What is the premium on a policy of insurance amounting to £3,500, at the rate of £2 15*s*. 6*d*. per cent. ?

16. What will a broker charge for purchasing goods to the amount of £390 at 2*s*. 6*d*. per cent. ?

17. A watchmaker sells a watch for £13 10*s*., thereby making a *profit* of 35 per cent. ; what did the watch cost him ?

18. A town had a population of 120,000; in ten years it increased to 150,000; what was the increase per cent.?

19. If an army, after losing 35 per cent. of its men, contained 32,500, what was its original number?

20. Last week there were 35 boys in the first class of a school; this week the number is increased to 42; calculate the increase per cent.

21. A town contains a population of 35,500, which shows an increase of 25 per cent. on the population 5 years ago; what was the population 5 years ago?

22. A boy gave 1*s.* 7*d.* for a knife, which was 5 per cent. less than the retail price; what was the retail price?

23. A school has on the books the names of 540 children; it has increased 20 per cent. in two years; how many children were in the school two years ago?

24. A woman bought oranges at 9*d.* per doz. and sold them at the rate of 2 for 2½*d.*; what was her gain per cent.?

25. What will the commission on £63,943 6*s.* 8*d.* amount to, at ⅛ per cent.?

26. A bought goods for £350 and sold them to C at a gain of 5 per cent.; C sold them to D and lost 4 per cent; what did D pay for them?

27. By selling goods for 10*s.* I lost 15 per cent.; for what must I sell them to gain 2 per cent.?

28. A tradesman loses 2*s.* 6*d.* by selling an article for £1 15*s.* 6*d.*; what is the loss per cent., and at what price must he sell it to gain 10 per cent. on the cost price?

29. By selling goods for £773 10*s.* 3½*d.* a gain is made of 3¼ per cent. on the prime cost; find the prime cost.

30. If after deducting 12½ per cent. from my money I have £525 10*s.* left, what had I at first?

31. A grocer sold 84 lbs. of tea for 14 guineas, thereby making a profit of 20 per cent.; what was the cost price of the tea per lb.?

32. A watch cost £4, and was sold for £4 10*s.*; find the gain per cent.

33. Out of an army of 35,400 men 15½ per cent. are invalided; required the effective force.

34. By selling goods for £817 19*s.* a merchant lost 9 per cent.;

find at what price he should have sold them to have gained 10½ per cent.

35. A grocer by selling sugar at 4½*d.* per lb. loses 10 per cent.; find the cost price per cwt.

36. By selling a horse for £116 17*s.* I lose 5 per cent.; what should I gain or lose per cent. by selling at £132 4*s.* 6*d.*?

37. What do I gain or lose per cent. by selling butter at 8½*d.* per lb. which cost £5 5*s.* per cwt.?

38. What must be paid yearly on an insurance for £9,626 11*s.* 3*d.* at 2·12 per cent.?

39. A school has 368 children "on the rolls," 92 are absent; find the per-centage number present.

40. A woman buys 2 pair of ducks for 7*s.* 6*d.* and sells them for 9*s.* 9½*d.*; find the gain per cent.

41. If eggs are bought at 5 for 3*d.*, at what price per hundred must they be sold to gain 20 per cent.?

42. I bought coffee for £7 9*s.* 4*d.* per cwt.; for what should I sell it per lb. to gain 25 per cent.?

43. I exchange ½ cwt. of sugar at 4½*d.* per lb. for 7 lbs. of tea at 3*s.* 6*d.* per lb.; find the gain or loss per cent.

44. By selling apples at 3 for a penny I gain 5 per cent.; what do I gain or lose per cent. by selling them at £1 per 1,000?

CHAPTER XLII.

SIMPLE INTEREST, DISCOUNT, ETC.

Example 1.—Find the interest on £350 at 4 per cent. for 3 years.

This is an example in compound proportion, and may be stated thus:—

$$\left.\begin{array}{r} 100 : 350 \\ 1 : \quad 3 \end{array}\right\} :: 4 : x$$

$$x = \frac{350 \times 4 \times 3}{100} = £42$$

Now in this equation let P stand for the principal, r the rate per cent., n the number of years, then—

$$\text{Interest} = \frac{P \times n \times r}{100} = \frac{P\,n\,r}{100}$$

But the amount is the principal and interest added together.

Hence, if M stands for the amount—

$$\text{Amount} = \text{Principal} + \text{Interest}.$$

$$M = P + \frac{P\,n\,r}{100}$$

The teacher must here illustrate by means of examples the truth of the two axioms:

(1) That if equals be added to equals the wholes are equal.

(2) That if equals be subtracted from equals the remainders are equal.

If $6 = 6$, then $6 + 4 = 6 + 4$, and $6 - 3 = 6 - 3$.

And because multiplication and division are short methods of addition and subtraction, it follows that *we may add to or subtract from, or multiply or divide, both sides of an equation, without changing its value.*

By means of the application of these axioms to the equation—

$$M = P + \frac{P\,n\,r}{100}$$

we may solve every question in simple interest. We can find the time, or rate, or principal, or amount.

Example 2.—(1) Find the amount of £500 at 5 per cent. in 5 years.

(2) Find what principal will amount to £625 in 5 years at 5 per cent.

(3) Find at what rate per cent. £500 will amount to £625 in 5 years.

(4) Find in how many years £500 will amount to £625 at 5 per cent.

(1) $$M = P + \frac{P\,n\,r}{100}$$

$$M = 500 + \frac{500 \times 5 \times 5}{100}$$

$$= 500 + 125 = £625. \textit{ Ans.}$$

(2) $$M = P + \frac{P\,n\,r}{100}$$

$$£625 = P + \frac{P \times 5 \times 5}{100}$$

$$\text{or } £625 = \frac{100\,P + 25\,P}{100} = \frac{125}{100}\,P.$$

Multiplying both sides of this equation by $\frac{100}{125}$

$$\frac{125}{100} \times \frac{100}{125}\,P = \frac{100}{125} \times 625$$

$$P = \frac{\overset{\overset{4}{\cancel{20}}}{\cancel{100}} \times \overset{125}{\cancel{625}}}{\underset{\underset{\cancel{5}}{\cancel{25}}}{\cancel{125}}} = £500. \textit{ Ans.}$$

(3) $$M = P + \frac{P\,n\,r}{100}$$

$$£625 = 500 + \frac{500 \times 5 \times r}{100}$$

Subtract £500 from both sides.

$$\begin{array}{c} 625 - 500 \\ \text{(or 125)} \end{array} = \frac{500 \times 5 \times r}{100}$$

Multiply each side by 100.

$$12500 = 2500 \times r$$

Divide both sides by 2500.

$$5 = r$$

or the rate per cent. is 5. *Ans.*

(4) $$M = P + \frac{P\,n\,r}{100}$$

$$625 = 500 + \frac{500 \times 5 \times n}{100}$$

As before, $12500 = 2500 \times n$

$$n = \frac{12500}{2500} = 5$$

Hence number of years = 5. *Ans.*

We may also, by means of this same formula, solve all examples in true DISCOUNT and PRESENT VALUE.

I have agreed to pay a man 100 guineas at the end of a year from the present time for a certain favour he had done me, but he wishes to receive payment *now.* If I pay him the £105 now, he will be able to put this out at interest, and if he gets 5 per cent., at the end of the year the £105 will be worth £105 + £5 5*s.* = £110 5*s.* But if I pay him now £100, and he puts this out at interest at 5 per cent., at the end of the year it will be worth £105. It is clear, therefore, that it will be right for me to pay him *now* not £105, but £100.

The £100 is the PRESENT VALUE of £105 due twelve months hence, and the £5 I deduct is called the DISCOUNT.

It is not usual, however, in commercial transactions to reckon discount in this way. A banker would have said £105 at 5 per cent. for one year gives £5 5*s.* interest, and he would have called this the discount, and would have deducted it from the £105. We see therefore that the banker gets an advantage of 5*s.* The discount obtained by the former method is the *true* discount. The discount obtained by the second method is the *banker's* discount.

Now let P in our formula stand for present value, and we can find the present value of £105 due 1 year hence at 5 per cent.

$$M = P + \frac{P\,n\,r}{100}$$

$$£105 = P + \frac{P \times 1 \times 5}{100}$$

$$£105 = \frac{100\,P}{100} + \frac{5\,P}{100}$$

$$£105 = \frac{105\,P}{100}$$

$$£10{,}500 = 105\,P$$

$$P = \frac{10500}{105} \quad £100. \textit{ Ans.}$$

Again: Find the present value of £143 5*s.* 4*d.* due 3 years hence at 4 per cent. per annum.

$$M = P + \frac{P\,n\,r}{100}$$

$$£143\ 5s.\ 4d. = \frac{100\,P + 12\,P}{100}$$

$$£143\ 5s.\ 4d. = \frac{112}{100}\,P = \frac{28}{25}\,P$$

$$P = \frac{£143\ 5s.\ 4d.}{28} \times 25$$

$$= £127\ 18s.\ 4d. \textit{ Ans.}$$

This result may be put to the proof by finding the amount of £127 18*s.* 4*d.* for 3 years at 4 per cent. per annum.

We may put the question in another form: What sum of money put to interest at 4 per cent. will amount to £143 5*s.* 4*d.* in 3 years?

And generally: There are four quantities in the formula, and if any three be given we can find the fourth.

EXERCISES.

1. Find the interest on £370 for 2 years at 3 per cent.

2. What principal will yield an interest of £480 in 2 years at 4 per cent. ?

3. What principal will become £480 in 2 years at 4 per cent. ?

4. What sum of money must you lend to receive as interest 14*s*. 6*d*. in 91¼ days at 3½ per cent. ?

5. What sum of money must be lent to obtain 13*s*. 6*d*. as interest in 5 months at 4½ per cent. ?

6. What is the difference in the interest of £1,000 at 3½ per cent., and at 5 per cent., for 150 days ?

7. I lent £380 for a year at 3½ per cent., and £530 for 1¼ years at 4 per cent ; what was the interest on the whole ?

8. I lent £182 10*s*. for 8 days, and received £1 as interest ; what is that per cent. ?

9. At what rate per cent. per annum must a guinea be lent for 15 days, to receive 2*s*. 6*d*. interest ?

10. At what rate per cent. per annum should I be paying if I give 3*s*. 6*d*. for the loan of £5 5*s*. for three weeks ?

11. I lent £80 for 3¼ years and received as interest £15 12*s*. at what rate per cent. per annum was I paid for the loan ?

12. In what time will a £5 note become £6, if put out to interest at 3½ per cent. ?

13. In what time will £186 amount to £198 11*s*. 1⅕*d*. if put out to interest at 2¼ per cent. ?

14. Find the time you would require to lend £450 to become £470 5*s*. at an interest of 2¼ per cent.

15. Find the time £360 15*s*. must be lent, to amount to £461 15*s*. 2⅖*d*. at 2⅓ per cent.

16. What principal will amount to £48 16*s*. 9⅗*d*. in 6 months at 3½ per cent. ?

17. What sum of money put out to interest at 5½ per cent. will become £815 10*s*. in 3 years ?

18. What principal must I have to yield an amount of £1,625 in 50 years at 4½ per cent. ?

19. What sum of money will amount to £80 10*s*. 4⅘*d*. if lent from March 18th to May 30th, at 3¼ per cent. ?

20. What will £369 amount to in 2 years and 25 days at 3 per cent. ?

21. What is the banker's discount on a bill of £500 drawn June 1st at 3 months at 5 per cent. ?

22. What is the true discount on a bill of £500 drawn June 1st at 3 months at 5 per cent. ?

23. Find the present value of a bill of £150 drawn Jan. 1st, 1878, at 4 months at 2½ per cent. ?

24. What sum of money will pay a bill of £150 now, which is not due for 5 months, money being worth 2¼ per cent. ?

25. What is the present value of a bill of £360, drawn Jan. 2nd, 1877, at 4 months at 2⅕ per cent. ?

26. What is the true discount on £154 due 3 months hence at 2⅛ per cent. ?

27. Find the true discount on a bill of £502 3*s*. 6*d*. drawn Aug. 4th at 5 months at 2$\frac{1}{10}$ per cent.

28. Find the simple interest on £16,904 7*s*. 6*d*. for 8 months at 3½ per cent.

29. Find the present value of £149 0*s*. 5*d*. due 11 months hence at 4 per cent.

30. Find the simple interest on £183 6*s*. 8*d*. for 1 year 146 days at 4¼ per cent.

31. What principal will produce £591 12*s*. 4*d*. as interest in 4 years at 2½ per cent. ?

32. At what rate per cent. will £91 13*s*. 4*d*. amount to £116 8*s*. 4*d*. in 4½ years ?

33. What sum of money will amount to £428 5*s*. 6*d*. in 3½ years at 4½ per cent. ?

34. Find the simple interest on 10 guineas for 117 days at 3½ per cent.

35. The sum of £533 6*s*. 8*d*. is received for £560 due 2 years hence ; at what rate per cent. is the discount calculated ?

36. Find the present value of £455 12*s*. 6*d*. due 4 years hence at 5⅜ per cent.

37. At what rate will £3,745 amount to £3,932 5*s*. in 2 years ?

38. Find the present value of £1,842 15*s*. due 3 months hence at 5 per cent.

39. In how many years will £300 amount to £373 10*s*. at 3½ per cent. ?

40. At what rate will £1,650 amount to £1,784 1*s*. 3*d*. in 2½ years ?

41. In what time will £2,175 amount to £2,338 2*s*. 6*d*. at 2½ cent. ?

42. At what rate will £1,315 amount to £1,545 2*s*. 6*d*. in 3½ years ?

43. A sum of £1,928 17*s*. is due 1¼ years hence; what amount should be accepted as present payment, reckoning 3½ per cent. ?

44. In what time will £2,220 double itself at 3¾ per cent. ?

CHAPTER XLIII.

COMPOUND INTEREST.

When the interest is added to the principal at the end of a certain time to form a new principal, and then at the end of another equal interval of time the interest on the new principal is added to the new principal to form again a new principal, and so on, we calculate by Compound Interest.

Thus let £100 be put out at compound interest for 3 years at 5 per cent. per annum.

At the end of the first year we add the £5 interest to the £100 principal, and this gives a new principal of £105 with which to begin the second year.

At the end of the second year the interest on £105 = £5 5*s*., and we again add this to the principal £105 to form a new principal, viz. £110 5*s*.

So we start a third year with £110 5*s*. at 5 per cent. Interest at the end of the third year on £110 5*s*. = £5 10*s*. 3*d*. Therefore the amount at the end of 3 years is £115 15*s*. 3*d*.

We may calculate the interest at any intervals of time.

To find the compound interest on a sum for a given time at a given rate per cent., we have then simply to find the simple interest at the end of each period and add to the last principal.

Example 1.—Find the compound interest and amount of £500 in 4 years at 5 per cent. per annum.

Int. at end of 1st year $= \dfrac{500 \times 5}{100} = \pounds 25$

Amt. at end of 1st year $= 500 + 25 = \pounds 525$

Int. at end of 2nd year $= \dfrac{\overset{105}{\cancel{525}} \times \cancel{5}}{\underset{\underset{4}{\cancel{20}}}{\cancel{100}}} = \pounds 26\frac{1}{4}$

Amt. at end of 2nd year $= 525 + 26\frac{1}{4} = \pounds 551\frac{1}{4}$

Int. at end of 3rd year =

$$\frac{551\frac{1}{4} \times 5}{100} = \frac{\overset{441}{\cancel{2205}} \times \cancel{5}}{4 \times \underset{\underset{4}{\cancel{20}}}{\cancel{100}}} = \frac{441}{16} = \pounds 27\tfrac{9}{16}$$

Amt. at end of 3rd year $= 551\frac{1}{4} + 27\frac{9}{16} = \pounds 578\frac{13}{16}$

Int. at end of 4th year =

$$\frac{578\frac{13}{16} \times \cancel{5}}{\underset{20}{\cancel{100}}} = \frac{9261}{16 \times 20} = \pounds 28\ 18s.\ 9\tfrac{3}{4}d.$$

Amt. at end of 4th year $= 578\frac{13}{16} + \pounds 28\ 18s.\ 9\frac{3}{4}d.$

£	s.	d.
578	16	3
28	18	9¾
£607	15	0¾

Therefore—

Interest £607 15*s.* 0¾*d* – £500 = £107 15*s.* 0¾*d.* *Ans.*

Amount = £607 15*s.* 0¾*d.* *Ans.*

By using decimal fractions we can shorten the process. Taking the same example—

500	
5	
25·00	= 1st year's Int.
500	
525	= Amt. at end of 1st year.
5	
26·25	= 2nd year's Int.
525	
551·25	= Amt. at end of 2nd year.
5	
27·5625	= 3rd year's Int.
551·25	
578·8125	= Amt. at end of 3rd year.
5	
28·940625	= 4th year's Int.
578·8125	
607·753125	= Amt. at end of 4th year.
20	
15·062500	
12	
·7500	
4	
3·00	

	£	*s.*	*d.*	
	607	15	0¾	*Ans.*
	500	0	0	
Int.	107	15	0¾	*Ans.*

By placing the figures two places to the right in the above example we shall have the decimal points under each other, and we can easily add the interest to principal without copying down the latter.

500	
5	
25·00	= Int. at end of 1st year.
525	= Amt. „ „ „
5	
26·25	= Int. „ 2nd year.
551·25	= Amt. „ „
5	
27·5625	= Int. „ 3rd year.
578·8125	= Amt. „ „
5	
28·940625	= Int. „ 4th year.
607·753125	= Amt. „ „

All the decimals after five places may be discarded.

Example 2.—What will £1 25 amount to in 2 yr. 6 mo. at 4 per cent., reckoning interest every 6 months?

4 p. c. per an. = 2 p. c. for 6 mo.

125	
2	
2·50	= Int. at end of 1st 6 months.
127·50	= Amt. „ „ „
2	
2·5500	= Int. „ 2nd „

$$\begin{array}{rl}
130{\cdot}0500 & = \text{Amt. at end of 2nd 6 months.}\\
2 & \\ \hline
2{\cdot}6010 & = \text{Int. ,, 3rd ,,}\\ \hline
132{\cdot}651 & = \text{Amt. ,, ,, ,,}\\
2 & \\ \hline
2{\cdot}65302 & = \text{Int. ,, 4th ,,}\\ \hline
135{\cdot}30402 & = \text{Amt. ,, ,, ,,}\\
2 & \\ \hline
2{\cdot}7060804 & = \text{Int. ,, 5th ,,}\\ \hline
138{\cdot}01010 & = \text{Amt. ,, ,, ,,}
\end{array}$$

£138 0*s.* 2·424*d.* *Ans.*

Example 3.—What sum of money will amount to £367 10*s.* in 2 years at 5 per cent. compound interest?

£100 in 2 years at 5 per cent. will amount to £110 5*s.* (See Ex. I.) That is, £110 5*s.* is produced by £100 in 2 years.

$$\therefore \text{ £1 is produced by } \frac{100}{\pounds 110\ 5s.} \text{ in 2 years.}$$

$$\therefore \text{ £367 10s. is produced by } \frac{100 \times \pounds 367\ 10s.}{\pounds 110\ 5s.} \text{ in 2 years.}$$

$$\frac{100 \times \pounds 367\ 10s.}{\pounds 110\ 5s.} = \frac{100 \times 367{\cdot}5}{110{\cdot}25} = \frac{\overset{4}{\overset{\cancel{20}}{\cancel{100}}} \times \overset{750}{\overset{\cancel{5250}}{\cancel{36750}}}}{\underset{9}{\underset{\cancel{63}}{\underset{\cancel{441}}{\underset{\cancel{2205}}{\cancel{11025}}}}}} =$$

$$\frac{3000}{9} = \pounds 333\ 6s.\ 8d. \textit{ Ans.}$$

EXERCISES.

1. Find the compound interest of £1,050 in 2 years at 3 per cent.

2. What will £800 amount to in 3 years at 4 per cent. compound interest ?

3. Find the compound interest of £2,500 in 3 years at 4 per cent.

4. What will £850 amount to in 2 years at 2½ per cent. compound interest ?

5. What is the compound interest of £2,500 in 3 years at 5 per cent. ?

6. Find the compound interest and amount of £1,550 for 2½ years at 3¼ per cent.

7. What will £380 amount to in 3 years at 3½ per cent. compound interest ?

8. Find the difference between the simple and compound interest of £1,005 3*s*. 4*d*. for 2½ years at 2¾ per cent. ?

9. What will £50 18*s*. 6*d*. amount to if put out to compound interest for 3 years at 3½ per cent. ?

10. What principal at compound interest will amount to £856 13*s*. 4*d*. in two years at 6 per cent. ?

11. Find the compound interest of £1,500 5*s*. for 2½ years at 4½ per cent.

12. What is the difference between the simple and compound interest of £756 6*s*. 8*d*. for 3 years at 6½ per cent. ?

13. Find what £350 will amount to in two years at 4 per cent. compound interest, payable half-yearly.

14. What sum at 5 per cent. compound interest will amount to £1,500 in 3 years ?

15. What sum will amount to £1,000 in 2½ years at 3¼ per cent. compound interest ?

16. What will £800 3*s*. 4*d*. amount to in 1½ years at 4½ per cent. compound interest, payable half-yearly ?

17. Find the compound interest of £450 for 1 year at 2¼ per cent., payable quarterly.

18. What will £805 amount to in 1¼ years at 2¾ per cent., com*pound interest* ?

19. What sum of money will amount to £1,000 if put out to compound interest for 1½ years at 3 per cent., payable half-yearly ?

20. Required the compound interest and amount of £350 10*s.* for 2½ years at 4½ per cent.

21. What sum of money put out to interest (compound) for 3 years at 2½ per cent. will amount to £5,000 ?

22. Find the simple and compound interest on £550 13*s.* 4*d.* for 2½ years at 7 per cent.

23. Find what £387 16*s.* 8*d.* will amount to in 2½ years at 4¼ per cent.

24. Find the difference in the amounts at simple interest and compound interest of £650 10*s.* 6*d.* for 1¼ years at 2¾ per cent.

25. Find the compound interest on £875 9*s.* 6*d.* for 2 years at 2½ per cent.

26. Find the compound interest and amount of £49,000 for 3¼ years at 7 per cent. per annum.

27. What is the compound interest on £5 for 5 years at 5 per cent. ?

28. Find the compound interest on £300 for 3 years at 4 per cent.

CHAPTER XLIV.

STOCKS AND SHARES.

STOCK is capital, or money loaned to a government, or to a trading company, on condition of receiving certain interest. The British Government has borrowed in this way, within the past 200 years, nearly eight hundred millions sterling. This constitutes the National Debt. Suppose the Government *issue a loan* (viz., give notice that they wish to borrow money) at 4 per cent. (viz., promising 4 per cent. for all money lent), and I determine to lend £100; I shall receive a document entitling me to interest, viz. £4 upon this £100 stock from year to year, until the Government choose to pay me the principal. *Funds* is another term for Government stock.

Again, supposing a company is being formed to make a new railway. I may choose to contribute a certain part of the money required, viz. buy *shares*, and I shall receive interest on these *shares* according to the profits of the concern.

Now these stocks and shares are bought and sold just like any other commodity, and the price constantly varies.

If I buy a £100 railway share when the interest or *dividend* is 3 per cent., I shall not care to sell the same for £100 when the dividend rises to 6 per cent.; or if the dividend falls to 2 per cent., I shall not readily find a purchaser to give me £100 for the share.

If the price of £100 stock is £100, the stocks are said to be *at par;* when the price is above £100, *above par*, or at premium; when below £100 *below par*, or at a discount.

Transactions in stocks and shares are usually conducted through brokers, who charge $\frac{1}{8}$th or 2*s.* 6*d.* per cent. to both buyer and seller.

Nearly all problems in stocks and shares can be solved by simple proportion; the great point to bear in mind is, that stock is not cash (£100 stock does not mean £100), and that there can be *no ratio between stock and cash.*

Example 1.—How much stock can be bought for £1,000 at $86\frac{1}{8}$, brokerage $\frac{1}{8}$th per cent.?

In this example I must add $\frac{1}{8}$th to the price, making $86\frac{1}{4}$.

There are three given terms, two of them sterling money and one £100 stock understood. Our answer will be in stock—hence $100 : x$ will constitute the second ratio of the proportion. £1,000 will buy more than £$86\frac{1}{4}$.

$$\therefore \quad 86\tfrac{1}{4} : 1000 :: 100 : x$$

$$x = \frac{1000 \times 100}{86{\cdot}25} = \frac{\begin{array}{c}80000\\ \cancel{400000}\\ \cancel{2000000}\\ \cancel{10000000}\end{array}}{\begin{array}{c}\cancel{8625}\\ \cancel{1725}\\ \cancel{345}\\ 69\end{array}}$$

$$= 1159\tfrac{29}{69}. \textit{ Ans.}$$

Another method :—

£$86\frac{1}{4}$ will buy £100 worth of stock.

£1 „ „ $\dfrac{100}{86\frac{1}{4}}$ „ „

£1000 „ „ $\dfrac{100 \times 1000}{86\frac{1}{4}}$ „.

Example 2.—I sell £1,000 stock at $98\frac{1}{2}$, brokerage $\frac{1}{8}$th; what money do I realise?

In this case I subtract $\frac{1}{8}$ from the price. I shall get only $98\frac{3}{8}$ per £100 stock.

£100 stock produces £$98\frac{3}{8}$

∴ £1 „ „ $\dfrac{98\frac{3}{8}}{100}$

∴ £1000 „ „ $\dfrac{98\frac{3}{8} \times 10\cancel{00}}{1\cancel{00}}$

$= 98{\cdot}375 \times 10 = 983{\cdot}75$

= £$983\frac{3}{4}$ or £983 15*s.* *Ans.*

Example 3.—Bought £600, 3 per cent. consols* at $95\frac{3}{8}$, and sold out at $97\frac{7}{8}$; what did I gain by the transaction? ($\frac{1}{8}$th allowed for brokerage.)

---

* "3 per cent. consols," *i.e.* 3 per cent. consolidated annuities.

My real price including brokerage is $95\frac{4}{8}$ or $95\frac{1}{2}$, my selling price is $95\frac{6}{8}$ or $95\frac{3}{4}$.

Hence my profit on each £100 stock $95\frac{3}{4} - 95\frac{1}{2} = \frac{1}{4}$.

$$\therefore 6 \times \tfrac{1}{4} = \tfrac{6}{4} = \text{£}1\ 10s. \text{ profit.}$$

Example 4.—What yearly income shall I derive by investing £1,240 in the 3 per cents. at 95?

£95 gives an income of £3

$$1 \quad ,, \quad ,, \quad ,, \quad \frac{3}{95}$$

$$\text{£}1240 \quad ,, \quad ,, \quad \frac{3\times\overset{248}{\cancel{1240}}}{\underset{19}{\cancel{95}}}$$

$$= \frac{3 \times 248}{19} = \text{£}39\ 3s.\ 1\tfrac{17}{19}d. \textit{ Ans.}$$

Example 5.—Which is the more profitable investment 3 per cents. at 90, or $2\frac{1}{2}$ per cents. at 80?

$$\text{In 3 per cents. £1 produces } \frac{3}{90} = \frac{1}{30}$$

$$,,\ 2\tfrac{1}{2} \quad ,, \quad ,, \quad ,, \quad \frac{2\frac{1}{2}}{80} = \frac{5}{160} = \frac{1}{36}$$

$$\text{and } \tfrac{1}{30} - \tfrac{1}{36} = 8d. - 6\tfrac{2}{3}d. = 1\tfrac{1}{3}d.$$

That is, the 3 per cents. is more profitable by $1\frac{1}{3}d.$ in every £.

Example 6.—I bought £10,000 stock at 95, at what price must I sell it to gain £475? [omit brokerage].

The first question here is, what did I pay for the stock?

£100 stock was bought for £95

$$1 \quad ,, \quad ,, \quad ,, \quad \frac{95}{100}$$

$$\text{£}10{,}000 \quad ,, \quad ,, \quad \frac{95\times 10{,}000}{100}$$

$$= \text{£}9500$$

Hence I must sell the whole for £9,975; that is, £10,000 stock brings £9,975; required to find what £100 stock brings.

$$\begin{array}{ccccccc} \text{stock.} & & \text{stock.} & & \text{cash.} & & \text{cash.} \\ 10{,}000 & : & 100 & :: & 9975 & : & x \end{array}$$

$$x = \frac{9975\cancel{00}}{100\cancel{00}} = 99{\cdot}75 = 99\tfrac{3}{4}$$

hence the selling price is $99\frac{3}{4}$.

A shorter method is to suppose the £10,000 sold for £475, and this price added to the purchase price must give the selling price.

$$\begin{array}{ccccccc} \text{stock.} & & \text{stock.} & & \text{cash.} & & \text{cash.} \\ 10{,}000 & : & 100 & :: & 475 & : & x \end{array}$$

$$x = \frac{45700}{10000} = 4{\cdot}75 = 4\tfrac{3}{4}$$

and $95 + 4\frac{3}{4} = 99\frac{3}{4}$. *Ans.*

Example 7.—What amount of stock in the 4 per cents. will produce the same income as £1,200 in the $3\frac{1}{2}$ per cents.?

$$\begin{array}{llll} (1) & \pounds 100 & \text{stock produces} & \pounds 3\tfrac{1}{2} \\ & 1 & ,, \quad ,, & \dfrac{3\frac{1}{2}}{100} \\ & 1200 & ,, \quad ,, & \dfrac{3\frac{1}{2} \times 12\cancel{00}}{\cancel{100}} \\ & & & = \pounds 42 \end{array}$$

$$\begin{array}{llll} (2) & \pounds 4 & \text{income requires} & 100 \text{ stock} \\ & \pounds 1 & ,, \quad ,, & \dfrac{100}{4} \quad ,, \\ & \pounds 42 & ,, \quad ,, & \dfrac{\overset{25}{100} \times 42}{4} \\ & & & = \pounds 1050 \text{ stock. } \textit{Ans.} \end{array}$$

Example 8.—If the current rate of interest be 4 per cent., at what price ought the 3½ per cents. to be quoted?

In other words, if it requires £100 stock to produce £4, how much will be required to produce £3½?

$$4 : 3\tfrac{1}{2} :: 100 : x$$

$$x = \frac{3\tfrac{1}{2} \times 100}{4} = \frac{3{\cdot}5 \times 100}{4}$$

$$= \frac{350}{4} = 87\tfrac{1}{2}.\ \textit{Ans.}$$

EXERCISES.

1. Find how much stock can be purchased for £5,800 at 90½; and for £3,800 at 86¼.

2. What stock can be purchased for £1,800 in the 3½ per cents. at 85½?

3. What money can be realised by selling out stock to the amount of £1,500 at 97¾?

4. What is the sterling value of £10,000 stock at 82¼, allowing ⅛ per cent. brokerage?

5. How much stock can be purchased in the 3 per cents. at 82½ for £5,000? What will be the income arising therefrom?

6. What will be the yearly income arising from the investment of £2,500 in the 3½ per cents. at 84½?

7. Invest £1,800 in the 4½ per cents. at 94¼, and find the income for the half-year.

8. Invest £2,180 in the 3½ per cents. at 89¼, and find the yearly income.

9. What sum of money must be invested in the 3 per cents. at 92½ to produce an income of £45 a year?

10. What sum of money will purchase in the 2½ per cents. stock at 84¼ a half-yearly income of £65?

11. What difference of income will a person find by investing £1,500 in the 3½ per cents. at 92 and the 2¾ per cents. at 85?

12. A person has an income of £185 a year from an investment in the 4½ per cents. at 94; by how much will it be increased or

diminished by selling out and investing in the 3¼ per cents. at 83 ?

13. The 4½ per cents. being at 85¼, what must be given for £2,000 worth of stock ?

14. What difference of income will arise from selling out £4,000 worth of stock in the 3½ per cents. at 84½ and investing in the 4½ per cents. at 90¼ ?

15. Which is the better investment—in the 3½ per cents. at 75 or the 4¼ at 93 ?

16. The sum of £1,560 is invested in the 4 per cents. at 82 ; how much must be invested in the 3 per cents. at 96 to produce the same income ?

17. What must be the price of the 4 per cents. stock, so that by investing the sum of £24,800 my income may be £1,008 a year ?

18. I invested £3,000 in the 3 per cents. stock, and obtained an income thereby of £120 a year ; what was the price of stock ?

19. I buy £1,500 worth of stock for £1,000, what is the value of stock ?

20. How much stock can be bought in the 4 per cents. at 84¼ for £8,050, and what will be the half-yearly dividend ?

21. What sum of money will buy enough stock in the 3 per cents. at 81 to produce an income of £1,080 a year ?

22. A banker lends £8,960 at 3 per cent., and invests a similar amount in railway shares at £120 which yield 5½ per share ; which is the more profitable investment ?

23. A person buys £1,500 worth of stock in the 4½ per cents. at 92, and sells it to gain £50 ; find the selling price.

24. What sum must be invested in the 4¼ per cents. at 92 to produce £270 a year ?

25. A person bought enough stock in the 3¾ per cents. at 76 to produce an income of £180 a year ; how much stock did he buy, and what did he give for it ?

26. A person bought £1,800 stock at 79, and sold it to gain £40 ; at what price did he sell it ?

27. What amount of stock in the 4 per cents. will produce £350 per annum, and what will it cost at 90⅖ ?

28. A person buys £5,000 stock at 90, and sells it at 94¼ ; find his profit.

29. I invested £955 in the 4½ per cents. at 95½; find the rate of interest of money and the half-yearly dividend.

30. Find the rate of interest of money invested in the 3¼ per cents. at 69.

31. Find the rate of interest of money invested in the 5 per cents. at 102½.

32. Find the price of the 4 per cents. stock, when it yields the same interest as the 4½ per cents. at 90.

33. How much stock must a person buy in the 3 per cents. to produce £1,000 a year, and what will it cost at 93 per cent.?

34. A person buys £1,000 in the 3 per cents. stock at 96¼, and sells out at 88½; how much does he lose thereby?

35. A person transfers £1,000 stock from the 4 per cents. at 90 to the 3 per cents. at 70; find the change in his income.

36. If the 3 per cents. consols be at 93⅛, what sum of money must be expended in the purchase of stock to produce an income of £552 a year?

37. A man invests £840 in the 3 per cents. at 90, sells out at 81, and reinvests in the 5 per cents. at 108; find his income.

CHAPTER XLV.

MISCELLANEOUS EXERCISES.

PER-CENTAGES, EQUATION OF PAYMENTS, FURTHER QUESTIONS IN COMPOUND INTEREST, ETC.

Example 1.—In a school of 500 children, 5 per cent. are working in the 6th Standard, 10 per cent. in the 5th, 15 per cent. in the 4th, 20 per cent. in the 3rd, 25 per cent. in the 2nd, and the remainder in the 1st Standard; what is the actual number of children working in each Standard?

Out of each 100 boys 5 are in Standard VI.
" " 10 " " V.
" " 15 " " IV.
" " 20 " " III.
" " 25 " " II.
" " 25 " " I.
100

Therefore in each Standard there will be 5 times these numbers.

$$\left.\begin{array}{llcr} \text{Or Standard} & \text{VI.} & = & 25 \\ \quad " & \text{V.} & = & 50 \\ \quad " & \text{IV.} & = & 75 \\ \quad " & \text{III.} & = & 100 \\ \quad " & \text{II.} & = & 125 \\ \quad " & \text{I.} & = & 125 \\ & & & \overline{500} \end{array}\right\} \textit{Ans.}$$

Example 2.—There are 410 boys, 356 girls, and 520 infants attending a certain school. Of the boys 10 per cent. are absentees, of the girls 12 per cent., and of the infants 15 per cent.; what is the average attendance in each department of the school?

Average attendance per 100 scholars =
(1) Boys $100 - 10 = 90$
(2) Girls $100 - 12 = 88$
(3) Infants $100 - 15 = 85$

$$\therefore \text{ Boys in } 410 = \frac{90 \times 410}{100} = 369$$

$$\text{and Girls in } 356 = \frac{88 \times 356}{100} = 313{\cdot}28$$

$$\text{and Infants in } 520 = \frac{85 \times 520}{100} = 442$$

Example 3.—In England and Wales during the year ending 31st August, 1876, 1,142,612 scholars were presented for examination in Public Elementary Schools; of these 995,046 passed in Reading, 907,452 in Writing and Spelling, and 801,497 in Arithmetic. Find the per-centage of passes in each subject.

	presented.		passed.	
(1) Reading	1,142,612 : 100	::	995,046 : x	
(2) Writing and Spelling	1,142,612 : 100	::	907,452 : x	
(3) Arithmetic	1,142,612 : 100	::	801,497 : x	

$$\left.\begin{array}{ll}\text{Reading} & = 87{\cdot}08 \\ \text{Writing and spelling} & = 79{\cdot}42 \\ \text{Arithmetic} & = 70{\cdot}14\end{array}\right\} \textit{Ans.}$$

Example 4.—A tradesman sells goods for £512 10*s.* 2*d.*, and thereby gets a profit of $10\frac{1}{4}$ per cent.; what did the goods cost him?

For every £110 worth sold he pays £100
„ £1 „ „ $\frac{100}{110} = \frac{10}{11}$
„ £512 10*s.* 6*d.* „ $\frac{10}{11}$ of £512 10*s.* 2*d.*
= £465 18*s.* 4*d.* *Ans.*

Example 5.—A grocer bought sugar at £1 3*s.* 4*d.* per cwt.; at what price per lb. must he sell it to lose 10 per cent., and at what price to gain 10 per cent.?

(1) Sugar bought for £100 to lose 10 per cent. must be sold for £90.

Bought for £1 = $\frac{90}{100}$ selling price.

For £1 3*s.* 4*d.* or $1\frac{1}{6} = \dfrac{90 \times \frac{7}{6}}{100} = \frac{63}{60} = \frac{21}{20} =$ £1 1*s.* 0*d.*
per cwt., *i.e.* at $2\frac{1}{4}$*d.* per lb. *Ans.*

(2) Sugar bought for £100 to gain 10 per cent. must be sold for £110.

Hence for £1 = $\frac{110}{100}$ and £$1\frac{1}{6} = \dfrac{110 \times \frac{7}{6}}{100} = \frac{77}{60} =$
£1 5*s.* 8*d.* per cwt. = $2\frac{3}{4}$*d.* per lb. *Ans.*

Example 6.—A merchant has goods worth £100; he sells one-fourth so as to lose 10 per cent., for what must he sell the remainder to gain 15 per cent. on the whole?

To gain 15 per cent. on the whole he must sell for £115. But he sells $\frac{100}{4}$ = £25 worth at 10 per cent. loss.

Sale of £100 worth of goods gives £90.
∴ Sale of £25 worth of goods gives £22½.

That is, the merchant must—

(1) Receive £115 for *all* the goods to gain 15 per cent. on the whole.

(2) Sell ¼ for £22 10*s*.

Hence the ¾ must be sold for £115 — £22 10*s*. = £92 10*s*. *Ans.*

Example 7.—A grocer buys a chest of tea at 2*s*. 6*d*. per lb. and another chest at 3*s*. 4*d*. per lb.; in what proportion must he mix the tea so as to sell at 3*s*. 6*d*. per lb. and gain 20 per cent. on the mixture?

We must find first what price should be given for tea sold at 3*s*. 6*d*. to gain 20 per cent.

£120 will gain £20
1 „ $\frac{20}{120} = \frac{1}{6}$
3*s*. 6*d*. „ $\frac{1}{6}$ of 3*s*. 6*d*. = 6*d*.

That is, tea sold for 3*s*. 6*d*. with 20 per cent. profit was bought for 3*s*.

On every lb. of tea bought at 2*s*. 6*d*. he will gain 6*d*.

On every lb. of tea bought at 3*s*. 4*d*. he will lose 4*d*.

Therefore on 2 lbs. of cheaper tea his gain is equivalent to the loss on 3 lbs. of the dearer tea.

Hence he must mix in the ratio of 2 to 3.

Example 8.—A man insured his life for £500 at a yearly premium of 6 per cent.; immediately after paying his fourth premium he died; what did the company lose by the transaction, reckoning the value of money to be 4 per cent. compound interest?

£500 at 6 per cent. = £30 annual premium; £30 paid at the commencement of the first year will be worth (at 4 per cent.) £30 + £1 4*s.* = £31 4*s.* at the end of 1st year.

£30 paid at the commencement of the second year, together with the £31 4*s.*, will be worth £61 4*s.* + £2 8*s.* 11·52*d.* = £63 12*s.* 11·52*d.* at the end of second year.

£30 paid at the commencement of the third year, together with £63 12*s.* 11½*d.*, will be worth £93 12*s.* 11½*d.* + £3 14*s.* 11*d.* = £97 7*s.* 10½*d.* at the end of the third year.

Therefore £30 paid at the beginning of the fourth year + £97 7*s.* 10½*d.* = £127 7*s.* 10½*d.* will be all the money paid.

Loss = £500 − £127 7*s.* 10½*d.* = £372 12*s.* 1½*d.* *Ans.*

Example 9.—Taking the population of London on January 1st, 1877, to be 3,500,000, and supposing the increase to be 2 per cent. per annum, what will be the population on January 1st, 1880.

3500000
2
70000·00 increase the 1st year
3570000 pop. on Jan. 1st, 1878
2
71400·00 increase the 2nd year
3641400 pop. on Jan. 1st, 1879
2
72828·00 increase the 3rd year
3714228 pop. on Jan. 1st, 1880. *Ans.*

Example 10.—A piece of broadcloth is retailed at 8*s.* per yard; if the retailer, the wholesale merchant, and the manufacturer realise profits of 20, 10, and 20 per cent. *respectively*, what was the cost of production?

(1) £120 worth cost the retailer £100

1 ,, ,, $\frac{10\cancel{0}}{12\cancel{0}} = \frac{5}{6}$

∴ 8*s*. or $£\frac{2}{5}$,, $\frac{\cancel{5}}{6} + \frac{2}{\cancel{5}} = £\frac{1}{3}$

(2) £110 worth cost the merchant £100

1 ,, ,, $\frac{100}{110} = \frac{10}{11}$

$£\frac{1}{3}$,, ,, $\frac{10}{11} \times \frac{1}{3} = £\frac{10}{33}$

(3) £120 worth cost the manufacturer £100

1 ,, ,, ,, $\frac{100}{120} = \frac{5}{6}$

$\frac{10}{33}$,, ,, ,, $\frac{5}{\underset{3}{\cancel{6}}} \times \frac{\overset{5}{\cancel{10}}}{33} = \frac{25}{99}$

∴ $£\frac{25}{99}$ or 5*s*. $0\frac{20}{33}d.$ was the cost of production.

Example 11.—*Three persons, A, B, and C, rent a field for £20; A put in 50 horses for 6 months; B, 70 horses for 5 months, and C, 60 horses for 8 months; what ought each to pay?

50 horses for 6 months are equivalent to 1 horse for $50 \times 6 = 300$ months; 70 horses for 5 months are equivalent to 1 horse for $5 \times 70 = 350$ months. 60 horses for 8 months are equivalent to 1 horse for $8 \times 60 = 480$ months.

∴ 180 horses are equivalent to 1 horse for 1130 months.

Then (1) $1130 : 300 :: 20 : x =$ A's share,

and (2) $1130 : 350 :: 20 : x =$ B's share,

and (3) $1130 : 480 :: 20 : x =$ C's share.

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* This is an example in what is sometimes called "equation of payments."

$$(1)\ x = \frac{300 \times 2\cancel{0}}{113\cancel{0}} = \tfrac{600}{113} = \text{£}5\ 6s.\ 2\tfrac{38}{113}d.\ \textit{Ans.}$$

$$(2)\ x = \frac{350 \times 2\cancel{0}}{113\cancel{0}} = \tfrac{700}{113} = \text{£}6\ 3s.\ 10\tfrac{82}{113}d.\ \textit{Ans.}$$

$$(3)\ x = \frac{480 \times 2\cancel{0}}{113\cancel{0}} = \tfrac{960}{113} = \text{£}8\ 9s.\ 10\tfrac{106}{113}d.\ \textit{Ans.}$$

Example 12.—A man travelled 190 miles, of which he went 5 miles by water to 6 by rail, and 6 by water to 2 on on foot; what distance was travelled over by water, rail, and on foot respectively?

(1) 5 by water to 6 by rail,
or 1 „ „ $\frac{6}{5}$ by rail.
(2) 6 by water to 2 on foot,
or 1 „ „ $\frac{2}{6}$ or $\frac{1}{3}$ on foot.

Therefore the distances travelled are in the proportions of 1, $\frac{6}{5}$ and $\frac{1}{3}$,

or $\frac{15}{15}$, $\frac{18}{15}$, $\frac{5}{15}$, or 15, 18, 5.

Therefore out of 38 miles, 15, 18, and 5 respectively are gone over by water, rail, and on foot.

Hence (1) $38 : 190 :: 15 : x$
(2) $38 : 190 :: 18 : x$
(3) $38 : 190 :: 5 : x$

Or $$(1)\ x = \frac{\overset{5}{\cancel{190}} \times 15}{\cancel{38}} = 75 \text{ m. by water.}$$

$$(2)\ x = \frac{\overset{5}{\cancel{190}} \times 18}{\cancel{38}} = 90 \text{ m. by rail.}$$

$$(3)\ x = \frac{\overset{5}{\cancel{190}} \times 5}{\cancel{38}} = 25 \text{ m. on foot.}$$

Example 13.—A goods and a passenger train are running on parallel lines of metals in the same direction at rates of 20 and 40 miles per hour respectively; if the goods train is 80 yards long and the passenger 30 yards long, how long will it take the passenger to pass the goods train?

The passenger train will have to gain the whole length of both trains to get clear, viz. 80 + 30 or 110 yds.

But passenger train gains 20 m. in 1 hour.

or „ „ „ 1 m. in $\frac{1}{20}$ ho. $=$ 3 min.

or „ „ „ 1 yd. in $\frac{3}{1760}$ min.

or „ „ „ 110 yds. in $\dfrac{3 \times \overset{1}{\cancel{110}}}{\underset{16}{\cancel{1760}}}$

$= \frac{3}{16}$ min. or $11\frac{1}{4}$ sec. *Ans.*

Example 14.—A school-master counts his boys 4 at a time, 5 at a time, 6 at a time, and there is always one over; but if he counts them 7 at a time there are none over; find how many boys there are in the school. Is there more than one answer to the question?

60 is the L.C.M. of 4, 5, and 6, and therefore the smallest number which can be divided evenly into fours, fives, or sixes; add one to this, and whether we divide into groups of 4, 5, or 6, there will be always one over.

60 being the L.C.M. of 4, 5, 6, no other number but 60 or a *multiple* of 60 with one added can be divided in fours, fives, or sixes with one remainder.

Hence the required number must be a multiple of 60 with one added.

By trial—

$60 \times 2 + 1 = 121$ not divisible by 7

$60 \times 3 + 1 = 181$ „ „

$60 \times 4 + 1 = 241$ „ „

$60 \times 5 + 1 = 301$ *divisible by* 7.

Hence 301 is the required number.

By further trial it will be found that—

$$60 \times 12 + 1 = 721$$
$$60 \times 18 + 1 = 10801, \text{\&c.},$$

answer the required conditions.

Example 15.—A garden is 80 feet long and 50 feet wide; required the cost of making a gravel walk 4 feet wide, running all round inside at 9*d.* per square yard, and of turfing the central part at 2*d.* per square yard.

(1) Gravel walk—

$$\begin{aligned} \text{sides} &= (80 \times 4 \times 2) \text{ ft.} \\ \text{ends} &= (42 \times 4 \times 2) \text{ ft.} \\ &= 640 + 336 \\ &= 976 \text{ ft.} = 108\tfrac{4}{9} \text{ sq. yds.} \end{aligned}$$

$$\text{Cost} = 108\tfrac{4}{9} \times 9 = £4\ 1s.\ 4d.$$

(2) Grass plot—

$$72 \times 42 = 3024 \text{ ft.} = 336 \text{ sq. yds.}$$
$$336 \text{ sq. yds. at } 2d. = £2\ 16s.$$
$$\text{Total cost} = £6\ 17s.\ 4d.\ \textit{Ans.}$$

Example 16.—A room is 18 feet long, 14 feet wide, and 12 feet high; what will it cost to cover the floor with carpet 2 feet wide at 3*s.* 6*d.* per yard, and the walls with paper 2 feet 3 inches at ¾*d.* per yard (omit allowance for doors and windows)?

$$(1)\ 3s.\ 6d. \times \frac{18 \times 14}{2 \times 3} = £7\ 7s.$$

$$\begin{aligned} (2)\ & (18 \times 2 + 14 \times 2) \times 12 \\ & = (36 + 28) \times 12 \\ & = 64 \times 12 = 768 \text{ sq. ft. of wall} = \end{aligned}$$

$$\frac{768}{2\frac{1}{4}} = \frac{768}{1} \times \frac{4}{9} = \frac{3072}{9} = \frac{3072}{27} \text{ yds. required.}$$

$$\frac{3}{4}d. \times \frac{3072}{27} = \frac{256}{3}d. = 7s.\ 1\tfrac{1}{3}d.$$

Hence £7 7*s*. + 7*s*. 1⅓*d*. = £7 14*s*. 1⅓*d*. *Ans.*

MISCELLANEOUS EXERCISES.

1. What will be the cost of insuring a house for £3,060 at 5*s*. 9*d*. per cent.?

2. Find the simple interest on £348 10*s*. for 7 years at 4½ per cent.

3. A bankrupt owes £7,357 12*s*. and his assets amount to £3,065 13*s*. 4*d*.; find the dividend in the £.

4. Of what sum does the simple interest for 4½ years at 6 per cent. amount to £276 6*s*. 6¾*d*.?

5. Divide £380 15*s*. in the proportion of ½, ⅔, and ¾.

6. A bankrupt pays 15*s*. in the £; if he owes A £256 10*s*., B £300, and C £150 15*s*., find what each will receive.

7. Find the simple interest on £635 for 9 years at 3¼ per cent.

8. If I receive 12*s*. 9*d*. for the loan of £136 for 8 days, what is that per cent. per annum?

9. Find the present worth of a bill for £682 18*s*. 9½*d*. due 3 years 9 months hence at 4½ per cent. per annum.

10. What is the difference between the simple and compound interest on £15 15*s*. for 2½ years at 4¼ per cent.?

11. A man saves £50 a year and puts it out at the end of each year at 4 per cent. compound interest; what will he have at the end of the third year?

12. By selling cheese at 1*s*. per lb. I gain 20 per cent.; what was the cost price per lb.?

13. Of what sum is £86 19*s*. 10*d*. the interest for 150 days at 4⅘ per cent.?

14. How many square feet are there in a carpet 18 ft. long and 15 ft. 9 in. wide?

15. Find the income produced by investing £4,500 in the 3 per cents. at 90.

16. If 2½ per cent. be lost by selling bacon at 6½*d*. per lb., what did it cost per cwt.?

17. Divide £48 between A and B so that for every 2*s*. 6*d*. which A receives B may receive 3*s*.

18. What sum of money will amount to £100 in 3 years at 4 per cent. simple interest?

19. Find the compound interest on £150 for 3 years at 4 per cent.

20. How many acres are there in a field 359 yards long and 276 yards broad?

21. If by selling tea at 3*s*. per lb. a grocer loses 4 per cent., what did it cost him per lb.?

22. What is the difference between the simple and compound interest on £8,100 for 2½ years at 4 per cent.?

23. Find the premium on a cargo worth £126,315 at 8*s*. 8*d*. per cent.

24. Find in how many years £452 10*s*. will amount to £644 16*s*. 3*d*. at 4¼ per cent. per annum simple interest.

25. The population of a town in 1851 was 15,786; and in 1871, 19,670; find the increase per cent.

26. Find the area of a courtyard 18 ft. 3 in. by 9 ft. 4 in.

27. A man has £575 15*s*. after paying income tax of 5*d*. in the £; what is his income?

28. A man buys eggs at 11¼*d*. per dozen and sells them at 2*d*. each; what does he gain per cent.?

29. Find the simple interest on 150 guineas at 3½ per cent. from the 3rd of March to the 18th of September.

30. If the interest on £270 for 2¼ years be £14 5*s*., what will be the interest on £596 10*s*. for 11 years at the same rate?

31. Find the true discount on a bill for £498 18*s*. due in 10½ months at 4½ per cent.

32. In what time will £1,400 amount to £1,411 18*s*. at 4¼ per cent. per annum, simple interest?

33. Divide £85 16*s*. into 3 parts in the proportion of ⅕, ⅐, and ⅑.

34. If I buy sugar at £1 12*s*. 8*d*. per cwt., at what price must I sell it per lb. to gain 7½ per cent.?

35. Three persons rent a field for £60 10*s*. A puts in 5 sheep or 4½ months, B 8 sheep for 5 months, and C 9 sheep for 6¼ months; what must each pay of the rent?

36. I pay £2 15*s*. 6*d*. as premium on an insurance of property *to the* amount of £5,000; find the rate per cent.

37. In what time will £3,125 amount to £3,681 12*s*. 9¾*d* at 3¾ per cent.?

38. An account is £16 14*s*. 9*d*. after 2½ per cent. discount has been taken off; what was the original bill?

39. How much 4 per cent. stock at 94 can be bought for £1,000?

40. What principal must I have to amount to 2,050 in 15 years at 4¼ per cent. simple interest?

41. Required the discount per cent. when 19*s*. 10½*d*. is taken from £39 15*s*. in consideration of cash.

42. What will £960 amount to at compound interest at 4½ per cent. in 1¼ years, interest payable quarterly?

43. What sum will amount to £882 in 2 years at 5 per cent. compound interest?

44. A invests £350 in a business for 8 months and B £480 for 12 months; the profits at the end of the year amount to £370; find the share of each.

45. What will it cost to pave a yard 28 ft. by 20 ft. at 2*s*. 6*d*. per sq. yd.?

46. At what price should goods be sold, which cost £180, so as to gain 15½ per cent.?

47. Find the cost of carpeting a room 27 ft. long and 19 ft. wide with carpet 2 ft. 3 in. wide at 5*s*. per yard.

48. A tradesman loses 3*s*. 6*d*. by selling an article for a sovereign; find the cost price, the loss per cent., and the selling price to yield a profit of 10 per cent.

49. Thames water contains ·032 per cent. of organic matter; find the weight of organic matter in 36,000 tons.

50. I sold goods at a gain of 9½ per cent. for £156·159; what did they cost me?

51. A ball weighing 13 cwts. 1 qr. 15 lbs. is made of silver and copper in the proportion of 7 to 113; find the weight of each metal.

52. What is the present value of a bill amounting to £350 10*s*. due 10 months hence at 1 per cent.?

53. What profit shall I make by investing £1,000 in the 2½ per cents. at 75¼ and selling out when they are at 86½?

54. A debtor arranges to pay 3*s*. 6*d*. in the £ at once, and

3*s*. 6*d*. more in 6 months; what is the present value of the dividend reckoning 5 per cent. interest?

55. What amount of stock in the 4 per cents. yields an income equal to that derived from the interest of £5,000 at 3⅓ per cent.?

56. How much paper ¾ yd. wide will be required to paper a room 22 ft. long, 17 ft. wide, and 11½ ft. high?

57. Find the income derived from investing £1,500 in railway shares at 108½, yielding a dividend of 5 per cent.

58. What is the price of £4,000 4½ per cents. stock at 72½, brokerage ⅛ per cent.?

59. Divide a line 12 ft. 9 in. long into 2 parts in the proportion of ·46 and ·05.

60. A tradesman who allows 15 per cent. for cash, adjusts his prices accordingly; how much must be put on an article worth £5 10*s*. 6*d*.?

61. I invest £3,631 5*s*. in the 5 per cents. at 102½, brokerage ⅛ per cent.; find my half-yearly income.

62. How much stock at 82¼ must I sell to pay a debt of £1,080 15*s*.?

63. A room measures 21 ft. by 18 ft. 6 in. Brussels carpet ¾ yd. wide costs 4*s*. 6*d*. a yard; Kidderminster one yard wide costs 5*s*. 3*d*. a yard; which is the cheaper, and by how much?

64. The sum of £1,245 is invested in the 4 per cents. at 84; how much money must be invested in the 3½ per cents. at 98, to produce the same income?

65. I receive from an investment in the 3½ per cents. a half-yearly dividend of £170, and I then sell out at 87½, thereby sustaining a loss of £250 upon my purchase; what was the price of the 3½ per cents.?

66. I invest my capital in the 3½ per cents. at 96¼; at what rate per cent. do I receive interest, allowing ⅛ per cent. brokerage?

67. A room measures 18 ft. by 16 ft.; what will it cost to whitewash the ceiling at 1½*d*. per sq. yd., and to carpet the floor with Brussels carpet ¾ yd. wide, at 4*s*. 9*d*. per yard?

68. The proceeds of the sale of 3½ per cent. stock at 93⅛ *amounted to* £6,844 13*s*. 9*d*.; find the amount of stock.

69. A grocer buys coffee at £8 10*s*. per cwt. and chicory at £2 10*s*., and mixed in the proportion of 5 chicory to 7 coffee; at what rate per lb. must the mixture be sold to gain 16⅔ per cent. on the outlay?

70. What sum must I invest in the 4 per cents. at 95⅛ to have an income of £150 clear of income tax at 2*d*. in the £?

71. A room measures 21 ft. by 16 ft. There is a square carpet 12 ft. in the side; find the cost of polishing the uncovered part at 2*s*. 6*d*. the sq. yd.

72. A dealer bought goods for £665, and gained 5 per cent. by selling to another, who on his part lost 5 per cent. by selling to a third; what did the third dealer pay for the goods?

73. If by selling oranges at 18 for 1*s*. 8*d*. I gain 10 per cent., find the price per dozen to gain 21 per cent.

74. A horse is sold for 80 guineas, at a loss of 10 per cent. on the outlay; find the selling price to gain 10 per cent. on the outlay.

75. £1,037 10*s*. is invested in the 3 per cents. at 83, and when this fund has risen 1 per cent., the capital is transferred to the 4 per cents. at 96; find the alteration of income.

76. A, who travels at 3½ miles per hour, starts 2½ hours before B, who travels the same road at 4½ miles an hour; find when B will overtake A.

77. A £50 railway share, which cost me £55, pays a yearly dividend of £6; what is my rate of interest?

THE END.

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Crown 8vo.

APPENDIX.

ANSWERS TO THE EXERCISES.

ADDITION AND SUBTRACTION. Pages 30 to 32.

(1) 73
(2) 61
(3) 6
(4) 71
(5) 79
(6) 45
(7) 74
(8) 8
(9) 36
(10) 61
(11) 96
(12) 46
(13) 62
(14) 8
(15) 31
(16) 19
(17) 9*s.*; 11*s.*
(18) 33
(19) 36
(20) 45
(21) 18
(22) 38; 23
(23) 97; 59
(24) 35
(25) 5
(26) 38

ADDITION AND SUBTRACTION. Pages 42 and 43.

(1) 15; 126
(2) 4; 105
(3) 12 lbs.; 469 lbs.
(4) 6 lbs.; 9 lbs.
(5) 4 needles; 306 pins
(6) 6 yrs.; 39 yrs.
(7) 12*s.*; 727*s.*
(8) 186
(9) 42
(10) 232
(11) 126
(12) 19
(13) 519
(14) 347
(15) £55
(16) 365
(17) 8 yrs.; 15 yrs.
(18) £171; 3 yrs.
(19) £137
(20) 92
(21) 74
(22) 50
(23) 47
(24) 38

ADDITION AND SUBTRACTION. Pages 49 and 50.

(1) 106
(2) 14
(3) (*a*) 15*d.*; (*b*) 756*s.*
(4) (*a*) 3; (*b*) 148
(5) 1465
(6) (*a*) 5; (*b*) 175
(7) 135
(8) (*a*) 9; (*b*) 908
(9) 662
(10) 243
(11) 1262
(12) 1135

(13) 59
(14) 213
(15) (*a*) 11; (*b*) 233
(16) 64
(17) 21
(18) 402
(19) 693
(20) 762
(21) 208
(22) 654
(23) 11,115
(24) 103

Examination Papers, St. I. Pages 51 to 53.

I.

(1) 4307; 300
(2) 52
(3) 135
(4) 102

II.

(1) 5870; 70
(2) 1143
(3) 322
(4) 70

III.

(1) 2100; 21
(2) 900
(3) 280
(4) 1054

IV.

(1) 96; 90
(2) 249
(3) 165
(4) nine hundred and six.

V.

(1) Three thousand and forty; one thousand two hundred and seven.
(2) 4341
(3) 261
(4) 20 yards

VI.

(1) 695
(2) 20*s*.
(3) 26 gals.
(4) 279

VII.

(1) sum = 8675
rem. = 3646
(2) £17
(3) 289
(4) 1485

VIII.

(1) 6163
(2) 918
(3) 23; 11
(4) 2320

Short Multiplication. Pages 58 to 60.

(1) 5728
(2) 2685
(3) 1252
(4) 3384
(5) 7329
(6) 19,614
(7) 5112
(8) 252; 5280
(9) 21,658

(10) 4536; 34,263
(11) 1460; 2555; 5475
(12) 432; 480; 9144; 10,056
(13) 3443
(14) 4768
(15) 2421
(16) 4932
(17) 2817
(18) 33,880
(19) £260
(20) 861 grs.
(21) The latter by 4979
(22) 1983
(23) The latter by 865

LONG MULTIPLICATION. Pages 62 and 63.

(1) 640; 7600
(2) 56,900
(3) 960; 2400; 14,400
(4) 3650; 36500; 365,000
(5) 2100; 2800
(6) 3264; 4896
(7) 4158
(8) 119,500
(9) 123,480
(10) 32,940
(11) 1092; 2184; 5460; 7644
(12) 97,700
(13) 14,340
(14) 7200
(15) 5760
(16) 7280
(17) 10,332
(18) 4000
(19) 58,920
(20) 19,008
(21) 3180
(22) 12,024
(23) 648
(24) 88,880

LONG MULTIPLICATION. Pages 65 and 66.

(1) 12,792
(2) 288
(3) 8760
(4) 38,080 lbs.
(5) 63,360
(6) 458,052 lbs.
(7) 260
(8) 18,720
(9) 14,235
(10) £4560
(11) 5985
(12) 7776
(13) 4160
(14) 697 miles
(15) 6,462,016
(16) 11,340
(17) 109,446
(18) 119,520
(19) 94,080
(20) 1,183,200; 14,198,400
(21) 160,632
(22) 59136
(23) 3042
(24) 9; 25; 49; 81; 121; 256; 324; 361; 625

SHORT DIVISION. Pages 68 to 70.

(1) (*a*) 5; (*b*) 196
(2) (*a*) 5; (*b*) 403
(3) 218
(4) 266
(5) 210
(6) 1760
(7) 195
(8) 125
(9) 1982
(10) 881
(11) 1250
(12) (*a*) 744; (*b*) 1295
(13) 1188
(14) 52
(15) 44
(16) 3801
(17) (*a*) 5; (*b*) 2022
(18) (*a*) 3; (*b*) 335; (*c*) 250
(19) 1021
(20) (*a*) 6; (*b*) 15,512
(21) 6; 7; 117; 100
(22) 206
(23) 271
(24) 310

Miscellaneous Exercises. Pages 70 and 71

(1) 241
(2) 903*s.*
(3) 133
(4) 56
(5) 227
(6) £2677
(7) 24
(8) 66
(9) £51; 17 weeks
(10) £1052
(11) £65
(12) 9990
(13) 1539
(14) 1460
(15) 133
(16) 510,255
(17) 4357
(18) 687
(19) 15*s.*
(20) 477
(21) 12,212
(22) 2164
(23) £225; £175
(24) 1100

Examination Papers, St. II. Pages 72 to 74.

I.

(1) 477
(2) 42,336
(3) £281
(4) 52,592

II.

(1) 16
(2) 4,313,428
(3) 130
(4) 694

III.

(1) 862
(2) 56 and 18
(3) 5250
(4) 468

IV.

(1) 32
(2) 2817 pence
(3) 1875
(4) 240

V.

(1) 1027
(2) 754
(3) The latter by 81
(4) 61,532

VI.

(1) £135
(2) 9 days
(3) 38,864
(4) 66,049

VII.

(1) 2275
(2) 670
(3) 5 days, and 85 pairs
(4) The latter by 7

VIII.

(1) 85
(2) 5,933,320
(3) 169
(4) 3,047

IX.

(1) 108
(2) 97
(3) 313,110
(4) 154

X.

(1) 431 boys
(2) 48 pence
(3) 1015 parts and £4060
(4) 43,807

Long Division. Pages 79 to 81.

(1) (*a*) 3; (*b*) 281
(2) 154
(3) 139
(4) 221
(5) 448
(6) 58
(7) 620 + 20
(8) 735
(9) 88
(10) 14
(11) (*a*) 9; (*b*) 27
(12) 311
(13) 25
(14) 13
(15) (*a*) 7; (*b*) 12
(16) (*a*) 3 times; (*b*) 45 times
(17) £22
(18) 30 from the cask, 240 from the cistern
(19) (*a*) 3; (*b*) 321
(20) 200
(21) 3 and 91
(22) The former by 1
(23) 37 trains; the last dispatched at 10.5
(24) £62
(25) £3
(26) 192
(27) 105
(28) 634
(29) 52
(30) 52

Addition of Money. Pages 89 and 90.

(1) £1 8*s.* 7½*d.*
(2) £4 2*s.* 0*d.*
(3) £11 18*s.* 9*d.*
(4) £1 17*s.* 10½*d.*
(5) £1 14*s.* 0*d.*
(6) £3 16*s.* 6*d.*
(7) £12 3*s.* 5*d.*
(8) £32 18*s.* 3¼*d.*
(9) £34 16*s.* 0*d.*
(10) £4 3*s.* 5*d.*
(11) 7*s.* 6½*d.*
(12) Yes, exactly
(13) £99 3*s.* 7*d.*
(14) £5 0*s.* 0*d.*
(15) £176 9*s.* 1*d.*
(16) £5 3*s.* 8¼*d.*
(17) £5 7*s.* 4*d.*
(18) £2 17*s.* 3½*d.*
(19) £413 15*s.* 4*d.*
(20) £8 17*s.* 9*d.*
(21) £6866 15*s.* 11*d.*
(22) £1897 2*s.* 7¼*d.*
(23) £1677 6*s.* 9¼*d.*
(24) £19,360 19*s.* 11¼*d.*

SUBTRACTION OF MONEY. Pages 93 and 94.

(1) £1 11*s.* 2¾*d.*
(2) £1 13*s.* 4*d.*
(3) 3*s.* 9*d.*
(4) £24 13*s.* 6*d.*
(5) £2 3*s.* 6*d.* lost
(6) £11 1*s.* 2*d.* gained
(7) £20 6*s.* 8*d.*
(8) 12*s.* 6½*d.*
(9) £15 18*s.* 6*d.*
(10) £1 2*s.* 7*d.*
(11) 6*s.* 6*d.*
(12) £4 16*s.* 5½*d.*
(13) £82 10*s.* 8*d.*
(14) £3 7*s.* 4*d.*
(15) £6 2*s.* 7*d.*
(16) 9*s.* 3½*d.*
(17) £10 0*s.* 1*d.*
(18) £414 18*s.* 6*d.*
(19) £124 7*s.* 6*d.*
(20) £9010 3*s.* 8*d.*
(21) £24 13*s.* 4*d.*
(22) 19*s.* 1½*d.*
(23) £49 14*s.* 3½*d.*
(24) £147 0*s.* 8½*d.*

ADDITION AND SUBTRACTION OF MONEY. Pages 94 to 96.

(1) £126
(2) £44 19*s.* 2*d.*
(3) £1 17*s.* 5½*d.*
(4) £14 5*s.* 6*d.*
(5) 9*s.*
(6) £1 5*s.* 9*d.*
(7) £4 14*s.* 4*d.*
(8) £8 5*s.* 9*d.*
(9) £84 3*s.* 4*d.*
(10) £3 6*s.* 1*d.*
(11) £2 4*s.* 1*d.*
(12) £2 1*s.* 7½*d.*
(13) £593 17*s.* 6*d.*
(14) £5 7*s.* 8*d.*
(15) £247 12*s.* 0*d.*
(16) £2537 11*s.* 2*d.*
(17) £91 19*s.* 10*d.*
(18) £59 13*s.* 4*d.*

REDUCTION OF MONEY. Pages 99 and 100.

(1) 60; 180; 144; 3780
(2) 3*s.*; 4*s.*; 8*s.*; 60*s.*
(3) 100*s.*; 200*s.*; 3300*s.*
(4) £1; £1 15*s.*; £3 5*s.*; £7; £150 10*s.*
(5) £14 13*s.* 0*d.*
(6) 5840*s.*
(7) 862 sixpences
(8) 1890 pence
(9) 1399 pence
(10) 540*s.*
(11) 1723*s.*
(12) £33
(13) 3797 pence
(14) 149 pence
(15) 8; 24; 40; 800
(16) £8 1*s.* 0*d.*
(17) £79 1*s.* 3*d.*
(18) 4*s.*; 45*s.*
(19) 8*s.* 4*d.*
(20) £1 2*s.* 1*d.*
(21) £5
(22) 12*s.* 6*d.*
(23) £99 11*s.* 8*d.*
(24) £2 0*s.* 1*d.*
(25) 4*s.* 0¼*d.*
(26) 4 times
(27) £149 16*s.* 10½*d.*
(28) £2 16*s.* 8*d.*
(29) The latter by £3 13*s.* 4*d.*
(30) £1 5*s.* 9¼*d.*

EXAMINATION PAPERS, ST. III. Pages 100 to 104.

I.

(1) 17*s.* 11½*d.*
(2) 277 bags and 28 nuts
(3) £183 11*s.* 7½*d.*
(4) £3 17*s.* 0*d.*

II.

(1) £1 5*s.* 1½*d.*
(2) 142
(3) £10 7*s.* 9*d.*
(4) £54 2*s.* 3*d.*

III.

(1) 13*s.* 10½*d.*
(2) 43; 1258
(3) £5 2*s.* 1*d.*; £1 17*s.* 11*d.*
(4) £66 9*s.* 2*d.*

IV.

(1) £3802 5*s.* 11*d.*
(2) 15
(3) £2 6*s.* 11*d.*
(4) £3 12*s.* 7½*d.*

V.

(1) £60 11*s.* 7¾*d.*
(2) 299 pence
(3) £10 1*s.* 6*d.*
(4) £3 16*s.* 4*d.*

VI.

(1) 0 rem.
(2) 2
(3) 3*s.* 6*d.*
(4) £7 3*s.* 0*d.*

VII.

(1) £1163 1*s.* 11¼*d.*
(2) 48
(3) £5 14*s.* 0*d.*; £950
(4) £96 10*s.*

VIII.

(1) £15 13*s.* 10*d.*
(2) 193 + 30
(3) 65
(4) £62

IX.

(1) £10 18*s.* 10½*d.*
(2) 17
(3) £219 10*s.* 6*d.*
(4) £2 13*s.* 11*d.*

X.

(1) 750 pence
(2) 144
(3) £13 3*s.* 1*d.*
(4) 3*s.*

XI.

(1) £99 11*s.* 8*d.*
(2) 501,000
(3) £312 10*s.*; £302
(4) 16*s.* 8*d.*

XII.

(1) 6*s.* 4½*d.*
(2) 37
(3) £361 15*s.* 3*d.*
(4) John 1*s.* 5*d.*

XIII.

(1) £4533 0s. 4¾d.
(2) 49
(3) Saves £2 4s.; spends 10s.
(4) £38 6s. 6d.

XIV.

(1) £95 2s. 1d.
(2) 9 tons
(3) 17s. 4d.
(4) £6 0s. 10¾d.

MULTIPLICATION OF MONEY. Pages 111 to 113.

(1) £9 5s. 6d.
(2) £1473 3s. 0d.
(3) 9s.
(4) 15s.
(5) £1 6s. 3d.
(6) £1 1s 6½d.
(7) 7s. 4d.
(8) £26 15s. 10½d.
(9) £9 19s. 0d.
(10) 6s.
(11) 13s.
(12) £27 10s. 9d.
(13) £16 15s. 6d.
(14) £3 0s. 4d.
(15) £16 13s. 0d.
(16) £7 17s. 6d.
(17) £174 6s. 0d.
(18) £26 6s. 6d.
(19) £22 0s. 6½d.
(20) £5 14s. 9d.
(21) £14 14s. 0d.
(22) £19,106 5s. 0d.
(23) £360 11s. 5½d.
(24) £438 10s. 8d.
(25) £674 1s. 0d.
(26) £1 13s. 1¾d.
(27) £232 13s. 9d.
(28) £371 13s. 9d.
(29) £2976 2s. 3d.
(30) £330 8s. 0d.
(31) £1 5s. 6d.
(32) £125 4s. 0d.
(33) £21 11s. 3d.
(34) £63 15s. 8d.
(35) £196 3s. 9d.
(36) £5 10s. 5d.
(37) £3
(38) £110 3s. 9d.
(39) £10 2s. 6d.
(40) £10,184 1s. 6d.
(41) £9 15s. 7½d.
(42) £1695 2s. 11d.
(43) £48 2s. 6d.
(44) £6 8s. 4½d.
(45) £365 15s. 0d.
(46) £2 11s. 4½d.
(47) £21 15s. 0d.
(48) £1420 15s. 6d.
(49) £3 5s. 0d.
(50) £8 6s. 8d.

DIVISION OF MONEY. Pages 117 to 119.

(1) 1s. 9d.; 3s. 9d.; 7s. 6d.; 1s. 8d.; 8s. 9d.; 11s. 3d.; 13s. 9d.
(2) 7½d.; 11¼d.; 1s. 3d.; 1s. 10½d.; 3s. 9d.; 4s. 4½d.; 8s. 10½d.; 9s. 4½d.
(3) 2s. 3d.; 5s. 9½d.; 6s. 8d.; 9s. 2d.; 10s. 8d.; 16s. 3¼d.
(4) £2 17s. 0¾d.
(5) £2 19s. $5\frac{6}{11}$d.
(6) £2 12s. 6⅜d.
(7) £39 3s. 0¾d.
(8) 12s. 6d.
(9) 11s. 8d.
(10) 2s. 11½d.
(11) 10 lbs.
(12) £445 7s. 2⅜d.
(13) 7s. $5\frac{2}{7}$d.

(14) 9*d.*
(15) 5*s.* 3*d.*
(16) (*a*) A 1*s.*; B 6*d.*;
(*b*) A £140 12*s.* 4*d.*;
B £70 6*s.* 2*d.*
(17) (*a*) A 7*d.*; B 1*s.* 2*d.*;
(*b*) A £92 5*s.* 5¼*d.*;
B £184 10*s.* 10½*d.*
(18) (*a*) 2*s.* and 1*s.*
(*b*) The man £346 3*s.* 0½*d.*; the woman £173 1*s.* 6¼*d.*
(19) 3*s.* 3*d.*
(20) 5*s.*
(21) 3*s.* 6*d.*
(22) 1½*d.*
(23) 2*d.*
(24) 2¼*d.*
(25) 8½*d.*
(26) (*a*) 1*s.*, (*b*) £87 5*s* 11*d.*
(27) £400
(28) £130 12*s.* 6*d.*
(29) 15*s.*
(30) 7*s.* 6*d.*
(31) £24 13*s.* 2½*d.*
(32) £2 10*s.* 6*d.*
(33) 1*d.*
(34) 4*d.*
(35) 64 yards
(36) 144 weeks
(37) £12
(38) 72 times
(39) 24
(40) 15

Miscellaneous Exercises. Pages 120 and 121.

(1) £59 19*s.* 10*d.*
(2) £35 19*s.* 4*d.*
(3) £5
(4) £39 2*s.* 6*d.*
(5) £12 7*s.* 9½*d.*
(6) £1 15*s.* 0*d.*
(7) 52 loaves; 17*s.* 4*d.*
(8) £140 16*s.*
(9) £4 4*s.* 0*d.*
(10) £41 9*s.* 0*d.*
(11) £34 0*s.* 3*d.*
(12) £9 19*s.* 3*d.*; £18 6*s.* 0*d.*
(13) 190,060; £791 18*s.* 4*d.*
(14) 3*s.* 8*d.*
(15) £1 15*s.* 1¼*d.*
(16) £40
(17) £2 4*s.* 0*d.*
(18) £1
(19) Father £2; son 10*s.*
(20) £32 12*s.* 1*d.*
(21) £407 2*s.* 9½*d.*

Reduction. Pages 124 and 125.

(1) 3684
(2) 21 half-crowns; 210 threepences
(3) 756
(4) 33 cwts. 1 qr. 26 lbs.
(5) 1*s.* 8*d*
(6) 2 m. 1480 yds.
(7) 1000
(8) 21,622
(9) 1659
(10) £10 14*s.* 8¼*d.*
(11) £357
(12) 2,240,000
(13) 2880 sheets; £6
(14) 1092
(15) 205,200
(16) £14
(17) 1664 days
(18) 1 lb.
(19) 259
(20) 512,000
(21) 13 yrs. 87 days
(22) 2512
(23) 1032
(24) 936
(25) 20 m. 800 yds.

Miscellaneous Exercises. Pages 125 to 128.

(1) 400
(2) £3 8*s.* 10*d.*
(3) 562
(4) 814
(5) £5 7*s.* 11*d.*
(6) £4 3*s.* 4*d.*
(7) 81
(8) £249 14*s.* 9½*d.*
(9) 35
(10) £305 6*s.* 8*d.* each to two; £389 6*s.* 8*d.* to the third.
(11) 390 miles
(12) 21
(13) 810½
(14) 10
(15) 3960
(16) 220
(17) A £170; B £190
(18) 3921
(19) £145
(20) 114,002
(21) 173,136
(22) 159
(23) 540 lbs.
(24) June 22nd
(25) 748
(26) 2100
(27) £15 1*s.* 0½*d.*
(28) 7 boys
(29) £15 6*s.* 8*d.*; £11 10*s.* 0*d.*
(30) 245 lbs.
(31) 72; 242 qrs.; 16 sheets
(32) £2 0*s.* 6*d.*
(33) 14 times
(34) 96
(35) £58 10*s.*
(36) 21 bushels, 43 pints
(37) £16 8*s.* 5*d.*
(38) 3424
(39) 3660¼ sq. yds.
(40) 475
(41) 1210
(42) 432
(43) 121
(44) 7*s.* 6*d.*
(45) 5062 lbs. 8 ozs., or 45 cwts. 22 lbs. 8 ozs.
(46) 6*s.*
(47) £14; £4 13*s.* 4*d.*
(48) 141 lbs.
(49) 156
(50) 10*d.*; £2 1*s.* 8*d.*
(51) 48
(52) 4*s.* 2*d.*; 8*s.* 4*d.*; 17*s.* 6*d.*

Examination Papers, St. IV. Pages 129 to 132.

I.

(1) 6616
(2) 63,360
(3) 16*s.* 6*d.*
(4) 100

II.

(1) £154 10*s.*
(2) 192
(3) £11
(4) 32,400

III.

(1) £1928 11*s.*
(2) Coffee by 3*s.* 4*d.*
(3) £1 5*s.* 0*d.*
(4) 2880

IV.

(1) £48 16*s.* 2¼*d.* + 6
(2) 160
(3) 1*s.* 6*d.*
(4) £125 2*s.* 6*d.*

V.

(1) No change
(2) 5*d.*
(3) 1¼*d.*; 15*s.*
(4) 149,348 ozs.

VI.

(1) 9*s.* 2½*d.*
(2) 10*s.* 7½*d.* gain
(3) 13*s.* 3¾*d.*; 24
(4) 45 weeks

VII.

(1) 59151
(2) 4*s.*
(3) £4 10*s.* 0*d.*
(4) 24

VIII.

(1) £73,326 8*s.* 3*d.*
(2) 12
(3) 11 days
(4) £1 5*s.* 0*d.*

IX.

(1) 23,994½
(2) 28
(3) The former by 3*s.* 4*d.*
(4) £14 8*s.* 10*d.*

X.

(1) 15,750
(2) 156
(3) 320
(4) £14 18*s.* 8*d.*

XI.

(1) 12*s.* 6*d.*
(2) 6 tons 5 cwts. 2 qrs. 9 lbs.
(3) £14 11*s.* 8*d.* gain
(4) 10*d.*

XII.

(1) 1520
(2) £169 5*s.* 10*d.*
(3) £2 1*s.* 8*d.*
(4) William £56 10*s.* 4*d.*; John £28 5*s.* 2*d.*

XIII.

(1) 1080
(2) £299 8*s.* 4*d.*
(3) C £123 6*s.* 8*d.*; A and B £83 6*s.* 8*d.* each
(4) £1 10*s.* 0*d.*

XIV.

(1) Subtract £517 10*s.*
(2) 75
(3) 6*d.*
(4) £108

XV.

(1) £6 15*s.* 7½*d.*
(2) 280
(3) 13 cwts.
(4) $54\frac{6}{11}$ miles

XVI.

(1) 91½ lbs.
(2) 1015
(3) 2*s.* 10*d.*
(4) 16; 5*s.* gain

Bills of Parcels. Pages 135 to 138.

1. (1) 11*d.*
(2) 9¾*d.*
(3) 7*d.*
(4) 3¾*d.*
(5) 6¼*d.*
(6) 1*s.* 8¼*d.*

2. (1) 7*s.* 8¾*d.*
(2) 12*s.* 6¾*d.*
(3) 7*s.* 6*d.*
(4) 4*s.* 9¾*d.*
(5) 5*s.* 5*d.*
(6) 10*s.* 3*d.*

3. (1) 16*s.* 3¼*d.*
(2) £2 4*s.* 2½*d.*
(3) £8 3*s.* 6¼*d.*
(4) £6 15*s.* 10½*d.*

4. (1) £4 0*s.* 1*d.*
(2) £9 13*s.* 11*d.*
(3) £1 5*s.* 11¾*d.*
(4) £2 18*s.* 10½*d.*
(5) £1 16*s.* 8*d.*
(6) 13*s.* 1*d.*

5. (1) £3 4*s.* 5¼*d.*
(2) £2 5*s.* 9¼*d.*

6. Corrected page.

£	*s.*	*d.*
2	11	6½
	8	5¾
	11	10½
		6¾
	4	5¾
	1	4½
	8	3
		5¼
	4	4½
	1	5½
4	12	10

7. £3 3*s.* 6½*d.*

8. Corrected accounts.

	£	*s.*	*d.*
(1)	4	4	4½
	1	15	0
	1	6	3
	7	5	7½

	£	*s.*	*d.*
(2)		4	2
		7	10
		14	0
	1	6	0

	£	*s.*	*d.*
(3)	3	8	3
	2	8	4½
	1	3	7½
	7	0	3

Simple Practice. Pages 143 to 145.

1. (1) 10*s.* 5*d.*; 5*s.* 2½*d.*; 15*s.* 7½*d.*; £1 0*s.* 10*d.*
(2) £26 17*s.* 0*d.*; £17 18*s.*; £13 8*s.* 6*d.*; £8 19*s.*; £4 9*s.* 6*d.*
(3) £1 3*s.* 10*d.*; £2 7*s.* 8*d.*; £2 19*s.* 7*d.*; £4 3*s.* 5*d.*

(4) £2; £2 8s.; £2 16s.; £3 4s.; £3 12s.
(5) £2 3s. 3¾d.; £2 11s. 2¼d.; £2 19s. 0¾d.; £3 6s. 11¼d.; £3 14s. 9¾d.
(6) £4 10s.; £5 8s.; £6 6s.; £7 4s.; £8 2s.
(7) £6 12s. 6d.; £7 5s. 9d.; £3 9s. 6¾d.; £3 12s. 10½d.; £3 16s. 2¼d.
(8) £7 5s. 2d.; £8 7s. 6d.; £9 9s. 10d.; £10 12s. 2d.; £11 14s. 6d.
(9) £10 2s. 7¼d.; £11 15s. 0¼d.; £13 7s. 5¼d.; £14 19s. 10¼d.; £16 12s. 3¼d.
(10) £25 18s. 0¾d.; £29 14s. 9¾d.; £33 11s. 6¾d.; £37 8s. 3¾d.; £41 5s. 0¾d.
(11) £22 14s. 8d.; £23 5s.; £23 15s. 4d.; £24 5s. 8d.; £18 1s. 8d.
(12) £91; £60 13s. 4d.; £45 10s.; £36 8s.; £30 6s. 8d.
(13) £76 8s.; £63 13s. 4d.; £47 15s.; £31 16s. 8d.; £38 4s.
(14) £80 16s. 8d.; £40 8s. 4d.; £48 10s.; £97; £161 13s. 4d.
(15) £125; £187 10s.; £62 10s.; £93 15s.; £31 5s.
(16) £118 16s.; £158 8s.; £138 12s.; £178 4s.; £59 8s.
(17) £369; £344 8s.; £328; £307 10s.; £410
(18) £525; £364 11s. 8d.; £328 2s. 6d.; £510 8s. 4d.; £306 5s.
(19) £218 10s. 8d.; £193 14s.; £307 17s. 4d.; £178 16s.; £173 16s. 8d.
(20) £170 0s. 10d.; £160 15s. 4d.; £139 2s. 6d.; £216 8s. 4d.; £231 17s. 6d.
(21) £366 15s.; £611 5s.; £489; £298 16s. 8d.; £373 10s. 10d.
(22) £276 3s.; £552 6s.; £328 15s.; £256 8s. 6d.; £341 18s.
(23) £267 12s.; £594 13s. 4d.; £334 10s.; £475 14s. 8d.; £327 1s. 4d.
(24) £119 18s. 6d.; £55 7s.; £230 12s. 6d.; £215 5s.; £169 2s. 6d.

2. (1) £10 9s. 3d.; £12 15s. 9d.
(2) £69 1s. 3d.; £54 0s. 7½d.
(3) £44 13s. 9¾d.; £55 10s. 4½d.
(4) £98 7s. 7½d.; £129 0s. 2¼d.
(5) £79 17s. 3½d.; £93 2s. 2½d.
(6) £167 1s. 3d.; £174 5s. 7½d.
(7) £103 19s. 8¼d.; £109 14s. 3¼d.

(8) £172 17s. 8d.; £139 3s.
(9) £147 0s. 2d.; £99 19s. 6¾d.
(10) £195 13s.; £206 10s.
(11) £550 13s. 7d.; £486 2s. 3d.
(12) £513 14s. 5d.; £521 16s. 7d.

3. (1) £498 6s. 8d.; £1443 5s. 0d.
(2) £1803 3s. 4d.; £3332 19s. 0d.
(3) £14,736 0s. 0d.; £1303 16s. 0d.
(4) £44,590 10s. 0d.; £5455 7s. 0d.
(5) £3452 5s.; £2148 7s. 6d.
(6) £11,063 2s. 3d.; £4884 11s. 6d.
(7) £2792 7s. 6d.; £3669 16s. 7d.
(8) £8401 18s. 9d.; £16,993 13s. 5d.
(9) £5466 6s. 0d.; £2659 1s. 0d.
(10) £10,224 0s. 0d.; £5123 5s.
(11) £31,989 8s. 4d.; £28,009 15s. 5d.
(12) £1808 2s. 0d.; £1343 6s.

4. £1057 2s. 4d.
5. £260 4s. 9d.
6. £411 1s. 6d.
7. £88 13s. 4d.
8. £8 1s. 0d.
9. £253 2s. 6d.
10. £558 14s. 0½d.
11. £121 12s. 0d.
12. £196 17s. 6d.
13. £1380 15s.
14. £1397 10s.
15. £293 6s. 8d.
16. £1732 10s.
17. £48,749 5s.
18. £613 7s. 4¼d.

Compound Practice. Pages 147 and 148.

(1) £8 3s. 0½d.
(2) £7 10s. 5¼d
(3) £13 11s. 8¼d.
(4) £7 16s. 1d.
(5) £31 2s. 4½d.
(6) £5 10s. 2d.
(7) £352 10s. 3¾d.
(8) £34 9s. 7d.
(9) £14 7s. 1¼d.
(10) £205 1s. 5d.
(11) £2 18s. 5¾d.
(12) £1 16s. 3d.
(13) £61 8s. 9¼d.
(14) 17s. 7½d.
(15) £3 3s. 9¾d.
(16) £38 14s. 4½d.
(17) £13 6s. 0d.
(18) £4 7s. 8¼d.
(19) £10 2s. 6d.
(20) £4 14s. 6d.
(21) £13 6s. 3d.
(22) £7 0s. 5¼d.
(23) £16 3s. 5¼d.
(24) £32 3s. 6d.
(25) £3 0s. 1¾d.
(26) £60 12s. 1¾d.
(27) £3 15s. 5½d.
(28) £41 15s. 7½d.

(29) £78 11*s*. 0¾*d*.
(30) £155 2*s*. 9*d*.
(31) 4*s*. 6½*d*.
(32) £16 6*s*. 3*d*.
(33) £103 0*s*. 11¼*d*.
(34) £44 13*s*. 10½*d*.
(35) £353 0*s*. 0*d*.
(36) £6 2*s*. 11*d*.
(37) £323 17*s*. 8½*d*.
(38) £28 19*s*. 6*d*.
(39) £8 16*s*. 9*d*.
(40) £11 16*s*. 3*d*.
(41) £7 0*s*. 0¾*d*.
(42) £95 7*s*. 7½*d*.
(43) £53 9*s*. 3¼*d*.
(44) £43 12*s*. 8*d*.
(45) £77 3*s*. 9½*d*.

Miscellaneous Exercises. Pages 148 and 149.

(1) £394 18*s*. 4*d*.
(2) £16 15*s*. 3*d*.
(3) £20 13*s*. 3½*d*.
(4) £9 6*s*. 1¼*d*.
(5) £484 19*s*. 9*d*.
(6) £19 12*s*. 11*d*.
(7) £306 5*s*. 0*d*.
(8) £109 0*s*. 10½*d*.
(9) £1 13*s*. 5¼*d*.
(10) £133 17*s*. 1½*d*.
(11) £40 5*s*. 5½*d*.
(12) £576,875
(13) £46 9*s*. 3*d*.
(14) £391 11*s*. 2½*d*.
(15) £53 9*s*. 8¼*d*.
(16) £8 4*s*. 6*d*.
(17) £47 5*s*. 1*d*.
(18) £81 2*s*. 3*d*.
(19) £7851 1*s*. 10½*d*.
(20) £246 6*s*. 5*d*.
(21) £5333 2*s*. 6*d*.
(22) £6 2*s*. 11*d*.
(23) £8 7*s*. 4¾*d*.;
£2 2*s*. 4¾*d*.
(24) £140 3*s*. 0¾*d*.
(25) £2 6*s*. 1½*d*.
(26) £378; £154
(27) £1 2*s*. 6*d*.
(28) £14 4*s*. 11*d*.

Miscellaneous Exercises. Pages 157 to 161.

(1) 8*s*. 4*d*.
(2) £2553 15*s*. 0*d*.
(3) £19 8*s*. 6*d*.
(4) £9 11*s*. 7½*d*.
(5) £3 12*s*. 4*d*.
(6) £4 17*s*. 4*d*.
(7) 17*s*. 3*d*.
(8) £2 2*s*. 0*d*.
(9) £227 15*s*. 7½*d*.
(10) £163 19*s*. 1¼*d*.
(11) £1 17*s*. 6*d*.
(12) 8*s*. 1½*d*.
(13) £487 10*s*. 0*d*.
(14) John £7 10*s*. 0*d*.;
Henry £5 5*s*. 0*d*.;
Thomas £3 15*s*. 0*d*.
(15) £8 11*s*. 2*d*.
(16) £5 8*s*. 9*d*.
(17) £2 2*s*. 0*d*.
(18) £1 4*s*. 6*d*.

(19) £1 12*s.* 3*d.*
(20) 14*s.* 5½*d.*
(21) £2 9*s.* 0*d.*
(22) 3*s.* 6½*d.*
(23) £8 11*s.* 0*d.*
(24) 40
(25) £2025
(26) 224
(27) 15*s.* 9*d.*
(28) £75; £77 12*s.* 2*d.*
(29) £8 2*s.* 6*d.*
(30) 253⅓ lbs.
(31) 14*s.* 1*d.*
(32) 5760 grs.; £8 8*s.* 0*d.*
(33) 437½ grs.; 42½ grs.
(34) 5 dwts.
(35) 15 days
(36) 25 miles
(37) £7 11*s.* 8*d.*; £120;
£112 11*s.* 8*d.* more
(38) £15
(39) £14 17*s.* 6*d.*
(40) £95; £33 5*s.* 0*d.*
(41) £24 18*s.* 1*d.*;
£26 12*s.* 7*d.*
(42) 450 yds.
(43) £112 10*s.* 0*d.*
(44) £63
(45) £875
(46) 25 cwts.; £100
(47) £2432 5*s.* 10*d.*
(48) £319 19*s.* 3¾*d.*
(49) 126 lbs.
(50) 7½*d.*; £23 2*s.* 6*d.*
(51) 21 days
(52) £18 15*s.* 0*d.*;
£1500
(53) 2 cwts. 37⅓ lbs.
(54) £1 5*s.* 0*d.*
(55) £476
(56) £58 6*s.* 8*d.*
(57) 3⅜
(58) 4
(59) £124 1*s.* 5½*d.*
(60) £71 8*s.* 0*d.*
(61) 750

FRACTIONS. Pages 166 and 167.

(1) £1 8*s.* 4½*d.*;
11*s.* 2¼*d.*
(2) 11*s.* 10*d.*;
£3 10*s.* 1½*d.*
(3) 1600
(4) £708 15*s.* 0*d.*
(5) £119 7*s.* 9¾*d.*
(6) The latter by £4 6*s.* 8*d.*
(7) £173 16*s.* 0*d.*
(8) 13*s.* 6*d.*
(9) £1 2*s.* 8*d.*
(10) 5*d.*
(11) £27,000
(12) £583 6*s.* 8*d.*
(13) £882 0*s.* 0*d.*
(14) £4 12*s.* 0*d.*
(15) 4*s.*
(16) 1*d.*
(17) $\frac{11}{4}$; $\frac{41}{7}$; $\frac{23}{5}$; $\frac{34}{3}$; $\frac{139}{7}$
(18) $2\frac{3}{4}$; $5\frac{6}{7}$; $4\frac{3}{5}$; $11\frac{1}{3}$; $19\frac{6}{7}$
(19) $\frac{4}{5}$; $\frac{1}{5}$; $\frac{6}{7}$; $\frac{1}{2}$; $\frac{1}{7}$

RATIO AND PROPORTION. Pages 174 and 175.

1. $\frac{2}{3}$; 2$\frac{1}{3}$; $\frac{1}{21}$
2. (1) 18
   (2) 162
   (3) 38
   (4) 10$\frac{1}{2}$

3. 75
4. 9
5. 1080
6. (1) 18
   (2) 45
   (3) 60

7. 14
8. 18
9. 228

SIMPLE PROPORTION. Pages 182 to 184.

(1) 2*s*.
(2) £6 6*s*. 9*d*.
(3) £11 0*s*. 6*d*.;
£14 3*s*. 6*d*.
(4) 8*s*. 7$\frac{1}{8}$*d*.
(5) 7*s*. 8*d*.
(6) £1 18*s*. 4$\frac{1}{2}$*d*.
(7) £1 6*s*. 1$\frac{1}{2}$*d*.
(8) 1*s*. 1$\frac{3}{10}$*d*.
(9) 9$\frac{1}{2}$*d*.
(10) £6 13*s*. 0*d*.
(11) £17 0*s*. 9*d*.
(12) £459 5*s*. 11*d*.
(13) 13*s*. 6*d*.
(14) £12 15*s*. 0*d*.
(15) 16*s*.
(16) 78
(17) 5
(18) £31 5*s*. 0*d*.
(19) 18 months
(20) 54 feet
(21) 1 day
(22) £195
(23) 1080 miles
(24) 140 acres
(25) £218 15*s*. 0*d*.
(26) £3 0*s*. 0*d*.
(27) £296 0*s*. 0*d*.
(28) 12 lbs.
(29) 180 days; 36 days
(30) 16$\frac{4}{5}$ gals.
(31) £15
(32) 5$\frac{2}{3}$ days
(33) £38
(34) £857 2*s*. 10$\frac{2}{7}$*d*.
(35) 25
(36) £720 15*s*. 11*d*.
(37) 45 miles
(38) 19$\frac{4}{23}$ days
(39) £24
(40) £6 11*s*. 1$\frac{55}{149}$*d*.

EXAMINATION PAPERS, ST. V. Pages 185 to 190.

I.

(1) 217
(2) £56 8*s*. 0*d*.
(3) £1 1*s*. 11$\frac{3}{4}$*d*.
(4) £264 13*s*. 9*d*.

II.

(1) £85
(2) £442 9*s*. 2*d*.
(3) 271
(4) £5 10*s*. 3*d*.

III.

(1) 7
(2) £74 14*s.* $2\frac{1}{2}$*d.*
(3) 9 lbs. 5 ozs.
(4) 4*s.*

IV.

(1) 15 days
(2) £1 1*s.* $11\frac{1}{4}$*d.*
(3) £1 0*s.* 6*d.*
(4) $200\frac{1}{2}$ lbs.

V.

(1) 6*s.*
(2) £3 14*s.* $9\frac{3}{4}$*d.*
(3) £15 1*s.* 0*d.*
(4) £123 0*s.* 0*d.*

VI.

(1) £34 14*s.* $6\frac{3}{8}$*d.*
(2) £7 10*s.* 0*d.*
(3) £24 4*s.* 0*d.*
(4) £13 14*s.* $9\frac{1}{2}$*d.*

VII.

(1) $67\frac{13}{17}$ feet
(2) £18 19*s.* $8\frac{1}{4}$*d.*
(3) 656 miles
(4) £60 7*s.* 6*d.*

VIII.

(1) £1095
(2) £4 6*s.* 3*d.*
(3) $33\frac{6}{25}$ days
(4) £12 13*s.* 5*d.* gain

IX.

(1) 23 cwts.
(2) $110\frac{6}{17}$ days
(3) £14 14*s.* 9*d.*
(4) £1833 6*s.* 8*d.* daughter's share; £1333 6*s.* 8*d.* son's share

X.

(1) £248 8*s.* $9\frac{1}{2}$*d.*
(2) 44
(3) 7 hrs. 59 min. $2\frac{3}{8}$ secs.
(4) £10 7*s.* 6*d.* the first £7 3*s.* 9*d.* each of th others

XI.

(1) £979 2*s.* $7\frac{7}{10}$*d.*
(2) 107 lbs. $5\frac{1}{2}$ ozs.
(3) 10
(4) £1 17*s.* 4*d.*

XII.

(1) 6*d.* too little
(2) £2 17*s.* $8\frac{1}{8}$*d.*
(3) £31 12*s.* 6*d.*
(4) 24 minutes

XIII.

(1) £1 15*s.* 11*d.*
(2) each woman 3*s.* 2*d.*; each man 4*s.* 9*d.*
(3) £16 5*s.* $9\frac{1}{4}$*d.*
(4) 20 days

XIV.

(1) £1921 12*s.* 3*d.*
(2) £80 3*s.* 0*d.*
(3) $6\frac{9}{11}$
(4) 9850

XV.

(1) 14
(2) 7*s.* 9¼*d.*
(3) potatoes by 4*s.* 8*d.*
(4) 500

XVI.

(1) £295 6*s.* 3*d.*
(2) £440 12*s.* 6*d.*
(3) £19 17*s.* 6*d.*
(4) £11 18*s.* 5¾*d.*

XVII.

(1) Beef by 11*d.*
(2) 174
(3) 14*s.* 5¼*d.*
(4) £7 1*s.* 5¼*d.*

XVIII.

(1) 398
(2) 30 lbs.
(3) 100 miles
(4) the former by 6*d.* in 40 weeks

Greatest Common Measure. Pages 195 and 196.

1. (1) 4
(2) 9
(3) 8
(4) 7
(5) 10
(6) 108
(7) 8
(8) 15
(9) 6
(10) 1001
(11) 147
(12) 1834

2. (1) 4
(2) 8
(3) 4
(4) 12
(5) 19
(6) 108
(7) 12
(8) 24
(9) 3
(10) 601
(11) 147
(12) 8

3. (1) 9
(2) 1241
(3) 27
(4) 36
(5) 6014
(6) 4
(7) 721
(8) 8
(9) 1234
(10) 42
(11) 101
(12) 8

4. (1) 2
(2) 4; 2
(3) 3
(4) 2
(5) 4; 2
(6) 12; 6; 4; 3; 2

5. 539
6. 4
7. 69
8. 7 plots of 8 acres each
9. 7 of 9 gals.
10. 6; 8 and 5 of 7*d.* each
11. 21; 42; 63
12. 2

Least Common Multiple. Pages 200 and 201.

1. (1) 60
(2) 36
(3) 336
(4) 304
(5) 3120
(6) 210

2. (1) 42
(2) 920
(3) 60
(4) 112
(5) 60
(6) 2520

3. (1) 3990
(2) 756
(3) 17,136
(4) 9744
(5) 7072
(6) 1755
4. 1001
5. 46
6. 12 min.
7. £21
8. 10*s.*
9. 150
10. 60
11. 107
12. £10 10*s.* 0*d.*

REDUCTION OF FRACTIONS. Page 203.

1. (1) $\frac{7}{2}$; $\frac{19}{5}$; $\frac{41}{3}$; $\frac{117}{8}$
(2) $\frac{71}{4}$; $\frac{161}{10}$; $\frac{98}{5}$; $\frac{167}{8}$
(3) $\frac{280}{11}$; $\frac{129}{7}$; $\frac{385}{13}$; $\frac{188}{5}$
(4) $\frac{353}{21}$; $\frac{327}{11}$; $\frac{667}{5}$; $\frac{369}{127}$
(5) $\frac{3722}{513}$; $\frac{1129}{113}$; $\frac{65}{18}$; $\frac{61}{5}$
(6) $\frac{103}{21}$; $\frac{98}{27}$; $\frac{115}{8}$; $\frac{633}{317}$

2. (1) $30\frac{2}{9}$; $10\frac{24}{29}$; $8\frac{17}{21}$; $21\frac{25}{33}$
(2) $7\frac{21}{31}$; 24; $5\frac{15}{16}$; $15\frac{2}{13}$
(3) $13\frac{3}{14}$; $13\frac{97}{108}$; $37\frac{2}{5}$; $3\frac{108}{301}$
(4) $18\frac{8}{17}$; $37\frac{71}{108}$; $10\frac{5}{31}$; $17\frac{1}{3}$
(5) $40\frac{1}{2}$; $620\frac{3}{5}$; $116\frac{12}{17}$; $11\frac{1}{3}$
(6) $49\frac{2}{5}$; 36; $26\frac{1}{2}$; $28\frac{1}{5}$

3. (1) $\frac{4}{5}$; $\frac{1}{2}$; $\frac{1}{4}$; $\frac{1}{3}$
(2) $\frac{3}{7}$; $\frac{1}{3}$; $\frac{1}{9}$; $\frac{1}{7}$
(3) $\frac{5}{11}$; $\frac{3}{4}$; $\frac{2}{11}$; $\frac{4}{11}$
(4) $\frac{5}{11}$; $\frac{1}{3}$; $\frac{1}{2}$; $\frac{1}{13}$
(5) $\frac{1}{6}$; $\frac{25}{129}$; $\frac{1}{4}$
(6) $\frac{1}{2}$; $\frac{1}{5}$; $\frac{1}{16}$

4. (1) $\frac{1}{9}$; $\frac{1}{2}$; $\frac{5}{11}$; $\frac{3}{8}$
(2) $\frac{11}{21}$; $\frac{1}{5}$; $\frac{91}{95}$
(3) $\frac{7}{20}$; $\frac{7}{37}$; $\frac{3}{32}$
(4) $\frac{9}{20}$; $\frac{1}{7}$; $\frac{1}{2}$

ADDITION AND SUBTRACTION OF FRACTIONS. Pages 207 and 208.

1. (1) $1\frac{13}{20}$; $1\frac{55}{126}$
(2) $1\frac{32}{77}$; $\frac{293}{315}$
(3) $1\frac{31}{72}$; $2\frac{35}{36}$
(4) $4\frac{1}{30}$; $18\frac{49}{120}$
(5) $19\frac{20}{21}$
(6) $17\frac{5}{8}$
2. $\frac{1}{2}$; $\frac{7}{24}$
3. $\frac{2}{3}$
4. $\frac{13}{20}$
5. $\frac{8}{25}$
6. 10*d.*
7. $9\frac{17}{28}$
8. $11\frac{9}{55}$*d.*
9. $3\frac{1}{9}$*s.*
10. $157\frac{89}{140}$
11. $38\frac{77}{180}$
12. $\frac{17}{19}$
13. $\frac{19}{45}$
14. $\frac{3}{8}$
15. $\frac{1}{10}$
16. $3\frac{77}{500}$
17. $\frac{19}{30}$
18. $\frac{2987}{3500}$
19. $\frac{11}{56}$
20. $\frac{37}{120}$
21. $\frac{1}{6}$
22. $6\frac{3}{8}$; $\frac{5}{8}$
23. $\frac{18}{35}$
24. $\frac{59}{112}$
25. $\frac{7}{12}$
26. $\frac{29}{35}$
27. $\frac{7}{9}$

MULTIPLICATION OF FRACTIONS. Pages 211 and 212.

1. (1) $\frac{6}{35}$; $\frac{4}{15}$
(2) $\frac{5}{84}$; $\frac{2}{105}$
(3) $\frac{4}{99}$; $\frac{2}{105}$
(4) $\frac{4}{9}$; $\frac{4}{39}$
(5) $\frac{1}{210}$; $\frac{1}{3}$
(6) $\frac{2}{49}$; $\frac{7}{11}$

2. (1) $\frac{5}{8}$; $\frac{2}{15}$
(2) $\frac{21}{64}$; $\frac{2}{21}$
(3) $\frac{1}{18}$; $\frac{1}{42}$
(4) $\frac{24}{49}$; $\frac{29}{30}$
(5) $\frac{1}{120}$; $\frac{3}{11}$
(6) $1\frac{5}{9}$; $1\frac{1}{3}$

3. (1) $21\frac{67}{128}$
   (2) $1\frac{5}{9}$
   (3) $77\frac{1}{27}$
   (4) $1\frac{8}{15}$
   (5) $\frac{1}{147}$
   (6) $\frac{47861}{239616}$
4. $123\frac{1}{20}$
5. $32\frac{1}{6}$
6. £$11\frac{3}{7}$
7. £3; £12; £1 4*s.*; 2*s.* 8*d.*
8. $11\frac{1}{5}$
9. Equal
10. $1\frac{9}{11}$*s.*
11. 1485
12. $92\frac{7}{10}$

Division of Fractions. Pages 217 and 218.

1. (1) $\frac{3}{8}$; $\frac{2}{5}$
   (2) $4\frac{1}{2}$; $\frac{2}{7}$
   (3) $\frac{1}{30}$; $\frac{17}{19}$
   (4) $\frac{9}{10}$; $1\frac{17}{35}$
   (5) 1; $4\frac{21}{25}$
   (6) $\frac{1}{2}$; $6\frac{3}{7}$
2. (1) $30\frac{2}{3}$; $\frac{3}{92}$
   (2) $\frac{1}{16}$; 8
   (3) $7\frac{8}{13}$; $\frac{13}{99}$
   (4) 42; $\frac{1}{42}$
   (5) $2\frac{13}{14}$; $15\frac{5}{11}$
   (6) 10; $4\frac{61}{154}$
3. (1) $\frac{36}{343}$
   (2) $\frac{527}{69972}$
   (3) $\frac{5}{946}$
   (4) 2
   (5) $1155\frac{5}{9}$
   (6) $25\frac{1}{5}$
4. $\frac{3}{5}$; $\frac{1}{8}$
5. $4\frac{23}{33}$
6. $\frac{1}{144}$
7. £$2\frac{3}{16}$
8. £4 10*s.*; $1\frac{9}{11}$*s.*; 1*s.*
9. 138
10. Equal
11. $10\frac{3}{4}$ miles
12. 45

Complex Fractions. Pages 223 to 225.

1. (1) $1\frac{3}{4}$; $\frac{3}{10}$
   (2) $\frac{30}{83}$; $\frac{6}{25}$
   (3) $1\frac{17}{22}$; $\frac{1}{11}$
   (4) 27; $\frac{27}{56}$
   (5) $\frac{13}{108}$; $\frac{6}{65}$
   (6) $4\frac{124}{1089}$; $\frac{1}{285}$
   (7) $\frac{3}{11}$; $\frac{128}{6345}$
   (8) $2\frac{1}{21}$; $1\frac{4}{13}$
   (9) $1\frac{61}{375}$
   (10) $\frac{1}{9}$
   (11) $\frac{62}{175}$
   (12) 0
   (13) $\frac{70}{459}$
   (14) $\frac{481}{1065}$
   (15) $1\frac{68}{81}$
   (16) $\frac{259}{960}$
   (17) $20\frac{39}{40}$
   (18) $2\frac{4091}{5520}$
   (19) $\frac{16}{55}$
   (20) $193\frac{3}{5}$
   (21) $2\frac{31}{196}$
   (22) $1\frac{1}{16}$
   (23) $1\frac{241}{403}$
   (24) $4\frac{1}{2}$
   (25) $\frac{115}{3136}$
   (26) $2\frac{2}{5}$
   (27) $2\frac{11}{32}$
   (28) $7\frac{93}{385}$
2. $1\frac{3}{5}$
3. $\frac{98}{143}$; $\frac{32}{143}$; $\frac{15}{143}$
4. $\frac{72}{77}$
5. $\frac{342}{399}$; $\frac{336}{399}$; $\frac{378}{399}$
6. £7000
7. £21
8. 1
9. £1 10*s.*
10. $\frac{12}{35}$
11. £89 8*s.* 6*d.*
12. A, £18,000;
    B, £12,000;
    C, £12,000
13. $\frac{3}{80}$
14. 7 : 2
15. 1575
16. £7350
17. Each = $1\frac{1}{16}$
18. $214\frac{2}{7}$ gals.

Reduction of Fractions. Pages 228 and 229.

(1) $\frac{1}{3}$; $\frac{2}{15}$
(2) $\frac{1}{6}$; $\frac{1}{6}$
(3) $\frac{1}{3}$; $1\frac{19}{32}$
(4) $\frac{4}{7}$; $1\frac{2}{5}$
(5) $\frac{17}{19}$
(6) $\frac{5}{8}$
(7) $\frac{144}{175}$
(8) $\frac{1}{48}$; $\frac{1}{16}$; $\frac{1}{96}$; $\frac{1}{28}$
(9) 4*s.*; $3\frac{9}{17}$; $3\frac{7}{11}$; $7\frac{9}{13}$
(10) 160; 45; 45
(11) 4; $9\frac{9}{23}$; 144; $6\frac{1}{4}$
(12) $\frac{1}{16}$; $\frac{7}{216}$; 4; $\frac{1}{66}$
(13) $\frac{7}{480}$; $\frac{1}{60}$; $\frac{5}{24}$
(14) $\frac{3}{156850}$; $\frac{7}{40}$; $\frac{3}{80}$
(15) $\frac{24}{365}$; $\frac{24}{730}$
(16) $\frac{3}{8960}$
(17) $2\frac{13}{25}$; $\frac{7}{15}$; $1\frac{2}{5}$
(18) $\frac{4}{5}$; $\frac{4}{15}$; $\frac{12}{25}$
(19) $\frac{1}{30}$; $\frac{1}{30}$; $\frac{4}{25}$; $\frac{2}{315}$
(20) $33\frac{363}{469}$
(21) $\frac{1}{504000}$; $\frac{32}{63}$
(22) $\frac{7}{100}$
(23) $\frac{181}{240}$; $\frac{91}{320}$
(24) $\frac{1}{220}$
(25) $\frac{73}{441}$
(26) $\frac{1}{168}$
(27) $\frac{1}{288}$
(28) $\frac{5}{21}$
(29) $\frac{279}{646}$
(30) $\frac{1}{15}$
(31) $\frac{1}{3}$
(32) $\frac{16}{27}$
(33) $\frac{3}{400}$
(34) $49\frac{1}{2}$
(35) $\frac{1}{168}$
(36) $18\frac{3}{8}$*s.*; $17\frac{7}{8}$*s.*; $18\frac{3}{4}$*s.*

Miscellaneous Exercises.—Fractions. Pages 233 to 236.

(1) $21\frac{19}{1260}$
(2) $2\frac{13}{35}$
(3) $\frac{27}{28}$
(4) $\frac{1}{24}$
(5) $\frac{13}{20}$
(6) $\frac{7}{75}$
(7) $4\frac{4}{15}$
(8) $\frac{67}{84}$
(9) $\frac{1}{2}$
(10) 2*s.* $9\frac{1}{3}$*d.*
(11) $\frac{179}{480}$
(12) 49 yds.
(13) $70\frac{1}{2}$
(14) 13 acs. 2 rds. $14\frac{6}{11}$ pers.
(15) 3 tons $4\frac{9}{10}$ cwts.
(16) $\frac{8}{45}$
(17) $23\frac{3}{5}$ hours
(18) $\frac{5}{32}$
(19) 3,600
(20) 50
(21) £1 2*s.* $3\frac{1}{5}$*d.*
(22) £1653 15*s.*
(23) $2\frac{44}{225}$
(24) $62\frac{51}{56}$
(25) £727 9*s.* 4*d.*
(26) $3\frac{229}{518}$
(27) $5\frac{1}{14}$
(28) 14
(29) $\frac{7}{80}$
(30) £6 5*s.* $6\frac{14}{75}$*d.*
(31) £285 14*s.* $3\frac{3}{7}$*d.*
(32) 21 miles
(33) $1\frac{101}{319}$ days
(34) £261 18*s.* 4*d.*

(35) $610\frac{10}{53}$
(36) $\frac{7}{100}$
(37) $\frac{327}{1792}$; £54,500
(38) $\frac{32}{153}$ days
(39) $\frac{47}{84}$
(40) 8 days
(41) 42
(42) $67\frac{1}{20}$; $105\frac{9}{20}$
(43) $11\frac{27}{28}$
(44) 60 days
(45) 16 days
(46) $3\frac{3}{5}$ days
(47) $2\frac{81}{82}$ days
(48) $21\frac{9}{11}$ min, past 4 o'clock
(49) $16\frac{4}{11}$ min. past 6 o'clock
(50) 6 o'clock

Decimal Notation. Page 239.

(1) $\frac{3}{10}$; $\frac{3}{100}$; $\frac{3}{1000}$; $\frac{3}{10000}$; $\frac{3}{100000}$
(2) $\frac{3}{10}$; $\frac{2}{100}$; $\frac{7}{1000}$; $\frac{5}{10000}$; $\frac{6}{100000}$
(3) $\frac{1}{20}$; $\frac{3}{500}$; $\frac{7}{10000}$
(4) ·01; ·005; ·3; ·0002
(5) $\frac{17}{100}$; $1\frac{7}{10}$; $\frac{17}{1000}$
(6) $\frac{1}{20}$; $\frac{1}{2}$; $\frac{1}{200}$; $\frac{1}{20}$; $\frac{1}{20}$
(7) $\frac{4}{100}$; $\frac{5}{1000}$; no change; $\frac{56}{10000}$; no change
(8) Divides by 10; multiplies by 10; no change; no change
(9) ·5; ·06; 10·62; 660·21; 2071
(10) ·005; ·0201; ·02401; ·01; ·001; ·0001; ·0101
(11) ·7; 210·5; 3001; 82; ·04; ·001
(12) ·007; 2·105; 30·01; ·82; ·0004; ·00001

Addition and Subtraction of Decimal Fractions. Page 241.

(1) 293·86
(2) 1192·5804
(3) 282·813
(4) 223·15
(5) 119·988
(6) ·1998
(7) 1·8388
(8) 22·694
(9) £48·06
(10) £14·005
(11) 4·5; 14·85; 14·949; 284·35; 7·195
(12) 182·817; 104·895; ·9; 12·9987
(13) 364·64
(14) ·697
(15) £2·7; 2·97*s.*; 14·85 yds.
(16) ·45, or $\frac{9}{20}$
(17) £·145
(18) £7·192; 4·95*s.*

Multiplication of Decimals. Pages 243 and 244.

(1) ·16; 35·4; 781; 2000
(2) 3500; 35; 3·51; 240
(3) 234; 17,500; 351; 270
(4) 10; 10; 20
(5) 2; ·54; 27·3; 2; 6·3; 280·8

(6) 1155·06; ·0552; ·2328; ·003472
(7) 30·066
(8) ·0060873
(9) 28·61134; 113·484; 1722
(10) 310·86; 255·9408; 881·144
(11) 48·240288; 3·9295354; ·20448
(12) 30·585016; ·00033608; 2500
(13) 5518·54
(14) ·3898759996
(15) 27·9152
(16) ·3762
(17) ·159984
(18) £164·45639
(19) 594·55s.
(20) £889

DIVISION OF DECIMALS. Pages 247 and 248.

(1) ·5324; ·02006; ·3017; ·00001
(2) ·0082; ·82; ·000082; ·00003
(3) ·0054; 7·32; ·0732
(4) ·00082; ·084; 3·2
(5) ·0031; ·0037; ·0009; ·143
(6) ·001; ·0001
(7) 11·66; 231·25; 58·35
(8) 1775; 596·153; 339·583
(9) 390·61728; 2501·03448; 38·591; 77·305
(10) ·06008; ·001; ·01; 8·0924
(11) ·0009547; ·000984; ·000994; ·811
(12) ·0098301; ·3001; ·001123; ·00166
(13) 10·5308
(14) ·03785
(15) 2·027
(16) £·01825
(17) £2·1522
(18) 27·114
(19) 38·152
(20) 9·25; ·25025; 1·77625; 2·029

CONVERTING VULGAR TO DECIMAL FRACTIONS. Page 251.

(1) $\frac{1}{2}$; $\frac{2}{5}$; $16\frac{1}{4}$; $\frac{17}{32}$
(2) 5·25; 6·4; 9; ·296875; ·024
(3) ·52; ·8125; $1\cdot8\dot{3}$; $\cdot41\dot{6}$; 1·2
(4) ·01; ·003; ·0005; $\cdot009\dot{3}$
(5) $\cdot\dot{0}\dot{9}$; $5\cdot\dot{7}8\dot{3}$
(6) 6·5; $\cdot\dot{9}2\dot{5}$

RECURRING DECIMALS. Pages 254 and 255.

1. (1) $\frac{1}{3}$; $\frac{35}{99}$; $\frac{16}{37}$; $\frac{2}{11}$; $\frac{23}{900}$
(2) $3\frac{7}{9}$; $\frac{26}{99}$; $12\frac{289}{990}$; $\frac{13}{90}$
(3) $14\frac{307}{999}$; $21\frac{7}{45}$; $74\frac{104}{999}$; $3\frac{1}{6}$
(4) $\frac{1}{45}$; $17\frac{5}{999}$; $\frac{6}{9999}$; $18\frac{5}{9}$
(5) $13\frac{13}{99}$; $13\frac{13}{999}$; $13\frac{13}{9999}$; $13\frac{13}{9900}$
(6) $14\frac{11}{90}$; $\frac{38}{45}$; $192\frac{137}{990}$; $\frac{23}{30}$
(7) $31\frac{1482}{4995}$; $25\frac{2477}{9990}$; $4\frac{8383}{9999}$

(8) $4\frac{3566}{4995}$; $2\frac{233}{2250}$; $7\frac{4069}{4995}$

(9) $2\frac{5}{999}$; $17\frac{103}{330}$; $2\frac{73}{74}$

(10) $\frac{53}{99001}$; $\frac{7}{3300}$; $2\frac{29}{3300}$

(11) $5\frac{18}{55}$; $\frac{6}{55}$; $3\frac{29}{198}$

(12) $7\frac{67}{495}$; $\frac{23}{495}$; $\frac{353}{495}$; $\frac{4}{15}$

2. $14{\cdot}\dot{8}\dot{2}$
3. $1{\cdot}872\dot{4}\dot{5}$
4. $8{\cdot}35\dot{6}1\dot{8}$
5. $9{\cdot}\dot{3}\dot{9}$
6. ·03395
7. $10{\cdot}8\dot{7}$; $12{\cdot}16\dot{8}\dot{6}$
8. 6.2432; $\cdot8\dot{5}1178\dot{4}$
9. ·7624; 1·488

(13) $315\frac{7}{66}$; $\frac{3977}{9900}$; $3\frac{8}{495}$; $\frac{83}{330}$

(14) $374\frac{23}{198}$; $\frac{67}{4995}$; $181\frac{8}{4995}$

(15) $53\frac{1}{666}$; $4\frac{206}{495}$; $\frac{13}{990}$

10. $\cdot135511\dot{3}\dot{6}$; 2·2872
11. 1.2933 2·0879
12. 1·7358; 17·43209
13. 1·7023; 6·8848
14. $18{\cdot}03\dot{1}\dot{8}$
15. 2·69
16. $4{\cdot}\dot{1}08$

Reduction of Decimals. Pages 257 and 258.

1. (1) 3*s.* 8·64*d.*; 7*s.* 4·8*d.*; £4 7*s.* 4·8*d.*
   (2) 6*s.* 5·7*d.*; 6*s.* 10·8*d.*; 5·76*d.*; 3*s.*
   (3) 9*s.* 8·88*d.*; 1*s.* 5·76*d.*; 16*s.* 4·56*d.*
   (4) 16*s.* 8·16*d.*; 3·8*d.*; 16*s.* 4·08*d.*
   (5) 7*s.* 5·04*d.*; 2 qrs. 8 lbs. 15·36 ozs.; 9 lbs. 10·56 ozs.
   (6) 2 ozs. 14·56 drs.; 7 cwts. 2 qrs. 4 lbs. 7·68 ozs.; 9 lbs. 13·28 ozs.
   (7) 4 yds. 2·01 ft.; 3 roods 32·48 poles
2. 6 cwts. 1 qr. 19·6 lbs.
3. £6 16*s.* 10·8*d.*
4. £1 5*s.* 0·48*d.*
5. 11*s.* 7·968*d.*
6. 19 cwts. 2 qrs. 17·92 lbs.
7. 2 ft. 11·388 ins.
8. 29,537·68
9. 2 roods 4·16 poles
10. £8 14*s.* 1½*d.*
11. 4*s.* 5·4*d.*
12. £4 13*s.* 11·52*d.*
13. 17*s.* 11·26*d.*
14. £3 4*s.* 0·38*d.*
15. ·192; $\cdot33658\dot{3}$; $\cdot188541\dot{6}$
16. $\cdot\dot{4}2857\dot{1}$; ·1825; ·01; ·695
17. ·23125; $\cdot138541\dot{6}$; $\cdot0008708\dot{3}$
18. £1·178125; £$2{\cdot}93958\dot{3}$; £$1{\cdot}673958\dot{3}$
19. ·0030532; ·86875; ·2175
20. 7*s.* 5·62*d.*
21. £$2{\cdot}5208\dot{3}$
22. £$\cdot31041\dot{6}$
23. ·28928
24. $\cdot\dot{1}$
25. ·8446
26. $\cdot\dot{1}4285\dot{7}$
27. $\cdot\dot{3}$; 2·31428

MISCELLANEOUS EXAMPLES. Pages 261 to 264.

(1) $\frac{1073}{1204}$
(2) 53550
(3) 11*s.*
(4) 39·1125
(5) 19*s.* 1$\frac{37}{84}$*d.*
(6) $\frac{11}{12}$
(7) £2 12*s.* 4·48*d.*
(8) 17*s.* 6·69*d.*
(9) $\cdot\dot{2}6984\dot{1}$
(10) ·6875
(11) 21$\frac{3}{7}$
(12) 51$\frac{17}{168}$
(13) $\frac{23}{180}$
(14) ·1
(15) 4$\frac{7}{8}$
(16) $\frac{9}{1600}$
(17) 272·25
(18) 37$\frac{1}{25}$; 21$\frac{7}{10000}$; $\frac{371}{100000}$; $\frac{1}{200}$
(19) 14$\frac{3}{8}$
(20) 4*s.*
(21) £1 13*s.* 0$\frac{9}{10}$*d.*
(22) ·2365
(23) 2 roods 24 per.
(24) 19*s.* 3·952*d.*
(25) 5 cwts. 1 qr. 17$\frac{3}{7}$ lbs.
(26) £1 18*s.* 7·05*d.*
(27) 13*s.* 10·1136*d.*
(28) ·830357
(29) 1·03*d.*
(30) ·175476
(31) £371 11*s.* 4$\frac{8}{17}$*d.*
(32) 22$\frac{10}{81}$ lbs.
(33) £1 0*s.* 8·8*d.*
(34) 13*s.* 1·464*d.*
(35) £15 12*s.* 7·768*d.*
(36) 18*s.* 9*d.*
(37) 63 years
(38) $\frac{2}{15}$
(39) 25$\frac{9}{35}$
(40) 3$\frac{1}{3}$
(41) 2$\frac{7}{45}$
(42) 3·947
(43) 83
(44) ·2125
(45) 17*s.*
(46) 168·93
(47) 235$\frac{5}{42}$
(48) ·15363; ·15113
(49) 4
(50) 1$\frac{47}{48}$

COMPOUND PROPORTION. Pages 269 to 272.

(1) 15
(2) 59$\frac{3}{7}$
(3) 133$\frac{1}{3}$ miles
(4) 51
(5) 30$\frac{3}{8}$
(6) 128
(7) 46$\frac{68}{77}$ ozs.
(8) £1 0*s.* 1$\frac{1}{2}$*d.*
(9) 2 days
(10) 7 lbs. 14 ozs.
(11) 51
(12) £29 4*s.* 5$\frac{5}{7}$*d.*
(13) 36
(14) £625
(15) £153 15*s.* 10$\frac{10}{17}$*d.*
(16) 4 cwts. 2 qrs. 14 lbs.
(17) 4$\frac{2}{7}$
(18) 4 weeks 4$\frac{1}{7}$ days
(19) £49 19*s.* 7$\frac{1}{5}$*d.*
(20) 33

(21) £25,300
(22) $10\frac{5}{6}$
(23) $26\frac{2}{3}$
(24) 10*s*. 6*d*.
(25) 6 days
(26) 189
(27) $1\frac{3}{25}$
(28) 6
(29) $1\frac{2}{3}$
(30) £50 5*s*. 4*d*.
(31) $15\frac{633}{1204}$ days
(32) $10\frac{1}{2}$ hours
(33) £19 12*s*. $8\frac{8}{11}$*d*.
(34) £8 10*s*. 8*d*.
(35) £10 16*s*. 10·7*d*.
(36) 560
(37) £31 10*s*. 0*d*.
(38) £2·47
(39) 24
(40) £51 11*s*. $9\frac{3}{5}$*d*.

EXAMINATION PAPERS, ST. VI. Pages 272 to 276.

I.

(1) 4
(2) $\frac{1}{45}$
(3) $\frac{2}{9}$
(4) £250

II.

(1) 168
(2) $4\frac{51}{56}$
(3) £10 13*s*. 4*d*.
(4) 6 years

III.

(1) 10*s*.
(2) $\frac{77}{2745}$
(3) $\frac{1}{112}$
(4) 5

IV.

(1) 129
(2) $1\frac{8}{11}$
(3) £1 15*s*. $7\frac{13}{18}$*d*.
(4) 600

V.

(1) $\frac{11}{90}$
(2) The latter by 11*s*. 8*d*.
(3) $\frac{5}{17}$
(4) £1 10*s*. 9*d*.

VI.

(1) $\frac{24}{7}$; $\frac{17}{5}$; $\frac{10}{3}$
(2) £3
(3) £1 2*s*. 0*d*.
(4) 10

VII.

(1) £15,000
(2) £11 7*s*. $4\frac{10}{63}$*d*.
(3) $3\frac{3}{5}$
(4) $4\frac{4}{17}$ days

VIII.

(1) $1\frac{5}{7}$
(2) $\frac{1}{6}$
(3) The second greater by £1 0*s.* $7\frac{1}{2}$*d.*
(4) £78 15*s.* 0*d.*

IX.

(1) $\frac{1}{10}$
(2) $\frac{6}{7}$
(3) 8
(4) £51 12*s.* 0*d.*

X.

(1) £1·485
(2) $12\frac{2}{3}$
(3) 3795 min.
(4) $56\frac{8}{11}$ days

XI.

(1) 2 m. 5 fur. 46 yds. 2 ft.
(2) $193\frac{1}{4}$
(3) £·135
(4) 11

XII.

(1) $\frac{3}{4}$
(2) $\cdot 597\dot{2}$
(3) The former by 3·9862
(4) A, 220; B, 176

XIII.

(1) $7\frac{3}{8}$
(2) 1*s.* 6·144*d.*
(3) $3\frac{3}{4}$
(4) £50

XIV.

(1) 47,014; 4701·4; 470·14
(2) $44\frac{4}{5}$
(3) $\frac{1}{48}$
(4) 3 days

XV.

(1) 1 acre $28\frac{7}{40}$ poles
(2) $25\frac{44}{81}$
(3) 399·514
(4) £26 12*s.* $10\frac{1}{2}$*d.*

XVI.

(1) $\cdot 0002\dot{6}$ mile
(2) $\frac{9}{64}$
(3) $2\frac{8071}{8325}$
(4) 9

XVII.

(1) 318,375·58
(2) $\frac{767}{1476}$
(3) 24
(4) $551\frac{1}{4}$ yds.

XVIII.

(1) ·069
(2) ·8
(3) ·03
(4) A, 5 days; B, 4 days

COMMISSION, BROKERAGE, ETC. Pages 282 to 284.

(1) £1 1*s*. 3*d*.
(2) £8 12*s*. $4\frac{1}{2}$*d*.
(3) £2 5*s*. 9*d*.
(4) £297 19*s*.
(5) £65
(6) 50
(7) $31\frac{1}{4}$
(8) 5*s*.
(9) 14*s*.
(10) 4*s*. $6\frac{6}{11}$*d*.
(11) 25
(12) £48 12*s*. 6*d*.
(13) $4\frac{1}{2}$.
(14) 3*s*. $4\frac{1}{2}$*d*.
(15) £97 2*s*. 6*d*.
(16) 9*s*. 9*d*.
(17) £10
(18) 25
(19) 50,000
(20) 20
(21) 28,400
(22) 1*s*. 8*d*.
(23) 450
(24) $66\frac{2}{3}$
(25) £79 18*s*. 7*d*.
(26) £352 16*s*.
(27) 12*s*.
(28) $6\frac{11}{19}$
(29) £749 3*s*. 4*d*.
(30) £600 11*s*. $5\frac{1}{7}$*d*.
(31) 2*s*. 11*d*.
(32) $12\frac{1}{2}$
(33) 29,913
(34) £993 4*s*. 6*d*.
(35) £2 6*s*. 8*d*.
(36) $7\frac{1}{2}$
(37) $24\frac{4}{9}$ loss
(38) £204 1*s*. 7·95*d*.
(39) 75
(40) $30\frac{5}{9}$
(41) 6*s*.
(42) 1*s*. 8*d*.
(43) $16\frac{2}{3}$ gain
(44) $24\frac{2}{5}$

SIMPLE INTEREST, DISCOUNT, ETC. Pages 289 to 291.

(1) £22 4*s*.
(2) £6000
(3) £444 8*s*. $10\frac{2}{3}$*d*.
(4) £82 17*s*. $1\frac{5}{7}$*d*.
(5) £36
(6) £6 3*s*. $3\frac{33}{73}$*d*.
(7) £39 16*s*.
(8) 25
(9) $289\frac{43}{63}$
(10) $57\frac{59}{63}$
(11) 6
(12) $5\frac{5}{7}$ years
(13) 3 years
(14) 2 years
(15) 12 years
(16) £48
(17) £700
(18) £500
(19) £80
(20) £391 17*s*. 11·57*d*.
(21) £6 6*s*. 0·328*d*.
(22) £6 4*s*. 5·52*d*.
(23) £148 15*s*. $6\frac{12}{23}$*d*.
(24) £148 12*s*. 1·63*d*.
(25) £357 8*s*. 3·56*d*.
(26) 16*s*. 3·24*d*.
(27) £4 7*s*. 7·68*d*.
(28) £394 8*s*. $8\frac{1}{2}$*d*.
(29) £143 15*s*.
(30) £10 18*s*. 2*d*.
(31) £5916 3*s*. 4*d*.
(32) 6
(33) £370
(34) 2*s*. 4·27*d*.
(35) $2\frac{1}{2}$
(36) £375
(37) $2\frac{1}{2}$
(38) £1820

(39) 7 years
(40) $3\frac{1}{4}$
(41) 3 years
(42) 5
(43) £1848
(44) $26\frac{2}{3}$ years

COMPOUND INTEREST. Pages 296 and 297.

(1) £63 18*s*. $10\frac{4}{5}$*d*.
(2) £899 17*s*. 9·8*d*.
(3) £312 3*s*. 2·4*d*.
(4) £893 0*s*. $7\frac{1}{2}$*d*.
(5) £394 1*s*. 3*d*.
(6) £129 4*s*. 9*d*.; £1679 4*s*. 9*d*.
(7) £421 6*s*. 3*d*.
(8) £1 10*s*. 7*d*.
(9) £56 9*s*. 2·7*d*.
(10) £762 8*s*. 7·2*d*.
(11) £174 18*s*. 5*d*.
(12) £9 15*s*. 10*d*.
(13) £378 17*s*. 0*d*.
(14) £1295 15*s*. 1*d*.
(15) £923 0*s*. 9*d*.
(16) £855 8*s*. 0*d*.
(17) £18 5*s*. 5*d*.
(18) £832 16*s*. 6*d*.
(19) £956 6*s*. 4*d*.
(20) £40 17*s*. 4*d*.; £391 7*s*. 4*d*.
(21) £4642 19*s*. 11*d*.
(22) £96 7*s*. 4*d*.; £101 17*s*. 2*d*.
(23) £430 9*s*. $1\frac{1}{2}$*d*.
(24) 2*s*. $5\frac{3}{4}$*d*.
(25) £44 6*s*. 5*d*.
(26) £12,077 11*s*. 7·5*d*. ;
£61,077 11*s*. 7*d*.
(27) £1 7*s*. 7·5*d*.
(28) £37 9*s*. 2*d*.

STOCKS AND SHARES. Pages 302 to 304.

(1) £6408 16*s*. $9\frac{22}{181}$*d*. ;
£4405 15*s*. $11\frac{7}{23}$*d*.
(2) £2105 5*s*. $3\frac{2}{57}$*d*.
(3) £1466 5*s*. 0*d*.
(4) £8212 10*s*. 0*d*.
(5) £6060 12*s*. $1\frac{5}{11}$*d*. ;
£181 16*s*. $4\frac{4}{11}$*d*.
(6) £103 $11\frac{1}{189}$*s*.
(7) £42 19*s*. $4\frac{376}{377}$*d*.
(8) £85 9*s*. $9\frac{38}{51}$*d*.
(9) £1387 10*s*.
(10) £4381
(11) £8 10*s*. $8\frac{232}{391}$*d*.
(12) £33 13*s*. $7\frac{133}{249}$*d*.
(13) £1705
(14) £28 10*s*. $7\frac{233}{361}$*d*. gain
(15) The former by £$\frac{3}{31}$ on the purchase of £100 stock.
(16) £2435 2*s*. $5\frac{11}{41}$*d*.
(17) £$98\frac{26}{63}$
(18) £75
(19) £$66\frac{2}{3}$
(20) £9554 17*s*. $11\frac{256}{357}$*d*. ;
£191 1*s*. $11\frac{169}{337}$*d*.
(21) £29,160
(22) The latter by £141 17*s*. 4*d*.
(23) £1430
(24) £5844 14*s*. $1\frac{7}{17}$*d*.
(25) £4800 ; £3648
(26) £1462
(27) £8750 ; £7910

(28) £212 10*s.*
(29) $4\frac{136}{191}$; £22 10*s.*
(30) $4\frac{49}{69}$
(31) $4\frac{36}{41}$
(32) 80
(33) £33,333 6*s.* 8*d.*; £31,000
(34) £77 10*s.*
(35) £1 8*s.* $6\frac{6}{7}$*d.*
(36) £17,135
(37) £35

Miscellaneous Exercises. Pages 313 to 317.

(1) £8 15*s.* $11\frac{2}{5}$*d.*
(2) £109 15*s.* $6\frac{3}{5}$*d.*
(3) 8*s.* 4*d.*
(4) £1023 8*s.* 9*d.*
(5) £99 6*s.* $6\frac{6}{23}$*d.*;
£132 8*s.* $8\frac{8}{23}$*d.*;
£148 19*s.* $9\frac{9}{23}$*d.*
(6) A £192 7*s.* 6*d.*; B £225;
C £113 1*s.* 3*d.*
(7) £185 14*s.* 9*d.*
(8) £21 7*s.* $8\frac{3}{4}$*d.*
(9) £584 6*s.* 8*d.*
(10) 1*s.* 1·8*d.*
(11) £156 1*s.* 7·2*d.*
(12) 10*d.*
(13) £4409 19*s.* $10\frac{1}{18}$*d.*
(14) $283\frac{1}{2}$
(15) £150
(16) £3 2*s.* $2\frac{2}{3}$*d.*
(17) A £21 16*s.* $4\frac{4}{11}$*d.*;
B £26 3*s.* $7\frac{7}{12}$*d.*
(18) £89 5*s.* $8\frac{4}{7}$*d.*
(19) £18 14*s.* 7*d.*
(20) $20\frac{571}{1210}$
(21) 3*s.* $1\frac{1}{2}$*d.*
(22) £26 3*s.* 7*d.*
(23) £547 7*s.* $3\frac{3}{5}$*d.*
(24) 10 years
(25) 24·6
(26) 170 ft. 48 in.
(27) £588
(28) $113\frac{1}{3}$
(29) £3 0*s.* $1\frac{112}{365}$*d.*
(30) £153 18*s.* $2\frac{22}{27}$*d.*
(31) £18 18*s.*
(32) 73 days
(33) £37 16*s.*; £27; £21
(34) $3\frac{3}{4}$*d.*
(35) £11 5*s.*; £20; £29 5*s.*
(36) 1*s.* $1\frac{8}{25}$*d.*
(37) $4\frac{3}{4}$ years
(38) £17 3*s.* 4*d.*
(39) £1063 16*s.* $7\frac{7}{47}$*d.*
(40) £1251 18*s.* $2\frac{2}{181}$*d.*
(41) $2\frac{1}{2}$
(42) £1015 4*s.* 6*d.*
(43) £800
(44) £121 0*s.* $6\frac{78}{107}$*d.*;
£248 19*s.* $5\frac{29}{107}$*d.*
(45) £7 15*s.* $6\frac{2}{3}$*d.*
(46) £207 18*s.*
(47) £19
(48) £1 3*s.* 6*d.*; $14\frac{42}{47}$;
£1 5*s.* $10\frac{1}{6}$*d.*
(49) 11·52
(50) £142 12*s.* 2·6*d.*
(51) 3 qrs. $3\frac{63}{120}$ lbs.;
12 cwts. 2 qrs. $11\frac{67}{120}$ lbs.
(52) £347 $12\frac{8}{121}$*s.*
(53) £112 10*s.*
(54) 6*s.* $10\frac{40}{41}$*d.*
(55) £4166 13*s.* 4*d.*

(56) $132\frac{5}{6}$ yards

(57) £69 2*s*. $5\frac{187}{217}$*d*.

(58) £2895

(59) 11 ft. 6 ins. and 1 ft. 3 ins.

(60) 16*s*. $6\frac{9}{10}$*d*.

(61) £88 9*s*. $2\frac{170}{821}$*d*.

(62) £1313 19*s*. $7\frac{205}{329}$*d*.

(63) The latter by £1 12*s*. $4\frac{1}{2}$*d*.

(64) £1660

(65) $90\frac{5}{68}$

(66) $3\frac{487}{771}$

(67) 4*s*.; £10 2*s*. 8*d*.

(68) £7350

(69) 1*s*. 3*d*.

(70) £3600

(71) £2 13*s*. 4*d*.

(72) £663 6*s*. 9*d*.

(73) 1*s*. $2\frac{2}{3}$*d*.

(74) £97 15*s*. $6\frac{2}{3}$*d*.

(75) £6 5*s*. more

(76) $8\frac{3}{4}$ hours

(77) $10\frac{10}{11}$

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